

Security Design in Initial Public Offerings*

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Abstract

We investigate the security design problem in an initial public offering (IPO). In line with Rock (1986), we consider a situation in which some investors are better informed than others about the prospects of the firm, resulting in a winner's curse problem. To raise capital, the owners of the firm must underprice the securities they issue in order to compensate the less informed investors for their willingness to participate in the issue. In this context, we first show that firms can sometimes lower the cost of going public by using unit IPOs, in which equity and warrants are combined into a non-divisible package. Because warrants are less sensitive to low cash flow realizations, unit IPOs tend to be valuable to firms that face large downside risks or whose uncertainty revolves around the eventual performance of their assets in place. Second, we show that firms may be able to completely eliminate the winner's curse problem by making the warrants callable. Such a first-best scenario is possible when a firm's growth potential is sizeable even in bad states of the world, as the callability feature of the warrant allows the firm to dynamically create payoffs that are insensitive to the investors' private information. Our theory is consistent with the prominent use of unit IPOs and produces empirical implications that differentiate it from existing theories.

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Abstract

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1 Introduction

The winner's curse problem in initial public offerings (IPOs) is well-known. As described by Rock (1986), the potential presence of investors who are better informed than others implies that lesser-informed investors tend to end up with a larger (smaller) allocation of shares when the issue is overpriced (underpriced). Anticipating this possibility, uninformed investors require a lower price to subscribe to the issue in the first place. This discount and associated reduction in proceeds is costly to the firm, as empirically documented by Koh and Walter (1989), Keloharju (1993), Michaely and Shaw (1994), and numerous authors following them.¹

In this paper we study how optimally designed securities can limit, and possibly even eliminate, the amount of money that new issuers must leave on the table to ensure the success of their offering. Our solution comes in the form of package offerings consisting of an appropriate mix of equity and warrants that cannot be sold separately to new investors. These packages are similar to the unit offerings which, according to Schultz (1993) and Jain (1994), are used in 15-20% of IPOs in the United States.²

In our model, a fraction of outside investors hold some information about the eventual distribution of the firm's future value. This information can be related to various aspects of value. For example, investors may have better information about the profitability of the firm's future investment (i.e., the firm's growth potential), the future performance of the firm's existing assets, or the firm's liquidation values in the event that the firm goes bankrupt. As such, uninformed investors face a winner's curse problem with respect to the firm's upside potential, its downside risk, or both.

In this context, our paper establishes two sets of results. The first compares the sensitivity of cash flows from a warrant to the investors' private information, relative to that of equity. Because warrants pay off only when the firm realizes some of its growth potential, the value of these securities is less sensitive than that of equity for low realizations of firm value. Consequently, when the informed investors' information relates mainly to the firm's downside risk and/or the eventual performance of its assets in place, uninformed investors are less at an informational disadvantage when warrants are included in the IPO. This reduces both the negative effect of the winner's curse and the firm's cost of going public. In contrast, when the informational advantage of informed

¹See Ritter and Welch (2002) for an excellent survey of the literature on IPO underpricing and IPOs in general.

²The use of warrants in IPOs is not unique to firms in the United States. For example, the use of warrants in IPOs is also prevalent in Israel (Hauser and Levy, 1996; Kandel, Sarig and Wohl, 1999) and Australia (How and Howe, 2001; Lee, Lee and Taylor, 2003).

investors is about the upside potential of the firm, a standard equity-only IPO is better.

In either case, the reduction of the winner's curse is not perfect. Indeed, like the payoff of equity-only IPOs, the payoff of unit IPOs is strictly increasing in the firm's eventual value, and so the winner's curse problem can never be fully eliminated by packaging standard warrants with equity. The paper's second main result shows that this can be fixed by allowing warrants that are callable by the firm before they expire. Since any initial difference of information about the firm's future is likely to be revealed in the secondary market via trading and/or via observing the performance of the firm over time, call provisions that allow the firm to force early exercise enable it to dynamically create securities whose payoffs are not as valuable in high states of the world. More precisely, by choosing an appropriately large trigger value for calling the warrants, the firm commits to force their early conversion only when the firm's (interim) market value is consistent with good information initially held by informed investors. By setting the warrants' strike price so that the payoff from this early exercise of the option is low, the firm is able to extinguish a valuable option when this happens. On the other hand, when the interim market value of the firm is not sufficiently high, the firm cannot force exercise and investors do not exercise early. Since the probability of a valuable option being extinguished early is positively correlated with the initial information of informed investors, such restrictive call provisions effectively provide insurance from the winner's curse to uninformed investors at the time of the IPO. In fact, if the firm's potential profitability in the bad states of the world, even though lower than in the good states, is sufficiently large, such restrictive call provisions allow the firm to dynamically create a security whose eventual payoff is insensitive to the private information initially held by informed investors. Such a security completely eliminates the winner's curse problem and yields the first-best outcome.

Parlour and Rajan (2005) also study an IPO problem in which investors are differentially informed. They show that the firm's commitment to rationing before the IPO alleviates the winner's curse problem and allows the firm to generate more revenues from the issue. On the one hand, rationing is costly in that the highest bidder does not get the full equity allocation; on the other hand, rationing benefits the firm by reducing the investors' concern about the winner's curse and by making them more aggressive in their bidding on average. Their work builds onto the bookbuilding approach of Benveniste and Spindt (1989), under which the firm must underprice and preferentially allocate shares to informed investors in order to elicit truthful information revelation.³ In

³This bookbuilding approach has been empirically investigated by Hanley and Wilhelm (1995), and Cornelli and Goldreich (2001).

both these papers, the focus is on the mechanism by which the firm allocates shares and arrives at an issue price. That is, these papers minimize the impact of information asymmetries by designing allocation mechanisms that preclude informed investors from excessively gouging uninformed investors. Our approach is complementary, in that we also look to minimize the costs associated with the winner's curse, but focus more on the securities being sold rather than the mechanism by which they are sold. More specifically, our model keeps the allocation mechanism fixed and instead concentrates on the design of securities whose sensitivity to information asymmetries across investors is minimized.

To our knowledge, only two theories have been advanced for the use of warrants in IPOs. Schultz (1993) argues that unit IPOs serve the same purpose as staged equity financing in venture capital (e.g., Sahlman, 1990). The idea is that, by providing the firm with a series of capital infusions, unit offerings reduce the agency costs associated with free cash flows (e.g., Jensen, 1986). Chemmanur and Fulghieri (1997) argue that the inclusion of warrants in IPOs helps risk-averse insiders signal the high quality of their firm when outcomes are risky. The idea is that risk-averse entrepreneurs wishing to diversify their position value the risky high-state payoffs less than investors and so warrants that pay only in those states provide a cheaper way to signal firm quality than underpricing (e.g., Grinblatt and Hwang, 1989) or equity retention (e.g., Leland and Pyle, 1977).

Like these other two theories, our model predicts that young, small and risky firms will tend to include warrants in their IPOs. Our prediction about risk, however, is a bit sharper than that of these other theories, as it is downside risk and asymmetric information about that risk that prompts firms to include warrants in their IPOs. For the same total firm risk, only firms whose assets in place are riskier will tend to use unit IPOs. For example, firms with intangible assets and firms that are heavily invested in research and development (R&D) are predicted to include warrants in their security offering when they first go public. Of course, for the downside risk to matter and create a potential winner's curse problem, this risk has to be significant. This is why our model also predicts that firms using unit IPOs will tend to subsequently go bankrupt more often and more quickly than firms that go the equity-only route.

In our model, the presence of existing senior debt affects the relative importance of upside potential versus downside risk for the new claimants at the time of the IPO. This implies that the benefits of including warrants in the IPO will be affected by the firm's financing prior to the IPO. In particular, low levels of unsecured debt are unlikely to affect the balance between downside risk and upside risk, but high levels of debt mean that existing debtholders already absorb a large

portion of the downside risk. As a result, we predict that unit IPOs are more likely to be used by firms with low levels of existing debt.

Our model yields additional testable implications regarding the characteristics of the warrants that firms will include in their IPOs. First, our optimality result about the callability of warrants is new to the IPO literature, as neither Schultz (1993) or Chemmanur and Fulghieri (1997) consider the possibility that the packaged warrants are callable. It is also consistent with Schultz's (1993) finding that almost 90% of warrants in unit IPOs are callable. Second, we show that the inclusion of callable warrants makes unit offerings optimal when the option value of the firm is significant even after negative shocks. In fact, because warrant options can be quite valuable to investors in bad times, firms will tend to include a higher proportion of warrants (relative to shares of equity) in their IPO packages in an effort to balance the value of these packages in subsequent good and bad states of the world. Finally, our model shows that the warrants that are most efficient in reducing the winner's curse problem are characterized by low call prices and high exercise prices (i.e., they will be out of the money at issue).

Our analysis of unit IPOs in terms of information sensitivity of different securities is related to the large literature on security design under asymmetric information.⁴ For instance, Nachman and Noe (1994) and DeMarzo and Duffie (1999) study the adverse selection problem of a firm that has some information that outside investors do not have (Myers and Majluf, 1984). This is distinct from the winner's curse problem of a firm facing imperfect competition among differentially informed outside investors, as we study in this paper. Nevertheless, the conditions that we identify, under which a package of junior claims (like warrants) and senior claims (like equity) has lower information sensitivity relative to equity only, are similar to those under which the usual pecking order will be reversed in adverse selection environments. For instance, under the same conditions, a package of debt and equity (or warrants) will lead to less dilution than issuing only debt in the framework of Myers and Majluf (1984).

Our results about the inclusion of securities with path-dependent contingencies in IPOs share some similarities with Chakraborty and Yilmaz's (2006) results about the optimality of financing projects with convertible securities. Our results also share some conceptual similarities with those of Brennan and Kraus (1987) and Constantinides and Grundy (1989) who show that the firm's commitment to buy back debt or equity allows it to credibly signal and finance valuable projects.

⁴While mechanism design problems with correlated signals may in general allow (almost) costless information extraction (e.g., Crémer and McLean, 1988), the restrictions on the set of allowable contracts (e.g., limited liability constraints) do not allow this in our setting.

An important difference between their work and ours is that these authors assume that the security that is bought back by the firm is exogenous and has been previously issued. We also focus on a different informational problem in capital markets: the winner’s curse as opposed to adverse selection. Consequently, security design has no signaling role in our setting. Finally, Axelson (2007) studies the tradeoff between issuing debt and issuing equity in an auction setting, and relates the optimal security to the number of bidders. More specifically, his model shows that debt (equity) is the preferred choice when competition is low (high) among investors. In contrast, the number of participants to the IPO in our model is fixed, and our analysis focuses on the potential use of (callable) warrants in unit IPOs, depending on the properties of underlying cash flows.

The rest of the paper is organized as follows. In Section 2, we set up our model. In Section 3 we present our results on unit versus standard IPOs based on the information sensitivity of these securities. In Section 4, we generalize the model to accommodate the possibility for the firm to make the warrants callable. We show that this often allows the firm to fully eliminate the winner’s curse problem. The empirical implications of the model are discussed in Section 5. Finally, Section 6 summarizes and concludes. All proofs are contained in Appendix A.

2 The Setup

Consider a two-date economy in which the riskless discount rate is zero and all agents are risk-neutral. At the outset, an all-equity firm privately owned by an entrepreneur must raise some capital through an initial public offering (IPO) in order to undertake a new project.⁵ More specifically, the firm seeks to raise $C > 0$ in capital, the initial cost of the project.⁶ The firm’s total cash flows, from both the firm’s existing assets and the new project, are denoted by V . The IPO takes place at $t = 0$. At that time, the potential investors in the IPO are differentially informed about the likely distribution of V , giving rise to a winner’s curse problem that is costly to the firm. At the end of the period (time $t = 1$), the cash flows of the securities issued by the firm are realized and investors in these securities receive their payoffs.⁷

The uncertainty about the firm’s prospects is captured by two states of nature, $s = 2$ and $s = 1$,

⁵We consider the effect of existing debt on our results later in the paper.

⁶Equivalently, the firm’s existing shareholders want to cash out an amount C of their shares for exogenous reasons. In this paper we do not model the reason behind or the timing of an IPO, but rather focus on the question of optimal security design given that an IPO must take place. These other issues are studied more specifically by Zingales (1995), Chemmanur and Fulghieri (1999), and Subrahmanyam and Titman (1999).

⁷Until Section 4 in which we introduce callable warrants, we do not allow the firm to issue securities that can pay off at an intermediate date between $t = 0$ and $t = 1$.

that occur with probability λ and $1 - \lambda$ respectively, $0 < \lambda < 1$. While a state s contains pertinent information about cash flows, we assume that cash flows are uncertain even given knowledge of s . Specifically, let $\Phi_s(v)$ denote the probability that the uncertain future cash flow V is less than or equal to some realized value $v \geq 0$, given a state s . We assume that the distribution Φ_2 dominates Φ_1 in the sense of first-order stochastic dominance, i.e., $\Phi_2(v) \leq \Phi_1(v)$ for all $v \in [0, \infty)$, with a strict inequality for a subset of strictly positive measure, and that they have finite moments.⁸ Let us denote the expected value of V in state s by $V_s \equiv E[V | s]$.

Some of the firm's future cash flows in state s come from its existing assets and these cash flows, denoted by A , are realized whether or not the new project is undertaken. The rest, $V - A$, represents the cash flows of the project, which is assumed to have a positive net present value and is worth undertaking in both states of the world; that is, $V_s - A_s > C$ for $s = 1, 2$, where $A_s \equiv E[A | s]$. This implies that, in a first-best scenario without any information asymmetries, the firm will raise C and invest in the project regardless of the state of the world, with the choice of securities irrelevant for shareholder welfare. However, as we show, when potential investors are differentially informed about the state s at $t = 0$, the choice of securities matters.

We assume that a fraction $\mu > 0$ of the outside investors learn the state of the world s at $t = 0$, while the firm and all the other outside investors do not. Although some uncertainty remains through Φ_s , the informed investors' knowledge of s allows them to identify and stay away from overpriced issues, as in Rock (1986). In general, the informational advantage of informed investors depends on the differences between Φ_1 and Φ_2 .⁹

Given a price at which the firm offers the issue, each investor decides whether or not to subscribe. Since some investors are potentially informed, other uninformed investors face a winner's curse problem. Uninformed investors are more likely to obtain a larger allotment when informed investors choose not to subscribe to the issue, i.e., exactly when the true value of the claims sold is not worth the price given the informed investors' information.¹⁰ In order to guarantee the success of the IPO, the firm is forced to underprice the issue in its effort to mitigate this winner's curse problem facing

⁸This is the most general order that generates a non-trivial informational cost for standard securities such as equity and warrants. We discuss later how our results are tightened if one considers stricter orders such as the monotone likelihood ratio property (see, e.g., DeMarzo and Duffie, 1999).

⁹While this information structure is particularly simple and similar to Rock (1986), our qualitative results extend to other informational environments, e.g., the one used in Benveniste and Spindt (1989).

¹⁰We assume that there are many informed (as well as many uninformed) investors, so that competition forces informed investors to subscribe to the issue when the true value of the claims sold is worth the price, as subsequent gains from trading are expected to be small. See Chakraborty and Yilmaz (2003) for a model of IPOs where the participation decision of informed investors is endogenized similarly in the presence of secondary markets.

uninformed investors. Such underpricing imposes a deadweight loss on the existing stake-holders in the firm. We wish to characterize the set of securities that minimizes this cost.¹¹

We assume that the set \mathcal{P} of feasible securities available to the firm consists of any package of equity and call warrants.¹² Given that the firm is initially all-equity financed, a share $\alpha \in [0, 1)$ of equity entitles investors to cash flows of αV when total firm cash flows equal V . If $\beta \in [0, 1 - \alpha]$ denotes the additional equity share that the packaged warrants entitle investors to upon paying an amount X per warrant (for a total exercise price of βX), the additional cash flows from the warrants is given by $\beta \max\{V - X, 0\}$, where $\max\{V - X, 0\}$ is the payoff from each warrant. The total cash flow from the package is denoted by $P(\alpha, \beta, X) = \alpha V + \beta \max\{V - X, 0\}$.¹³ We denote the expected value of a warrant with an exercise price of X given a state $s = 1, 2$ by $w_s(X) = E[\max\{V - X, 0\} | s]$, and summarize the set of feasible packages of equity shares and warrants available to the firm by

$$\mathcal{P} \equiv \left\{ P(\alpha, \beta, X) : \alpha \geq \bar{\alpha}, \beta \geq 0, \alpha + \beta \leq 1, X > 0 \right\}.$$

The set \mathcal{P} is constrained to include only packages with a minimum number $\bar{\alpha} > 0$ of shares, where $\bar{\alpha}$ can be made arbitrarily close to zero. This assumption is made purely for realism purposes, as no firm ever goes public by issuing warrants only, the reason (e.g., liquidity, cost of issue, etc.) being outside the model. Frequently, this constraint will not bind for the optimal package. The constraint on X states that the warrants are non-trivial while the constraint $\alpha + \beta \leq 1$ states that at most the entire firm may be sold.

Since our main objective is to contrast standard IPOs (that include only equity) with unit IPOs (that include equity and warrants in a single package) in terms of how their pricing is affected by the winner's curse, our results revolve around the comparison between $\beta = 0$ and $\beta > 0$. In particular, in what follows, the unit IPO is said to dominate a standard IPO when there exists an equity-warrant package with $\beta > 0$ and $X > 0$ that dominates every equity-only package with

¹¹To be sure, if these costs are severe enough, the firm may decide to design an IPO that fails with some probability, a possibility of underinvestment and associated deadweight costs that we ignore in this paper. To ensure that underinvestment is suboptimal for the firm, it is enough to assume that the project's net present value in state $s = 1$ is large enough.

¹²While this focus is entirely empirically motivated, our approach and insight cover a larger class of securities, such as debt and convertibles.

¹³As shown in Appendix B, this accounting convention is without loss of generality, as it is equivalent to one in which shareholders are entitled to their share of the strike price that is paid to the firm when the warrants are exercised. This convention allows us to treat the equity share α separately from the additional share β the warrant holders are entitled to. Our results will be unchanged under alternative but analytically cumbersome conventions.

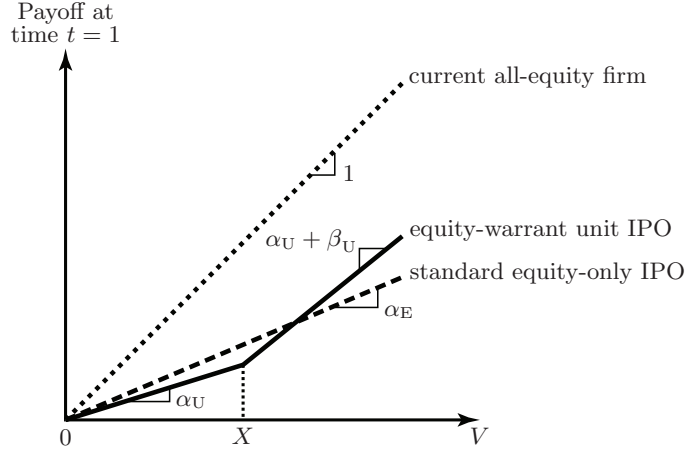


Figure 1: The above graph shows the cash flows that investors will receive as a function of the firm's cash flows when the IPO consists of equity only ($\alpha_E > 0$, $\beta_E = 0$) or a package of equity and warrants ($\alpha_U > 0$, $\beta_U > 0$, $X > 0$).

$\beta = 0$.¹⁴ Figure 1 depicts how the payoffs from the two financing strategies differ as a function of V .

Notice that the payoff of any package $P(\alpha, \beta, X) \in \mathcal{P}$ is increasing in the underlying cash flows V . Since the firm's total cash flows in state $s = 2$ stochastically dominate those in state $s = 1$, this implies that the expected value of the claims issued is always higher conditional on $s = 2$ than on $s = 1$, leading informed outside investors to stay away from a fairly priced issue (i.e, one whose price reflects only ex ante information) in state $s = 1$ but to subscribe to it in state $s = 2$. In other words, the issue is more likely to be oversubscribed in state $s = 2$. As a result, uninformed investors whose subscription choice cannot depend on s (as it is unknown to them) are more likely to be allocated the issue in state $s = 1$: they suffer from the winner's curse in that state. From their perspective, the effective probability that state $s = 2$ occurs, *conditional on being allocated the issue*, is smaller than the unconditional probability λ of that state occurring. We denote this effective probability by $\theta < \lambda$, where θ close to zero (to λ) corresponds to a large value of μ and thus a large (small) winner's curse problem for uninformed investors.

Since the firm can only invest if the issue succeeds, it designs the issue to ensure that uninformed investors are willing to subscribe and break even by paying C . It is then necessary that the expected value of the claims sold under the conditional beliefs θ , as denoted by \bar{P}_θ , must equal the required

¹⁴A similar terminology is used in section 4, in which we allow the warrants to be callable and the resulting packages are referred to as callable unit IPOs.

outlay C . But then the true (unconditional expected) value of the claims sold, \bar{P}_λ , derived from the prior beliefs λ , will typically exceed $\bar{P}_\theta = C$. The difference $\bar{P}_\lambda - \bar{P}_\theta = (\lambda - \theta)(P_2 - P_1) \geq 0$ is the (ex ante) expected informational cost to the firm associated with the offering,¹⁵ where $P_s \equiv E[P | s]$ denotes the expected value of the package in state s . Clearly, this informational cost can be alleviated via a reduction in $\lambda - \theta$ or in $P_2 - P_1$. The former approach effectively corresponds to that followed by Benveniste and Spindt (1989) and Parlour and Rajan (2005), who study how an efficient allocation of shares across investors can reduce the extent of information asymmetries.¹⁶ In contrast, we fix the share allocation mechanism, and focus instead on a security design approach that characterizes the package $P \in \mathcal{P}$ of equity and (callable or non-callable) warrants that minimizes the cost $P_2 - P_1$.

3 Optimal IPOs with Non-Callable Warrants

3.1 The General Case

To minimize $P_2 - P_1$, the firm must choose the package $P \in \mathcal{P}$ that has the lowest payoff sensitivity to the private information held by informed investors at time $t = 0$. Proposition 1 identifies a general condition under which this is achieved by a unit IPO as opposed to a standard IPO. To state the result, we use $D_s(X) \equiv E[\min\{X, V\} | s] = V_s - W_s(X)$, which is the expected portion of firm value that does not belong to warrant holders when α and β are arbitrarily close to zero and one respectively. This quantity can also be interpreted as a zero-coupon debt claim with a face value of X written on the firm's final cash flow. Of course, this debt claim is purely fictitious here, as debt is never packaged in an IPO.¹⁷

Proposition 1. *There is a unit IPO that strictly dominates the standard IPO if and only if there exists $X > 0$ such that*

$$\frac{D_2(X)}{V_2} > \frac{D_1(X)}{V_1}. \quad (1)$$

The intuition for this result is as follows. The value of a call warrant with a strike price of X in state s depends only on the distribution of upside cash flows (i.e., the portion of V above X). If

¹⁵It is also equal to the ex ante expected profits of informed investors, as uninformed investors earn zero expected profits in equilibrium.

¹⁶Note that this correspondence is not perfect, since their multiple-states models do not map directly into ours.

¹⁷Interestingly, a few of the Israeli IPOs in the sample used by Kandel, Sarig and Wohl (1999) pool debt along with the new equity issue. As far as we know, this is never done elsewhere. Also, as we show in section 3.2, the presence of previously issued debt does have an impact on our results.

(1) does not hold for any X , the asymmetric information about expected cash flows on the upside (as captured by the ratio $\frac{W_2(X)}{W_1(X)}$) is more severe than that involving the downside (as captured by the ratio $\frac{D_2(X)}{D_1(X)}$) for all X . In such cases, informational asymmetry about overall or average cash flows (as captured by the ratio $\frac{V_2}{V_1}$) is also lower than that about the upside. This implies that the winner's curse cost of equity is lower than that of warrants, and so the firm favors a standard IPO that consists entirely of equity. If however, $\frac{D_2(X)}{V_2} > \frac{D_1(X)}{V_1}$ for some $X > 0$, then the asymmetry of information is less severe on the upside (cash flows above X) than on the downside (cash flows below X). In this case, including warrants with a strike price equal to X in the IPO will lower the overall winner's curse cost, as less money must then be raised via equity.

We now characterize the insight of Proposition 1 in terms of the properties of the distribution of cash flows in a setting where $\Phi_s(\cdot)$ admits a continuous density $\phi_s(\cdot)$ that is positive on $[0, \infty)$. We consider the case with discrete support in section 3.2. To state the result concisely, we let $r(v) \equiv \frac{1-\Phi_2(v)}{1-\Phi_1(v)} \geq 1$. The behavior of $r(v)$ captures the informational advantage of informed investors as it pertains to the upside versus the downside.¹⁸

Proposition 2. *Suppose that, for $s = 1, 2$, $\Phi_s(\cdot)$ admits a continuous density $\phi_s(\cdot)$ that is positive on $[0, \infty)$. (a) If $r(v)$ is non-decreasing, the standard IPO strictly dominates any unit IPO. (b) If $\lim_{v \rightarrow \infty} r(v) < \frac{V_2}{V_1}$, there is a unit IPO that strictly dominates the standard IPO.*

When $r(v)$ is non-decreasing, the two distributions become ‘increasingly different’ as v gets larger. This means that the informational advantage of informed investors increases for high cash flows. Consequently, warrants have larger winner's curse costs than equity does, and a standard IPO is optimal. Notice that $r(v)$ is non-decreasing if and only if Φ_2 dominates Φ_1 in the sense of hazard rates, i.e., $\frac{\phi_2(v)}{1-\Phi_2(v)} < \frac{\phi_1(v)}{1-\Phi_1(v)}$ (e.g., see Krishna, 2002). This last condition is stronger than our assumed condition of first-order stochastic dominance, but weaker than the likelihood ratio order (under which $\frac{\phi_2(v)}{\phi_1(v)}$ is assumed to be increasing) that is frequently used in the literature. Interestingly, therefore, part (a) of Proposition 2 establishes that unit IPOs are never optimal when only likelihood ratio orders are considered in the winner's curse problem. As part (b) of Proposition 2 demonstrates, this is not the case when the distributions are ordered in the weaker sense of first-order stochastic dominance.

To understand part (b), recall that the ratio $r(v) = \frac{1-\Phi_2(v)}{1-\Phi_1(v)}$ effectively measures the informational advantage of informed investors with respect to cash flows above v . The condition that

¹⁸The assumption that Φ_s admits a continuous density is used only for Proposition 2. Since all distributions can be approximated by continuously differentiable ones, the same qualitative results obtain in general.

$\lim_{v \rightarrow \infty} r(v) < \frac{V_2}{V_1}$ then amounts to this informational advantage being less severe for high v than it is on average, as measured by $\frac{V_2}{V_1} = \frac{\int_0^\infty [1 - \Phi_2(v)] dv}{\int_0^\infty [1 - \Phi_1(v)] dv}$. For instance, if the distributions, Φ_2 and Φ_1 are identical above some cutoff v^* , then $r(v) = 1$ for $v > v^*$ and warrants with a strike price of v^* have no winner's curse costs associated with them. Including such warrants in a unit IPO will then be strictly better than a standard IPO.¹⁹

3.2 Binary Distributions

To highlight the intuition behind our results and to more precisely characterize the empirical implications of our model, we revisit the results derived so far using simple binary distributions for Φ_1 and Φ_2 . Specifically, let us assume that in state s , V is equal to H_s with probability $\phi_s \in (0, 1)$ and to L_s with probability $1 - \phi_s$, where $H_s > L_s > 0$. Cash flows in state $s = 2$ first-order stochastically dominate those in state $s = 1$, i.e., $H_2 \geq H_1$, $L_2 \geq L_1$ and $\phi_2 \geq \phi_1$, with at least one strict inequality. As one interpretation of this setup, we may think of $A_s = L_s$ as the state- s value of the firm's assets in place at time $t = 1$. Then, $H_s - L_s$ is the potential payoff of the growth opportunity associated with the new project, and $\phi_s(H_s - L_s)$ its expected value.

This specification captures in a simple way the different kinds of informational advantage that informed investors may have over uninformed ones. For instance, when $H_1 = H_2$ and $\phi_1 = \phi_2$, their informational advantage revolves around the profitability of the firm's existing assets and is measured by $L_2 - L_1$. Similarly, when $L_1 = L_2$ and $H_1 = H_2$, the information is about the viability of the new project. Finally, when $L_1 = L_2$ and $\phi_1 = \phi_2$, they know more about the potential scale of the new project. One can also think of s as the state of the industry upon which the firm's future profitability depends. That is, state $s = 2$ ($s = 1$) represents an industry that is likely to perform well (poorly), but the success of any one firm is still affected by idiosyncratic factors, as captured by ϕ_2 (by ϕ_1).

We start our treatment of this binary specification with the analogue of Proposition 2, in terms of the various parameters introduced above.

Proposition 3. *Suppose that Φ_1 and Φ_2 are binary distributions. If $H_1 \leq L_2$, the standard IPO is strictly optimal. If $H_1 > L_2$, there is a unit IPO that strictly dominates the standard IPO if and*

¹⁹The conditions in parts (a) and (b) of Proposition 2 are both sufficient. However, if attention is restricted to distributions for which the hazard rates cross at most once, then it can be shown that the condition in part (b) is also necessary for the unit IPO to dominate the standard IPO.

only if

$$\frac{L_2}{\phi_1 L_2 + (1 - \phi_1) L_1} > \frac{\phi_2 (H_2 - L_2)}{\phi_1 (H_1 - L_2)}. \quad (2)$$

When $H_1 \leq L_2$, the informational advantage of the informed investors is ordered in the sense that they know whether cash flows will be high (at least L_2) or low (at most H_1). In this case, including warrants that are valuable if and only if cash flows are high only worsens the winner's curse problem. It is then optimal for the firm to have a standard IPO.

In the more interesting case with $H_1 > L_2$, the informational advantage of informed investors is no longer ordered. In this case, it is possible that a package offering a larger fraction of its payments for $V \in [L_2, H_1]$ dominates the standard IPO, e.g., one that includes warrants with an exercise price of L_2 . Including these warrants is only optimal if (2) holds, that is, when the asymmetry of information is more severe for downside cash flows (i.e., below L_2) than for upside cash flows (i.e., above L_2). Indeed, the left-hand side of (2) is the ratio of the value of a (fictitious) debt claim with a face value L_2 in state $s = 2$ over that in state $s = 1$, while the right-hand side of (2) is the ratio of the warrant's value in these two states. In the proof of Proposition 3, we show that it is necessary and sufficient to consider packages with $F = L_2$ to verify (1) and establish (2).

More insights come from (2) when we consider some special cases. Suppose first that $L_1 = L_2 = L$. In this case, investors do not have any informational advantage as far as the cash flows below L are concerned. Indeed, a promised debt payment of L would be repaid with probability one in both states of the world, and so the left-hand side of (2) is equal to one in this case. Because the right-hand side of (2) is always above one, it is better for the firm to issue as few claims that are informationally sensitive to V being above L as possible. This is achieved by issuing equity only.

Consider next the case where $H_1 = H_2 = H$. In this case, manipulations of (2) tell us that the unit IPO dominates the standard IPO if and only if the ratio L_2/L_1 is larger than the ratio $\frac{\phi_2/(1-\phi_2)}{\phi_1/(1-\phi_1)}$. When the firm's upside potential is known to be H , the decision to include warrants in its IPO revolves around the relative size of the firm's assets in place in the two states of the world. When L_1 and L_2 are close, learning the state of the world does not provide investors with much downside information, and so issuing warrants, which are informationally sensitive to the firm's upside potential (as captured by the size of ϕ_2 relative to that of ϕ_1), is suboptimal. However, when learning the state of the world removes a great deal of uncertainty about assets in place, issuing claims that are less sensitive to this information is valuable, and so unit IPOs become attractive to the firm.

Finally, when $\phi_1 = \phi_2 = \phi$, a unit IPO dominates the standard IPO if and only if ϕ is

small enough. The intuition is similar to that for the previous case. When ϕ approaches one, information about assets in place (i.e., L_1 vs. L_2) is irrelevant, as the high cash flows are likely to be realized. Thus claims that are sensitive to downside information can be issued without much of an informational rent, and this is why issuing only equity is better for the firm. When ϕ is small however, the uncertainty revolves primarily around assets in place, as high cash flows do not get realized frequently in either state of the world. Claims that are sensitive to the downside cash flows are then expensive as they come with a large informational rent. As a result, the firm is better off including warrants in its IPO.

We conclude this section by considering the possibility that the firm already has outstanding debt and showing how this affects the choice between standard and unit IPOs. For simplicity and ease of exposition, we present our results only in the context of the simple binary specification, although a similar result obtains more generally. Specifically, let us assume that the firm has existing debt of face value $F_0 \in (0, H_2)$ and that this debt is senior to any new claims issued by the firm. Because the current bondholders are promised the first tranche of F_0 out of the firm's total cash flows at time $t = 1$, this existing debt affects the size of the cash flows that are available to new investors and, as we show, the tradeoff between standard and unit IPOs.

Consider first the case where the debt is risky in both states of the world, that is, $F_0 > L_2$ and so the firm can only possibly afford its debt when $V = H_s$ is realized. In this case, the firm cannot promise any cash flow to new investors when $V = L_1$ or $V = L_2$ are realized. As such, the firm never gains from including warrants in its IPO, as there is no asymmetry of information regarding downside cash flows as far as the new investors are concerned: they get zero when V turns out to be L_s , no matter what the state of the world is. In this case, a unit IPO never dominates a standard IPO.

At the other extreme, when the existing debt is riskless, $F_0 < L_1$, more of it can only make the inclusion of warrants in the IPO more advantageous to the firm. This is because increasing the existing riskless debt increases the sensitivity of the downside cash flows to the informed investors' information. Indeed, as F_0 increases from zero to L_1 , the ratio of assets in place that are available to investors in state $s = 2$ over state $s = 1$, $\frac{L_2 - F_0}{L_1 - F_0}$, goes from $\frac{L_2}{L_1}$ up to infinity. This makes securities that are not sensitive to downside risk, like warrants, more appealing. This leads to the following result.

Proposition 4. *There exists a cutoff $F^* \in (L_1, L_2]$, such that existing debt of face value F_0 makes a unit IPO more attractive (relative to the case with no debt) if and only if $F_0 < F^*$.*

Proposition 4 shows that the firm’s preference for a standard IPO over a unit IPO may be affected by the amount of existing debt even after controlling for asset characteristics. For instance, a slight preference for a unit IPO given little or no extant debt could be reversed if such debt obligations are sufficiently large.

4 Optimal IPOs with Callable Warrants

So far, we have characterized optimal IPOs ignoring the possibility that the warrants may be callable. In this section, we return to our general setup of Section 2 and expand the set of feasible securities \mathcal{P} by including this possibility. To accommodate some information about the firm to flow into the market before warrants can be called, we assume that the securities purchased by investors at the time of the IPO ($t = 0$) are traded prior to the end of the period, at $t = \frac{1}{2}$, in efficient financial markets. We implicitly assume that competition among investors gets their information incorporated into market prices at date $\frac{1}{2}$; that is, the state s is revealed during the trading round. The warrants are callable in the sense that the firm may reserve the right to call back each warrant at a pre-specified unit call price $k \geq 0$ (for a total price of βk) at $t = \frac{1}{2}$, provided that the state-contingent value of the firm at that date exceeds a threshold value ν (i.e., provided that $V_s \geq \nu$). As we discuss in Section 5, such call provisions are frequently seen in practice. We use the term callable unit IPO in what follows in order to distinguish a unit IPO involving callable warrants from one where the warrants are non-callable.

Call provisions may have value since the securities are publicly traded at time $t = \frac{1}{2}$ in efficient financial markets. Specifically, the market price of any traded security at $t = \frac{1}{2}$ is equal to the expected payoff of that security given state s . The length of time between $t = 0$ and $t = \frac{1}{2}$ can be thought of as the time it takes for the private information about the firm to get aggregated into market prices.²⁰ The call provisions on the warrant can be designed in such a way that the firm will, in equilibrium, be able to call the warrants and force investors to exercise them at date $t = \frac{1}{2}$ when the state is $s = 2$. In contrast, the call restriction ν can be set to a value that is sufficiently large to prevent the firm from forcing conversion in state $s = 1$, and allow the warrant holders to hold onto the valuable options in that state.

²⁰In reality, because the securities included in the IPO start trading separately immediately after the IPO, the assumption amounts to one in which competing informed traders quickly push prices to their efficient level through their trading. The risk neutrality of investors and market-clearing at time $t = \frac{1}{2}$ are sufficient to achieve this. Our qualitative results depend only on the assumption that the date 1 market prices are sufficiently (and not necessarily perfectly) informative about the differential information held at $t = 0$.

At $t = 0$, the subscribers to the IPO rationally anticipate that they will end up holding only equity whenever $s = 2$, and equity plus warrants whenever $s = 1$. Furthermore, since a warrant is less likely to be extinguished via a forcing call when $s = 1$, the expected *equilibrium* value of a warrant at $t = 0$ is higher conditional on $s = 1$ than on $s = 2$. That is, the warrants are more likely to be valuable exactly when the equity claims are less so. Notice that the equivalence of a call decision when $s = 2$ and early exercise at $t = 1$ is an endogenous equilibrium relation. We show below that it can be exploited in such a way that the variations in the expected values of equity and warrant components exactly offset each other, i.e., the total equilibrium value of the package does not depend on s . This eliminates the winner's curse problem and the need for underpricing.

To understand more clearly the call and exercise decisions facing investors at date $t = \frac{1}{2}$ when market prices reveal the state $s = 1, 2$, notice first that the warrants are worth more to investors alive than dead since $w_s(X) \geq V_s - X$.²¹ As a result, investors will not want to exercise the warrants early unless the firm forces them to do so by calling the warrants, i.e., unless $V_s - X \geq k$, surrendering the warrants for the call price otherwise. On the other hand, the firm will want to call the warrants only when they are worth more dead than alive to the firm, that is, the firm will prefer to call whenever $w_s(X) \geq k$ and not to call otherwise. To prevent the firm from extinguishing option values, the callable warrant contract specifies that the firm cannot call to force exercise unless the market value of the firm is higher than the trigger value ν . The value of a callable warrant at time $t = 1$ in state s is then given by $w_s(X)$ if $V_s < \nu$ or $w_s(X) < k$, and by $\max\{V_s - X, k\}$ otherwise. Our next result identifies a general necessary and sufficient condition for a package of equity and callable warrants to completely eliminate the negative impact of information that is differentially held across investors.

Proposition 5. *There is a callable unit IPO that eliminates any loss from the winner's curse if and only if*

$$w_1(V_2) \geq \frac{V_2 - V_1}{V_2 - C} C. \quad (3)$$

The proof of Proposition 5 shows that the firm will take advantage of the call provision to force exercise when good news arrives to the market at time $t = 1$. This allows it to make the overall package less sensitive to informational differences across investors, and to eliminate all winner's curse costs, whenever the upside potential of the firm, as measured by $w_1(V_2)$, is large enough.

²¹Since the warrants considered in this section are callable, we interpret $w_s(X)$ as the expected payoff to a warrant in state s , provided the warrant is not called and/or exercised prior to $t = 1$.

In general, there can be multiple packages of equity and callable warrants that fully eliminate the winner's curse. However, Proposition 5 also pins down a unique contract that is sure to achieve first-best when any other contract achieves first-best. As shown in the proof, this contract is given by

$$\alpha = \frac{C}{V_2}, \quad \beta = \frac{V_2 - V_1}{V_2 - C} \frac{C}{w_1(V_2)}, \quad X = V_2, \quad k = 0, \quad \nu \in (V_1, V_2). \quad (4)$$

With this contract, the warrants are called by the firm to force exercise at time $t = \frac{1}{2}$ when $s = 2$. When this happens, the warrants are at the money and the call price is zero so that the investors are left holding equity only. In contrast, when $s = 1$, the call restrictions prevent the firm from forcing investors into exercising or surrendering their warrants. As such, the investors keep holding a package of equity and warrants.

Given this equilibrium behavior at time $t = \frac{1}{2}$, the expected value of the contract given $s = 1$ is seen, using (4), to be $\alpha V_1 + \beta w_1(V_2) = C$, while that given $s = 2$ is seen to be $\alpha V_2 = C$ as well. Since the expected value of the contract equals C regardless of s , informed investors do not have any informational advantage over uninformed investors, eliminating the winner's curse costs completely.

When condition (3) does not hold, no package of equity and callable (or non-callable) warrants can achieve first-best. Nonetheless, including callable warrants of the type specified by (4) can still reduce the winner's curse problem. This is the object of our next result.

Proposition 6. *There is a callable unit IPO that strictly dominates a standard (or unit) IPO if $w_1(V_2) > 0$.*

The intuition behind this result is similar to that underlying the previous. Whenever the firm's upside potential is large enough in state $s = 1$ (in the precise sense that $w_1(V_2) > 0$), including callable warrants that will be called and exercised at time $t = \frac{1}{2}$, if and only if $s = 2$, reduces the effect of the winner's curse at the time of the IPO.²² If this upside potential is negligible, then call provisions do not have much value. In such cases, unit IPOs (involving non-callable warrants or, equivalently, callable warrants that are never called) may still be optimal if the uncertainty about V mostly concerns the downside risk, as shown in the previous section. Of course, relative to a straight equity issue, the gains from including callable or non-callable warrants may be small (e.g.,

²²It can be shown that the converse of the last result also holds, i.e., if $w_1(V_2) = 0$ then a standard IPO dominates any callable unit IPO in which the warrants are sometimes called in order to force exercise. The details are available from the authors upon request. Of course a standard IPO may still be dominated by a callable unit IPO in which the warrants are never called or exercised early, a situation identical to that covered by Proposition 2.

when $w_1(V_2)$ is close to zero). If including warrants imposes higher (unmodelled) transactions costs on the firm, then a straight equity issue will be optimal in such cases as well.

To finish, note that with more than two states of uncertainty, the state-contingent package that achieves first-best will in general be more complicated. More specifically, the first-best scenario can then only be implemented by allowing multiple classes of warrants that differ in their equity shares, strike prices and call provisions. Regardless, a single class of warrants with a fixed strike price and prespecified equity share will still help reduce the winner's curse.

5 Empirical Implications

In this section, we present and discuss the empirical predictions that our model yields. Some of these predictions are consistent with existing empirical evidence, and we do mention the appropriate references in those cases. The other predictions are novel, and they should provide a way not only to assess the validity of our model but also to test it against the theories advanced by Schultz (1993) and by Chemmanur and Fulghieri (1997).

Propositions 1-3 imply that a firm is more likely to include warrants as part of its initial offering of publicly traded securities when there is substantial uncertainty about downside values. Indeed, in the binary case, $H_s - L_s$ may be interpreted as the value created by the firm's investment in successful projects and L_s as the value of the firm's existing assets. Since information differences about a firm's existing assets (i.e., the downside) are likely higher for firms with intangible assets, we have the following prediction.

Implication 1. *Firms with intangible assets are more likely to use unit IPOs than other firms.*

The evidence provided by Schultz (1993) supports this prediction. He finds that unit IPOs are rarely used by firms in the mining, transportation, construction and retail industries. On the other hand, unit IPOs seem to be highly favored by firms in business services (which include computer software), engineering, health services and personal services. Similar evidence that unit IPOs tend to be used by firms in service-oriented and high-technology industries is provided by Jain (1994), by Lee, Lee and Taylor (2003) and by Garner and Marshall (2005). Also supporting this prediction is the evidence provided by Garner and Marshall (2005) that firms whose asset return volatility is higher, issue a larger number of warrants per share as part of their unit IPO. In other words, these firms avoid issuing securities whose cash flows are sensitive to the value of assets in place as much as possible.

For the uncertainty about downside values to matter, it has to be the case that there is a sizeable chance that these downside values occur for a given firm. For instance, as discussed in Section 3.2 with $\phi_1 = \phi_2 \equiv \phi$ for binary distributions, it must be the case that ϕ is low (so that $\Pr\{V = L_s\}$ is large) for a unit IPO to be optimal.

Implication 2. *Firms that use unit IPOs are more likely to experience negative performance shocks after they go public than firms that use standard (equity-only) IPOs.*

Various measures of firm performance could be used to assess the validity of this prediction, including firm profits, cash flows, return on assets, and stock price. Of course, when these variables decline over time (or decline relative to competitors), the firm is also more likely to find itself in financial trouble and perhaps even to close its doors. The empirical evidence provided by Garner and Marshall (2005), showing that firms that use unit IPOs tend to go bankrupt more often and more quickly than firms that use regular IPOs, appears to be consistent with this prediction.²³

In Proposition 4, we find that firms with low levels of risky debt are more likely to include warrants in their IPOs. This is because some uncertainty remains about the portion of firm value that belongs to equity holders in the low states of the world, since the firm may not be bankrupt in all such states. Lee, Lee and Taylor (2003) and Howe and Olsen (2006) provide evidence that indeed unit IPOs are used by firms with less outstanding debt than regular IPOs.

Implication 3. *Firms with low levels of debt use unit IPOs more than firms with high levels of debt.*

We now turn to the empirical predictions of our model that come from the callability feature of the warrants, as described in Propositions 5 and 6. In particular, these propositions establish that callable warrants are especially useful when warrants retain some option value even in the bad states of the world, that is, when $w_1(V_2)$ is large or at least positive. Since options are more valuable when the volatility of the underlying asset is large, the stock's volatility conditional on low firm performance, negative industry shocks, or overall bear markets becomes an important element of the firm's decision to include callable warrants in its unit IPO.

Implication 4. *The warrants included in unit IPOs are more likely to be callable for firms whose cash flows and performance are highly volatile when the firm, the industry or the economy experiences negative shocks.*

²³Because we do not model bankruptcy per se, this prediction only makes sense when the level of debt and debt riskiness are controlled for, especially given implication 3 below. Also, it is worth mentioning that Jain (1994) finds that, after controlling for size, risk and underwriter reputation, the survival rate between the two groups is similar.

Because the callability of warrants does not play a role in Chemmanur and Fulghieri's (1997) model and is not affected by conditional volatility in Schultz's (1993) model, this prediction is novel. Also, although Schultz (1993) documents that the majority of warrants in unit IPOs are callable (87% of his sample), no information is provided about the determinants of the callability decision of issuing firms. As such, this prediction remains to be tested. As before, the firm's performance can be measured using profits or the firm's stock price, and so a cross-sectional comparison of the volatility of this measure between firms that do and do not make their IPO warrants callable would provide a direct test of Implication 4.

Our second prediction concerns the number of warrants to be included in the IPO package. Although we do not demonstrate this formally, it is easy to verify, using (4), that the ratio of warrants to shares of equity that are included in the issue, $\frac{\beta}{\alpha}$, decreases with V_1 , and increases with V_2 . As such, large differences in the firm's performance in good and bad states, as proxied by the unconditional volatility of the aforementioned performance measures, is likely to lead the firm to include more warrants in its IPO. Furthermore, it is easy to verify that $\frac{\beta}{\alpha}$ is increasing in C , and so we would expect firms that raise more capital at the time of their IPO to include more warrants in their securities offering.

Implication 5. *A larger ratio of warrants to shares of equity are included in the unit IPOs of firms with large (unconditional) performance volatility and firms that sell a larger portion of their value when they first go public.*

Again, little existing empirical evidence can be used to assess the validity of this prediction. The one exception is the work of Garner and Marshall (2005) who find that the proportion of firm value sold as warrants increases with the firms' riskiness (as measured by the volatility of the post-IPO stock price, return on assets, or earnings per share), and with the ratio of warrants to equity shares in the unit IPO package. Because these findings are consistent with the signaling hypothesis underlying Chemmanur and Fulghieri's (1997) model however, they cannot be taken as exclusively supportive of our model.

The last direct implications of our model relate to the call and exercise prices (k and X) of the warrants included in the IPO. First, as discussed in the discussion following Proposition 5 and as shown in (4), the call price that makes the equity-warrant package efficient in most scenarios is zero, and so we would expect the call price that firms set for their warrants to be low. As far as we know, only Schultz (1993) provide any evidence on this front, and he finds that the average warrant call price is quite low, at about \$0.05 per warrant.

Implication 6. *The call price that firms set for the warrants included in their IPO is predicted to be low.*

Finally, as discussed earlier, the exercise price of the warrants must be large enough so that the option's payoff to investors is not so large when the firm forces conversion. Again, this is consistent with the evidence provided by Schultz (1993) that the exercise price is more than 25% above the IPO's price per share. Looking at (4), X increases with V_2 , and so we also expect the exercise price of the warrant to be larger for firms whose growth potential is extreme in some states of the world, another prediction that has yet to be tested.

Implication 7. *The exercise price of the option is predicted to be large relative to the stock price at the time of the issue, and it is expected to be larger when the firm's growth prospects are large.*

6 Conclusion

When some investors possess information about the prospect of a firm that other investors do not possess, a winner's curse exists at the time the firm decides to go public. In particular, uninformed investors are more likely to receive shares when informed investors know something negative about the firm and stay away from the new issue. To protect themselves, the uninformed investors require a discount in order to participate in the issue, and this is costly to the issuing firm. To resolve this problem, our model proposes a security design approach which allows firms to pool warrant options along with equity shares in the security offering. As we show, the inclusion of warrants becomes advantageous when the downside risk of the firm is large and subject to information asymmetries across investors. Moreover, the presence of warrants in the IPO package sometimes allows the firm to eliminate the winner's curse altogether. For this to be possible, the firm must make the warrants callable in high states of the world and it must be the case that the warrants are still valuable in low states of the world. Indeed, this combination dynamically removes the monotonicity of the packaged securities as a function of the firm's eventual value, and lowers the effective informational advantage of informed investors down to zero.

Our model considers the possibility that the issuing firm has some private debt outstanding and shows how this can affect its choice of financing at the time of the IPO. However, it does not consider the possibility of pooling a debt issue with the firm's IPO, as this is never done by firms in the United States. Still, the use of debt could improve the firm's security design when the inclusion of warrants in IPO units is either insufficient to eliminate the winner's curse

problem or simply infeasible for other exogenous reasons. Indeed, because debt contracts are less sensitive to information asymmetries related to the firms' upside potential, it will be the case that firms with high downside risk and potentially large information asymmetries across investors for downside values issue public equity before they issue public debt. Interestingly, this result has been empirically documented in a paper by Berkovitch, Gesser and Sarig (2006). More precisely, in their sample of privately held firms who access the public market for financing for the first time, 23% of them do so via a public debt issue as opposed to a public equity issue. They also find that these firms have a higher asset base and invest less in R&D, both consistent with lower downside risk and smaller information asymmetries across investors.

Although our theory is explained in the context of an IPO, its point is more general. Indeed, any situation in which financing is subject to the winner's curse problem can be approached from a security-design perspective. This, for example, could be the case for seasoned equity offerings (SEOs). Because the information asymmetries that prevail at the time of the IPO are likely to persist past the outcome of the IPO, we would expect later issues to also be affected by the winner's curse problem, although probably not to the same extent as the IPO. Because warrants are used when information asymmetries about downside values are large, we would expect firms that use unit IPOs to keep issuing securities that are less sensitive to downside outcomes. Indeed, Datta, Iskandar-Datta and Patel (2000) find that publicly traded firms with a larger asset base are significantly more likely to access public debt markets for the first time (i.e., go through an initial public debt offering, or a "debt IPO"). Moreover, Byoun, and Moore (2003) find that stock-warrant units are used in seasoned offerings by younger, riskier firms that did not receive venture-capital financing and whose offering was intermediated by a less reputable underwriter.

Another situation in which a security-design approach to the winner's curse could be useful is that of an entrepreneur seeking private financing from a venture capital firm (VC). The asymmetric information problem then arises from the fact that any one VC does not know whether it was approached before the other VCs by the entrepreneur. A VC that is approached second is clearly at a disadvantage since, presumably, the first VC passed on the deal for a reason, i.e., the prospects of the firm are not as good conditional on the fact that the first VC decided not to finance. Because of this possibility, VC firms should demand a discount on their shares of the firm, unless an appropriately chosen set of securities can be put together in one financing contract. As we show in this paper, the option features embedded in the securities included in these contracts may affect their effectiveness in solving the winner's curse problem.

7 Appendix A

Proof of Proposition 1

Take any package $P(\alpha, \beta, X) = \alpha V + \beta \max\{0, V - X\} \in \mathcal{P}$. For any random variable Ω that depends on V , we denote the expected value conditional on state $s = 1, 2$ as $\Omega_s \equiv E[\Omega | s]$ and the unconditional expected value under probability $q = \theta, \lambda$, as $\bar{\Omega}_q \equiv q\Omega_2 + (1 - q)\Omega_1$. Since any issue must be valued at C by uninformed investors in order to guarantee the investment in the new project, we must have

$$\alpha \bar{V}_\theta + \beta \bar{w}_\theta(X) = C. \quad (5)$$

From the firm's perspective, however, this package is worth $\alpha \bar{V}_\lambda + \beta \bar{w}_\lambda(X) > C$ at the time of the issue, as the probability of state $s = 2$ is then $\lambda > \theta$. The difference represents the informational rent R captured by the informed investors which, using (5) to replace α , is given by

$$R = \alpha (\bar{V}_\lambda - \bar{V}_\theta) + \beta [\bar{w}_\lambda(X) - \bar{w}_\theta(X)] = \frac{\bar{V}_\lambda - \bar{V}_\theta}{\bar{V}_\theta} [C - \beta \bar{w}_\theta(X)] + \beta [\bar{w}_\lambda(X) - \bar{w}_\theta(X)].$$

Since the optimal package must minimize this informational rent, the choice between a regular IPO (that has $\beta = 0$) and a unit IPO (that has $\beta > 0$) depends on the sign of

$$-\beta \bar{w}_\theta(X) \frac{\bar{V}_\lambda - \bar{V}_\theta}{\bar{V}_\theta} + \beta [\bar{w}_\lambda(X) - \bar{w}_\theta(X)].$$

In particular, the regular IPO is preferred if and only if this expression is positive for all possible exercise prices above zero, that is if and only if

$$\frac{\bar{V}_\lambda - \bar{V}_\theta}{\bar{V}_\theta} < \frac{\bar{w}_\lambda(X) - \bar{w}_\theta(X)}{\bar{w}_\theta(X)}$$

for all $X > 0$. Simple algebra and the fact that $\lambda > \theta$ shows that this inequality is equivalent to

$$\frac{w_1(X)}{V_1} < \frac{w_2(X)}{V_2}. \quad (6)$$

Since $w_s(X) = V_s - D_s(X)$, (6) is equivalent to (1). This completes the proof. ■

Proof of Proposition 2

(a) Suppose that $r(v)$ is a non-decreasing function of v . By Proposition 1, we need to show that $d(X) \equiv \frac{D_2(X)}{D_1(X)} < \frac{V_2}{V_1}$ for all $X > 0$ where, as before, $D_s(X)$ can be interpreted as the value of (fictitious) debt of face value X in state s . For any $X > 0$, we have

$$D_s(X) = \int_0^X v \phi_s(v) dv + X [1 - \Phi_s(X)],$$

and so $D'_s(X) = 1 - \Phi_s(X)$. This means that

$$d'(X) = \frac{D'_2(X)D_1(X) - D'_1(X)D_2(X)}{D_1^2(X)} = \frac{D'_2(X) - D'_1(X)d(X)}{D_1(X)} = \frac{1 - \Phi_1(X)}{D_1(X)} [r(X) - d(X)],$$

Furthermore, for future use, we show that $\lim_{X \rightarrow 0} d(X) = 1$. Note that the limit is in indeterminate form as both the numerator and the denominator goes to zero as X goes to zero. Using l'Hôpital's rule once leads to $\lim_{X \rightarrow 0} d(X) = \frac{1 - \Phi_2(X)}{1 - \Phi_1(X)} = 1$.

We now claim that $d'(X) \geq 0$ for all $X > 0$. Contrary to our claim, suppose there exists $\tilde{X} > 0$ such that $d'(\tilde{X}) < 0$. First note that $d'(X) > 0$ if and only if $r(X) - d(X) > 0$. Furthermore, notice that we must have $r(X) \geq 1$ for all $X > 0$, because $\Phi_2(\cdot)$ first-order stochastically dominates $\Phi_1(\cdot)$. Consequently, we must have $d(\tilde{X}) > r(\tilde{X}) \geq 1$. Since $\lim_{X \rightarrow 0} d(X) = 1$, there must exist $\hat{X} < \tilde{X}$ such that $d'(\hat{X}) = 0$ (and so $r(\hat{X}) = d(\hat{X})$, as shown above) with $d(\hat{X}) > d(\tilde{X})$. Thus we have $r(\hat{X}) = d(\hat{X}) > d(\tilde{X}) > r(\tilde{X})$. However, this contradicts the monotonicity of $r(X)$, completing the argument that $d'(X) \geq 0$ for all $X > 0$.

The fact that $d'(X) \geq 0$ for all $X > 0$ implies that $d(\tilde{X}) \leq d(\hat{X})$ for any $\tilde{X} < \hat{X}$. Also, since $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$ are not everywhere identical, there exists $\tilde{X} > X$ such that $d(X) < d(\tilde{X})$. Therefore, for any $X > 0$, we have $d(X) < d(\tilde{X}) \leq \lim_{\hat{X} \rightarrow \infty} d(\hat{X}) = \frac{V_2}{V_1}$, establishing the result.

(b) Since $\lim_{X \rightarrow \infty} r(X) < \frac{V_2}{V_1} = \lim_{X \rightarrow \infty} d(X)$, it must be the case that, for all X large enough, $d(X) > r(X)$ and so $d'(X) < 0$, using the same arguments as in the proof of part (a). This in turn implies that, for all such X , $d(X) > \lim_{\tilde{X} \rightarrow \infty} d(\tilde{X}) = \frac{V_2}{V_1}$, yielding the result by Proposition 1. ■

Proof of Proposition 3

We know from Proposition 1 that, for the unit IPO to strictly dominate the standard IPO, it is necessary and sufficient to find an $X \in (0, H_2)$ such that

$$d(X) \equiv \frac{D_2(X)}{D_1(X)} > \frac{V_2}{V_1}.$$

Notice that for $X \leq L_1$, we have $d(X) = 1 < \frac{V_2}{V_1}$. Notice also that $d(X)$ is a non-decreasing function of X for all $X \in [H_1, H_2)$, and that $d(H_2) = \frac{V_2}{V_1}$.

Let us first consider the case in which $H_1 \leq L_2$. Then $d(X)$ is also increasing in X for $X \in (L_1, H_1)$, and so $d(X)$ is a non-decreasing function of X for all $X \in (L_1, H_2)$. This in turn implies that the standard IPO dominates the unit IPO, as $d(X)$ then increases from $d(L_1) = 1 \leq \frac{V_2}{V_1}$ to $d(H_2) = \frac{V_2}{V_1} \leq \frac{V_2}{V_1}$, without ever decreasing as X increases from zero to H_2 . This completes the proof for the case where $H_1 \leq L_2$. Accordingly, we focus on the case in which $H_1 > L_2$ for the rest of the proof. Notice then that for $X \in (L_1, L_2)$,

$$d(X) = \frac{X}{\phi_1 X + (1 - \phi_1)L_1},$$

which is an increasing function of X . Notice next that, for $X \in [L_2, H_1)$, we have

$$d(X) = \frac{\phi_2 X + (1 - \phi_2)L_2}{\phi_1 X + (1 - \phi_1)L_1}$$

and

$$d'(X) = \frac{\phi_2 D_1(X) - \phi_1 D_2(X)}{[D_1(X)]^2},$$

which is strictly negative if and only if $d(X) = \frac{D_2(X)}{D_1(X)} > \frac{\phi_2}{\phi_1}$ or, equivalently, if and only if $\frac{L_2}{L_1} > \frac{\phi_2/(1-\phi_2)}{\phi_1/(1-\phi_1)}$. Thus $d(X)$ is either entirely non-decreasing or entirely non-increasing for $X \in [L_2, H_1)$. If $d(X)$ is non-decreasing in this interval, then it is non-decreasing for all $X \in (0, H_2)$ and so, using the same reasoning as before, $d(X) \leq \frac{V_2}{V_1}$ for all $X \in (0, H_2)$. As such, for the unit IPO to strictly dominate the standard IPO, it is necessary that $d(X)$ be non-increasing in the interval $[L_2, H_1)$. Indeed, it is necessary that the local maximum of $d(X)$ at $X = L_2$ be global, that is, it is necessary that

$$d(L_2) = \frac{L_2}{\phi_1 L_2 + (1 - \phi_1)L_1} > \frac{V_2}{V_1} = \frac{L_2 + \phi_2(H_2 - L_2)}{L_1 + \phi_1(H_1 - L_1)}.$$

The denominator of the last term can be rewritten as $L_1 + \phi_1(L_2 - L_1) + \phi_1(H_1 - L_2)$. Rearrangement of this term yields $\phi_1 L_2 + (1 - \phi_1)L_1 + \phi_1(H_1 - L_2)$. Therefore, we can rewrite the inequality as

$$\frac{L_2}{\phi_1 L_2 + (1 - \phi_1)L_1} > \frac{L_2 + \phi_2(H_2 - L_2)}{\phi_1 L_2 + (1 - \phi_1)L_1 + \phi_1(H_1 - L_2)}.$$

This inequality in turn simplifies to

$$\frac{L_2}{\phi_1 L_2 + (1 - \phi_1)L_1} > \frac{\phi_2(H_2 - L_2)}{\phi_1(H_1 - L_2)},$$

which is necessary for the unit auction to strictly dominate the standard auction. The sufficiency of this last inequality for the unit auction to strictly dominate the standard auction follows directly from Proposition 1. ■

Proof of Proposition 4

Observe first that if the face value of the existing debt is $F_0 > L_2$, then from the perspective of new investors, the firm's cash flows in state s are $L'_s \equiv 0$ or $H'_s \equiv H_s - F_0$, with probability $1 - \phi_s$ and ϕ_s respectively. As such, using the notation (and strategy) introduced in the proof of Proposition 1b, we have $d(F) = \frac{\phi_2}{\phi_1}$ for all $F \in (0, H_1 - F_0)$. Since $\frac{V'_2}{V'_1} = \frac{\phi_2(H_2 - F_0)}{\phi_1(H_1 - F_0)}$, this implies that the standard auction strictly dominates if $H_2 > H_1$ (since we then have $d(F) < \frac{V'_2}{V'_1}$), while the two IPO formats are identical otherwise.

Observe next that if the face value of the existing debt is $F_0 < L_1$, then from the perspective of new investors, the firm's cash flows in state s are $L'_s \equiv L_s - F_0$ or $H'_s \equiv H_s - F_0$, with probability

$1 - \phi_s$ and ϕ_s respectively. Replacing L_s by L'_s and H_s by H'_s in (2), we obtain

$$\frac{L_2 - F_0}{\phi_1 L_2 + (1 - \phi_1)L_1 - F_0} < \frac{\phi_2(H_2 - L_2)}{\phi_1(H_1 - L_1)}.$$

Since the left-hand side of this expression is monotonically increasing in F_0 , the inequality becomes more difficult to satisfy as F_0 increases, and so unit IPOs become more attractive. This result now follows directly via the intermediate value theorem and continuity. ■

Proof of Proposition 5

Sufficiency. We show that if (3) is satisfied, one can construct a callable unit IPO that achieves first-best. Consider a package $P(\alpha, \beta, X)$ of equity and callable warrants with a call price of k and a trigger value $\nu \in (V_1, V_2)$. Then the firm can call the warrants at time $t = \frac{1}{2}$ if and only if $s = 2$. Suppose that $k = 0 = V_2 - X$ so that, when $s = 2$ at time $t = \frac{1}{2}$, the warrant is at the money and the warrant holders exercise if the warrant is called. Furthermore, set α and β such that the value of the (optimally exercised and called) package in each state is equal to C , that is,

$$\alpha V_2 = C, \quad \text{and} \quad (7)$$

$$\alpha V_1 + \beta w_1(V_2) = C. \quad (8)$$

Such a contract, if it exists, has no information rents associated with it, since the expected value of the security equals C , state by state. We look for conditions which guarantee existence. To this end, note first that the solution to (7) and (8) is $\alpha = \frac{C}{V_2}$ and $\beta = \frac{C}{w_1(V_2)} \left(1 - \frac{V_1}{V_2}\right)$. Because $V_2 > V_1$ (by the first-order stochastic dominance of $\Phi_2(\cdot)$ over $\Phi_1(\cdot)$), it is clear that $\beta > 0$. It just remains to show that $\beta \leq 1 - \alpha$, or that

$$\frac{C}{w_1(V_2)} \left(1 - \frac{V_1}{V_2}\right) \leq 1 - \frac{C}{V_2},$$

as required by the definition of \mathcal{P} . This last inequality is equivalent to (3).

Necessity. Suppose that a package $P(\alpha, \beta, X) \in \mathcal{P}$ together with call provisions $\{k, \nu\}$ achieve first-best, with $\alpha \in (\bar{\alpha}, 1)$, $\beta \in [0, 1 - \alpha]$, and $X > 0$. Because equity-only financing (i.e., $\beta = 0$) cannot eliminate the winner's curse problem, it must be the case that β is strictly positive. We proceed in steps.

Step 1: The warrants are called if and only if $s = 2$, that is, $\nu \in (V_1, V_2)$.

By first-order stochastic dominance, it is immediate that the first-best cannot be achieved if (i) the warrants are never called, or (ii) the warrants are always called and always turned in (unexercised), or (iii) the warrants are always called and always exercised. If the warrants are always

called, and they are sometimes exercised and sometimes turned in, then they will be exercised when $s = 2$ (requiring that $V_2 - X \geq k$) and turned in when $s = 1$. To achieve first-best, we need

$$\begin{aligned}\alpha V_2 + \beta(V_2 - X) &= C, \quad \text{and} \\ \alpha V_1 + \beta k &= C.\end{aligned}$$

However this is impossible, since $V_2 > V_1$ (by first-order stochastic dominance) and $V_2 - X \geq k \geq 0$. We conclude that the warrants will be called at time $t = \frac{1}{2}$ if and only if $s = 2$, that is, we have $\nu \in (V_1, V_2)$.

Step 2: Without loss of generality, $k \geq V_2 - X$.

We show that for any first-best package with a warrant strike price that satisfies $k' < V_2 - X$, then there exists another package that also achieves first-best, that differs only in the call price k , and satisfies $k \geq V_2 - X$. To see this suppose that $V_2 - X > k' \geq 0$ so that the warrants are exercised when called in state $s = 2$. Since first-best is achieved, we must have

$$\begin{aligned}\alpha V_2 + \beta(V_2 - X) &= C, \quad \text{and} \\ \alpha V_1 + \beta w_1(X) &= C.\end{aligned}$$

Consider another warrant contract that differs only in the call price, which is set at $k = V_2 - X$. Clearly, with such a contract, the warrant holders are indifferent between exercising their warrants and turning them into the firm unexercised. Also, the contract achieves first-best as it satisfies

$$\alpha V_2 + \beta k = C, \quad \text{and} \tag{9}$$

$$\alpha V_1 + \beta w_1(X) = C, \tag{10}$$

together with the constraints that

$$k \geq 0, \quad k \geq V_2 - X, \tag{11}$$

$$\bar{\alpha} \leq \alpha \leq 1, \quad 0 \leq \beta \leq 1 - \alpha. \tag{12}$$

Thus, without loss of generality, the contract can be such that warrant holders weakly prefer to surrender the warrants when called, and such that it satisfies (9)-(12).

Step 3: Without loss of generality, $k = 0$.

Notice first that it must be the case that $k < w_1(X)$. For if $k \geq w_1(X)$, since $\alpha, \beta > 0$ and $V_2 > V_1$, we cannot then have (9)-(10) hold. We show next that if $k > 0$ in a package that achieves first-best, then there exists another package that also achieves first-best with $k' < k$. To see this, suppose that $k > 0$ in a package that achieves first-best. From (11) and $\beta > 0$, we have $\alpha, \beta \in (0, 1)$. Furthermore, from (9)-(10), we have $\beta > 0$ which in turn implies that $\alpha < 1$. Consider a differential

change in k of amount $dk < 0$, together with changes in X , α and β , such that $dX = -dk > 0$, and further, (9)-(10) continue to hold, i.e.,

$$V_2 d\alpha + k d\beta + \beta dk = 0, \quad \text{and} \quad (13)$$

$$V_1 d\alpha + w_1(X) d\beta + \beta w_1'(X) dX = 0, \quad (14)$$

where $w_1'(X) \equiv \frac{dw_1(X)}{dX} \in (-1, 0)$. Since $dX = -dk$, the second inequality in (11) will continue to hold after the change. We need to guarantee that second one in (12) also does, i.e., that $d\alpha + d\beta \leq 0$. Using $dX = -dk$ in (13)-(14) and solving for $d\alpha$ and $d\beta$ in terms of dk , we obtain

$$d\alpha + d\beta = \beta \frac{[(V_1 - w_1(X)) + w_1'(X)(V_2 - k)]}{V_2 w_1(X) - V_1 k} dk.$$

Since $dk < 0$, $\beta > 0$, $V_2 > V_1$ and $w_1(X) > k$, it follows that $d\alpha + d\beta \leq 0$ if and only if

$$V_1 - w_1(X) \geq -w_1'(X)(V_2 - k).$$

But this follows from observing that

$$V_1 - w_1(X) = D_1(X) \geq (1 - \Phi_1(X))X = -w_1'(X)X$$

and, from the second inequality in (11), that $X \geq V_2 - k$. This concludes step 3.

Step 4: It must be the case that (3) is satisfied.

Since, without loss of generality, we have $k = 0$ (from step 3) and $k \geq V_2 - X$ (from step 2), it follows that $X \geq V_2$. Using this in (9), we have $\alpha = \frac{C}{V_2}$. Since the warrant value is non-decreasing in X , for (10) to hold it is necessary that

$$C \frac{V_1}{V_2} + \beta w_1(V_2) \geq C \quad (15)$$

for some $\beta \in (0, 1 - \frac{C}{V_2}]$. Clearly, since $V_2 > V_1$, the left-hand side of (15) is strictly smaller than C at $\beta = 0$. Since the left-hand side of (15) is monotonic in β , a necessary condition for (15) to hold is that the left-hand side be larger than C at $\beta = 1 - \frac{C}{V_2}$. That is, it is necessary that

$$C \frac{V_1}{V_2} + \left(1 - \frac{C}{V_2}\right) w_1(V_2) \geq C,$$

which is equivalent to (3). This completes the proof. ■

Proof of Proposition 6

If (3) holds, then we know from Proposition 5 that callable unit IPOs achieve first-best and so the result trivially holds. So suppose instead that (3) does not hold but that $w_1(V_2) > 0$. We show

that a standard IPO is dominated by package that includes callable warrants. The proof for the unit IPO is entirely analogous and so is left out.

Consider a package with $P(\alpha, \beta, X) \in \mathcal{P}$ that has $k = V_2 - X = 0$, $\nu \in (V_1, V_2)$. Since (3) does not hold, such a package does not achieve first-best, and so it must satisfy

$$\begin{aligned}\alpha V_2 &= C + \delta_2, \quad \text{and} \\ \alpha V_1 + \beta w_1(V_2) &= C - \delta_1,\end{aligned}$$

for some $\delta_1 > 0$ and $\delta_2 > 0$. The informational rent paid by the firm (and captured by the informed investors) are then given by

$$R = (\lambda - \theta)(\delta_1 + \delta_2) = (\lambda - \theta) \left[(V_2 - V_1)\alpha - \beta w_1(V_2) \right].$$

At the same time, since the issue raises C in capital for the firm, we must also have

$$\theta \alpha V_2 + (1 - \theta) \left[\alpha V_1 + \beta w_1(V_2) \right] = C. \tag{16}$$

Notice that if $\beta = 0$, such a package reduces to a straight equity issue. Consider a change to an equity-warrant package by increasing β from zero to a positive value, and adjusting α downwards so that (16) continues to hold. Then we have $\frac{d\alpha}{d\beta} < 0$, and so

$$\frac{dR}{d\beta} = (\lambda - \theta) \left[(V_2 - V_1) \frac{\partial \alpha}{\partial \beta} - w_1(V_2) \right] < 0. \quad \blacksquare$$

8 Appendix B

In this appendix, we show that the accounting convention used for the warrants throughout the paper is without loss of generality. More specifically, we show that our convention to assume that the warrant holders get a fraction β of the firm's value above X (the strike price of the warrant) is equivalent to one in which βX is added to the firm's value when the warrants are exercised.

To see this, let us assume that the firm initially has $n = 1$ shares outstanding, and suppose that the IPO is done via a package issue consisting of n_s shares and n_w warrants with a strike price of X . At expiration, the new claim-holders will own a fraction n_s of the firm's value, which we denote by V , if they choose not to exercise their warrants. If they exercise for a price of $n_w X$, then these new claim-holders will own a fraction $\frac{n_s + n_w}{1 + n_w}$ of the firm's value, which is now $V + n_w X$. Thus, the new claim-holders exercise if and only if

$$\frac{n_s + n_w}{1 + n_w} (V + n_w X) - n_w X > n_s V,$$

which is equivalent to $V > X$. Of course, the original shareholders own the remainder of the firm, that is, $(1 - n_s)V$ when the warrants are not exercised, and $\frac{1 - n_s}{1 + n_w} (V + n_w X)$ when the warrants are exercised.

Throughout the paper, we assume that the firm’s IPO consists of α shares and β warrants with a strike price of X , with the accounting convention that the new claim-holders get a fraction α of V plus a fraction β of $V - X$ when they exercise their warrants (i.e., when $V > X$). With this convention, the original shareholders get $(1 - \alpha)V$ when the warrants are not exercised and they get $(1 - \alpha)X + (1 - \alpha - \beta)(V - X) = (1 - \alpha - \beta)V + \beta X$ when the warrants are exercised.

This accounting convention is without loss of generality if we can show that there is a unique mapping between $\{n_s, n_w\}$ and $\{\alpha, \beta\}$ such that the cash flows received by the original shareholders and the new claim-holders are exactly the same in all scenarios. This will be the case if and only if we can find a unique $\{\alpha, \beta\}$ such the the following equalities hold:

$$(1 - \alpha)V = (1 - n_s)V, \tag{17}$$

$$(1 - \alpha - \beta)V + \beta X = \frac{1 - n_s}{1 + n_w}(V + n_w X), \tag{18}$$

$$\alpha V = n_s V, \tag{19}$$

$$\frac{n_s + n_w}{1 + n_w} - n_w X = \alpha V + \beta(V - X). \tag{20}$$

The first (last) two equalities ensure that the original shareholders (new claim-holders) receive the same cash flows when $V \leq X$ and when $V > X$. It is easy to verify that

$$\alpha = n_s \quad \text{and} \quad \beta = \frac{n_w(1 - n_s)}{1 + n_w} \tag{21}$$

is the unique solution to (17)-(20). Alternatively, our accounting convention is equivalent to one in which the strike price is added to the firm’s value when the warrants are exercised as long as we set

$$n_s = \alpha \quad \text{and} \quad n_w = \frac{\beta}{1 - \alpha - \beta}, \tag{22}$$

which is obtained by inverting (21). In short, our accounting convention is without loss of generality.

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