Financial Calculations on the Texas Instruments BAII Plus

This is a first draft, and may contain errors. Feedback is appreciated.

Compounding Assumptions

- The TI BAII Plus has built-in preset assumptions about compounding and payment frequencies.
- Compounding and Payment frequencies are controlled with the [P/Y] key.

Compounding Assumptions

- Press the [P/Y] key ([2nd][I/Y])
- Unless the settings have been changed, you will see the default, preset payment frequency: \( P/Y = 12.00 \) - 12 payments/year
- Using the down arrow [*] or up arrow [v] will scroll you to the next window, the number of times per year the interest is compounded: \( C/Y = 12.00 \) - 12 times/year

An Alternative

- One way to make the BAII Plus work very much like the Sharp EL-733A is to set the [P/Y] and [C/Y] to 1 and leave it there all the time.
- If you do this, some of the directions that follow will not work if the values of [P/Y] and [C/Y] are changed.

Clearing

- It is also very important to clear the Time Value worksheet before doing a new set of calculations.
- [CLR][TVM]
A Word on Rounding

- I set my BA II Plus to an artificially large number of decimals - usually 7 - which will rarely all be displayed.
- The BA II Plus will display the answer rounded correctly to the number of decimals available or as set by you, whichever is less.
- In these notes, $1/7 = 0.142857...$ may be written as $0.1428…$, where the “…” simply means that I have stopped writing down the decimals, but I have not rounded.

Future Values

$$FV_0 = PV_0(1 + k)^n$$

$$= 44,651.06(1.06)^5$$

$$= 44,651.06(1.33822...)$$

$$= 59,753.19$$

On the TI BAII Plus

- 44651.06 [PV]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -59,753.19

- To get the $FV_{k,n}$, simply use $PV = 1$
- 1 [PV]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -1.338225...

Present Values

- A contract that promised to pay you $59,753.19 in 5 years would be worth today, at 6% interest:

$$PV_0 = (FV_5) \cdot PVIF_{6\%, 5}$$

$$= 59,753.19(1.06)^{-5}$$

$$= 59,753.19(0.74725...)$$

$$= 44,651.06$$

On the TI BAII Plus

- 59753.19 [FV]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -44,651.06

- To get the $PV_{k,n}$, simply use $FV = 1$
- 1 [FV]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -0.747258...

Perpetuities

- Perpetuities, growing perpetuities and growing finite annuities must be done using the formulae as financial calculators do not have special functions for these cash flows
PV of Annuity Example

\[
PV_0 = \frac{10,600 \left(1 - (1 + k)^{-n}\right)}{k} = \frac{10,600 \left(1 - (1.06)^{-5}\right)}{0.06} = 10,600(4.212363...) = 44,651.06
\]

On the TI BAII Plus

- \[10,600 \text{ [PMT]}; 6 \text{ [I/Y]}; 5 \text{ [N]} \]
- \[\text{[CPT][PV]} \text{ Display = -44,651.06}\]
- To get the \(PVA_{k,n}\), simply use \(\text{PMT} = 1\)
- \[1 \text{ [PMT]}; 6 \text{ [I/Y]}; 5 \text{ [N]} \]
- \[\text{[CPT][PV]} \text{ Display = -4.21236...}\]

FV of Annuity Example

\[
FV_5 = \frac{10,600 \left((1 + k)^n - 1\right)}{k} = \frac{10,600 \left((1.06)^5 - 1\right)}{0.06} = 10,600(5.637092...) = 59,753.19
\]

On the TI BAII Plus

- \[10,600 \text{ [PMT]}; 6 \text{ [I/Y]}; 5 \text{ [N]} \]
- \[\text{[CPT][FV]} \text{ Display = -59,753.19}\]
- To get the \(FVA_{k,n}\), simply use \(\text{PMT} = 1\)
- \[1 \text{ [PMT]}; 6 \text{ [I/Y]}; 5 \text{ [N]} \]
- \[\text{[CPT][FV]} \text{ Display = -5.63709...}\]

Annuities Due

- To access the toggle that switches the annuity payments between regular (END) and due (BGN) you use the [BGN] key ([2nd][PMT])
- To toggle between the BGN and END setting, use [SET] ([2nd][ENTER]) and [QUIT] to return to the calculator mode
- If set for annuities due, you will see BGN in the display

PV of an Annuity Due

\[
PV_0 = \frac{10,000 \left(1 - (1 + k)^{-n}\right)(1 + k)}{k} = \frac{10,000 \left(1 - (1.06)^{-5}\right)(1.06)}{0.06} = 10,000(4.4651056...) = 44,651.06
\]
On the TI BAII Plus

- [BGN][SET] to set to BGN
- 10,000 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][PV] Display = -44,651.06

- To get the PVA_{k,n}, simply use PMT = 1
  - 1 [PMT]; 6 [I/Y]; 5 [N]
  - [CPT][PV] Display = -4.4651056...

FV of an Annuity Due

\[ FVA_{k,n}^{(Due)} = \text{PMT} \left(\frac{(1+k)^n - 1}{k}\right)(1+k) \]

On the TI BAII Plus

- [BGN][SET] to set to BGN
- 10,000 [PMT]; 6 [I/Y]; 5 [N]
- [CPT][FV] Display = -59,753.19

- To get the FVA_{k,n}, simply use PMT = 1
  - 1 [PMT]; 6 [I/Y]; 5 [N]
  - [CPT][FV] Display = -5.9753185...

Example, Uneven Cash Flows

- Valued at 6%

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>20,000</td>
<td>$18,667.92</td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
<td>$13,349.95</td>
</tr>
<tr>
<td>3</td>
<td>25,000</td>
<td>$20,990.48</td>
</tr>
<tr>
<td>4</td>
<td>30,000</td>
<td>$23,762.81</td>
</tr>
<tr>
<td>5</td>
<td>10,000</td>
<td>$7,472.58</td>
</tr>
</tbody>
</table>

Example, Uneven Cash Flows

- After the cash flows are entered, we use the [NPV] key
  - The first display is I = and is asking us to enter the interest or discount rate.
  - After entering the rate the [v] gives us the NPV = display. [CPT] will give us the net present value of the cash flows.

On the TI BAII Plus

- We use the [CF] key,
  - Initially, we see the Display: Cf0 = 0.00
  - The down arrow [v] and up arrow [u] allow us to scroll through the displays
  - Each Cnn is followed by Fnn to allow the user to enter multiple occurrences of a value
Example on the TI BAII Plus

- CF0 = 0.00 [v]
- C01 = 20000 [ENTER] [v] F1 = 1 [v]
- C02 = 20000 [ENTER] [v] F2 = 1 [v]
- C03 = 20000 [ENTER] [v] F3 = 1 [v]
- C04 = 20000 [ENTER] [v] F4 = 1 [v]
- C05 = 20000 [ENTER] [v] F5 = 1 [v]
- [NPV] Display: I = 6 [ENTER] [v]
- Display: NPV = [CPT]
- Display: NPV = 84,443.74

Look for hidden annuities

- Sometimes there will be annuities to simplify your calculations that are not so obvious

Example on the TI BAII Plus

- CF0 = 0.00 [v]
- C01 = 15000 [ENTER] [v] F1 = 2 [ENTER] [v]
- C02 = 20000 [ENTER] [v] F2 = 3 [ENTER] [v]
- [NPV] Display: I = 6 [ENTER] [v]
- Display: NPV = [CPT]
- Display: NPV = 75,080.31

Example

- Suppose that Consolidated Moose Pasture (CMP) borrowed $466,500 and promised to repay $1,000,000 eight years from now. There will be no intermediate interest payments. What is the implied rate of interest?

- 466500 [PV]; 1000000 [+/-] [FV]; 8 [N]
- [CPT][I/Y] Display = 10.00

Example - Annuities

- Suppose you have the choice to receive $100,000 now or $15,000 per year at the start of each of the next 10 years.

  - [BGN][SET] (to toggle to BGN)
  - 15000 [PMT]; 100000 [+/-][PV], 10 [N]
  - [CPT][I/Y] Display: 10.409
Converting from APR to EAR

- Consider $1 for 1 year 6% compounded
  - quarterly: 1.5% every quarter for 4 quarters
  - monthly: 0.5% every month for 12 months
  - daily: (6/365)% every day for 365 days

Effective Annual Rate

Quarterly FV = $1 \times (1.015)^4 = $1.06136
EAR = 6.136%

Monthly FV = $1 \times (1.005)^{12} = $1.061678
EAR = 6.1678%

Daily FV = $1 \times (1 + (6/365))^{365} = $1.061831...
EAR = 6.1831%

On the TI BAII Plus

To convert from a nominal (APR) to EAR
- You can do it by using the formulaic approach from the previous slide, or
- You can use the [ICONV] worksheet (above the numeral [2])
  - The first screen is NOM =
  - The second screen is EFF =
  - The third screen is C/Y =

On the TI BAII Plus

Using the [ICONV] worksheet
- NOM = 6 [ENTER], [>] C/Y = 4 [ENTER], [>] EFF = [CPT] Display = 6.136355...
- NOM = 6 [ENTER], [>] C/Y = 12 [ENTER], [>] EFF = [CPT] Display = 6.167781...
- NOM = 6 [ENTER], [>] C/Y = 365 [ENTER], [>] EFF = [CPT] Display = 6.183131...

Converting from EAR to APR

The account earns an EAR of 6%
- If the account compounds interest quarterly, what is the APR?
- If the account compounds interest monthly, what is the APR?
- If the account compounds interest daily, what is the APR?

Example

\[ q = \left(1 + \frac{\text{EAR}}{m}\right)^{m} - 1 \]

Quartely: \[ q = \left(1.06\right)^{\frac{1}{4}} - 1 \]
\[ = 5.8695\% \]

Monthly: \[ q = \left(1.06\right)^{\frac{1}{12}} - 1 \]
\[ = 5.841\% \]

Daily: \[ q = \left(1.06\right)^{\frac{1}{365}} - 1 \]
\[ = 5.827\% \]
On the TI BAII Plus

To convert from EAR to APR

- You can do it by using the formulaic approach from the previous slide, or
- You can use the same [ICONV] worksheet with the nominal being the value to be computed

Example

- Your older sister just had a baby. If she opens an RESP and puts $125/month into it for 18 years, how much will be available for the child if the rate of return is 8% per annum, 2/3% per month?
  - [P/Y] = 12 [ENTER] [QUIT]
  - 8 [I/Y]; 125 [PMT]; 216 [N] (or 18[2nd][N])
  - [CPT][FV] Display: -60,010.77

Continuous Compounding

- With continuous compounding, you must solve using the formula and the [ex] key (or [2nd][ln])
- Suppose you want to have $1,000,000 in your retirement account when you reach 65. 44 years from now. If a financial institution is offering you 7% compounded continuously, how much would you have to deposit now, while you’re 21?
  - 0.07 [x] 44[+/-][=] Display: -3.08[ex] Display: 0.045959… [x] 1000000 [=] Display: 45,959.26

Mortgage Example

- $120,000 principal (=PV)
- 25 year amortization (n=300 months)
- 8% five year term
  - EAR=8.16%
  - APR=7.87%
  - monthly=0.655819…%

Solution

\[
PV = C(PV_{a_{r_{8.16\%},n=300}}) \\
120,000 = C(PV_{a_{0.655819\%,300}}) \\
C = \frac{120,000}{PVA_{0.655819\%,300}} \\
= \frac{120,000}{131.024343...} = $915.86
\]
### On the TI BAII Plus

1. Enter the payment frequency, \([P/Y]\) 12 [ENTER] and compounding frequency, \([C/Y]\) 2 [ENTER]
2. Enter the mortgage parameters: The principal: 120000 [PV], nominal rate 8 [I/Y] and amortization term 300 [N]
3. Compute the payment [CPT][PMT]
   
   **Display:** PMT = -915.86

**MORE TO COME, DO NOT CLEAR**

### Renewal Balance

- The principal of a mortgage is always the PV of the payments that remain on the amortization
- After 5 years:

  \[
  \text{BAL}_{60} = \text{PMT} \times (PVA_{10\%,60}) = 915.86(120.720826...) = 110,563.38
  \]

### Other Questions

<table>
<thead>
<tr>
<th>Principal</th>
<th>$120,000.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Renewal</td>
<td>110,563.38</td>
</tr>
<tr>
<td>Principal Paid</td>
<td>9,436.62</td>
</tr>
<tr>
<td>Interest Paid</td>
<td>45,514.98</td>
</tr>
<tr>
<td>Total Paid</td>
<td>54,951.60</td>
</tr>
</tbody>
</table>

### On the TI BAII Plus

- The “AMORT” key gives us access to the amortization worksheet
- Once you have accessed the AMORT worksheet, the display should say P1 = 1
  - This is the first payment in the range
- Pressing the down arrow will give you P2 = something and you can specify the last payment in the range
- If you want to see each payment sequentially, use P1 = 1, P2 = 1; then P1 = 2, P2 = 2; and so on.

### On the TI BAII Plus

- We can jump to any payment
- This is one of the situations where the calculator takes its time - and appears to die - to do the calculation
On the TI BAII Plus

- Sixtieth Payment (just before the five-year renewal)
  P1 = 60 [ENTER], [v]
  P2 = 60 [ENTER], [v]
  BAL = 110,562.91, [v]
  PRN = -189.52,[v]
  INT = -726.34

Accumulated Values

- The [AMORT] worksheet will also allow us to determine how much principal and interest has been paid over a range of periods by specifying ranges for P1 = and P2 =

On the TI BAII Plus

- Over the first five years:
  P1 = 1 [ENTER], [v]
  P2 = 60 [ENTER], [v]
  BAL = 110,562.91, [v]
  PRN = -9,437.09,[v]
  INT = -45,514.28

Car Buying or Leasing

- Suppose you have decided on a new Bolero from National Motors. Its total cost before sales taxes (15%) is $23,500. You plan to put $3,500 down regardless whether you lease or buy. The buyback at the end of the 48 month lease is $9,000. The dealer is offering 4.8% APR financing and lease rates, both compounded monthly.

Buying - Loan

- $23,500 + 15%(v23,500) - 3,500 = $23,525
- 4.8% APR, r_{MON} = 0.4%
- Assume a 48 month loan
- PVIFA_{0.4,48} = 43,9542...

\[ \text{PMT} = \frac{\text{Principal}}{PVIFA_{r_{MON},N}} = \frac{23,525}{43.5942...} = \$539.64 \]
Lease

- On the lease, the sales tax does not get financed, but the payments are subject to sales taxes
- The present value of the lease payments, plus the present value of the buyback on the car must equal the cash price of the car
- Lease payments are made in advance, or at the beginning of each month

$20,000 = PMT\left(\text{PVIFA}_{0.4\%,48}\right) + \frac{9,000}{(1.004)^{48}}$

\[= PMT(43.7686\ldots) + \frac{9,000}{1.2112\ldots} = PMT(43.7686\ldots) + 7,430.61 = 12,569.39 = PMT(43.7686\ldots)\]

\[\text{PMT} = \frac{12,569.39}{43.7686\ldots} = 287.18\]

On the TI BAII Plus

- The [P/Y] and [C/Y] should both be set to 12 - this is what you get when you [CLR WORK] the [P/Y] key
- You also need to set the calculator to BGN: [BGN][SET]; BGN will display
- 48 [N], 20000 [PV], 4.8 [I/Y], -9000 [FV] [CPT] [PMT] Display: -287.178...

Bonds

- With Bonds, you can approach them two ways:
  - Using the Time Value functions, OR
  - Using the [BOND] worksheet

Regular Fixed Coupon Bond

\[PV = B_0 = P\left(\text{PVF}_{b,n}\right) + \frac{M}{(1 + k_b)^n}\]

Consider a 9%, 12 yr bond @7%

\[B_0 = 45(16.058\ldots) + \frac{1000}{(1.035)^{24}}\]

\[B_0 = 722.627 + 437.957 = 1160.58\]

Using the Time Value Functions

- \([P/Y] = 2 \text{ [ENTER]}\)
- \([N] = 24; [FV] = 1000; [PMT] = 45; [I/Y] = 7\)
- [CPT][PV] = -1,160.58
Using the [BOND] Worksheet

- The [BOND] worksheet requires starting and maturity dates
  - If none are given, make them up
  - Dates are entered in the form mm.ddyy
  - Jan 1, 2001 would be 1.0101
- The maturity value [RV] defaults to 100 and is fine if you keep the coupon as a percent
- You will also get a price that is a percent of par

Using the Default Value of RV = 100

- SDT = 1.0101 [ENTER] [v]
- CPN = 9 [ENTER] [v]
- RTD = 1.0113 [ENTER] [v] (Maturity Date)
- RV = 100 [ENTER] [v] (Maturity Value)
- ACT [v] (Actual day count)
- 2/Y [v] (2 coupons/yr)
- YLD = 7 [ENTER] [v]
- PRI = [CPT] Display: 116.058...

Using the [BOND] Worksheet

Using the Value of RV = 1000

- SDT = 1.0101 [ENTER] [v]
- CPN = 90 [ENTER] [v]
- RTD = 1.0113 [ENTER] [v] (Maturity Date)
- RV = 1000 [ENTER] [v] (Maturity Value)
- ACT [v] (Actual day count)
- 2/Y [v] (2 coupons/yr)
- YLD = 7 [ENTER] [v]
- PRI = [CPT] Display: 1,160.58...