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Microstructure Bluffing with Nested Information

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Chakraborty and Yılmaz (2004a, b) consider strategic models of repeated trading by an informed insider with long-lived private information. They show that, if market makers face uncertainty about the existence of informed trades in the order flow then, with sufficiently many periods of trading, the insider will bluff in every equilibrium in the precise sense ruled out in Albert S. Kyle (1985, 1323), i.e., "by first destabilizing prices with unprofitable trades made at the *n*th auction, then recouping the losses and much more with profitable trades at future auctions." By bluffing, the insider adds noise to the market's inference problem and makes prices less sensitive to his trades.

In this note, we demonstrate how the scope for profitable bluffing is enhanced by the presence of other rational informed traders in the market. These traders have superior information compared to the market makers, i.e., they know if there is any informed trading in the observed order flow, although like the market makers they do not know the precise nature of the information. Due to this advantage, they are better able to extract information from early period order flows, relative to market makers. This gives rise to profitable trading opportunities via mimicking, or *following,* the earlier trades of the insider in later periods. Competition among these followers then leads prices to quickly incorporate all private information, eliminating almost all trading profits for the insider in later periods of trading. We show that such competitive pressure from followers may lead the insider to *bluff* the followers in equilibrium, i.e., undertake unprofitable early trades in order to trade in the future at favorable expected prices.

More precisely, whenever the insider is not expected to bluff in equilibrium, followers will find it in their interest to trade in the direction of the first-period order flow in the second period. This allows the insider to bluff by buying in the first period an asset that he knows is overvalued. He then profitably unwinds his position in the second period by trading in the direction of his information, but against the direction of anticipated follower order flow. When the follower order flow is large, the profits from such bluffing will be greater than the profits from not bluffing, i.e., from always trading in the direction of his information in both periods. It follows that the insider must be expected to bluff in any equilibrium. By bluffing, the insider lowers the competitive pressure from followers on his trading profits.

Since the insider aims to add noise to the inference problems of the market makers and followers in our model, we use the term bluffing to describe the process, in line with definitions proposed by Kyle and S. Viswanathan (2008).[1] Similar phenomena are also analyzed by Douglas F. Foster and Viswanathan (1994), in a model of an asset market with a nested information structure. In terms of our terminology, they show that in a linear equilibrium the insider has an incentive to hide his exclusive information in early periods and trade only on the information held by followers about fundamentals. This may involve early trades by the insider that are against the direction of his own superior information. Such behavior reduces the competitive pressure the insider faces from followers by quickly revealing the information of the followers to the market. While the two papers are conceptually related, the followers in our model have no information about fundamentals and it is not in the interest of the insider (indeed, fatal for him) to reveal the followers' information to

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[1] Our results may also be thought of as an example of pure trade-based manipulation, along the lines of an earlier taxonomy proposed by Franklin Allen and Douglas Gale (1992).

the market, as this will eliminate all his trading profits. Furthermore, the insider not only tries to hide his own information from others in our setting, but also undertakes costly bluffing in *every* equilibrium.

Interpreted more broadly, our work is also related to the literature on the effect of disclosure regulations on market manipulation (Michael J. Fishman and Kathleen Hagerty 1995; Kose John and Ranga Narayanan 1997; Steven Huddart, John Hughes, and Carolyn Levine 2001), except that information about the insider's past trades is not driven by exogenous regulation in our model. Rather, it is generated endogenously via the interaction of differentially informed traders in the marketplace.

I. Model

We consider a market for one asset with risk-neutral agents and a continuum of possible trade sizes. The current price of the asset is scaled to zero. The long-term or fundamental value of the asset is $v \in \{-1, 1\}$, with equally likely priors.

There are three kinds of traders in the market. The first is a strategic informed trader (the insider or informed leader) who knows the realized value of v and arrives in the market with probability $\alpha \in (0,1)$. With probability $1 - \alpha$ the leader is a noise or liquidity trader whose trades are uncorrelated with fundamentals, represented by a probability density $g(x)$ and associated distribution $G(x)$, where x stands for a trading position. We suppose that $g(x)$ is continuous, strictly positive for all $x \in [-1, 1]$, and symmetric around zero. Indeed, in what follows, we assume for simplicity that g is the uniform distribution and that noise trades are distributed independently across periods.

Apart from the insider there is also a second class of rational traders in the market called followers, indexed collectively by F. Each follower is infinitesimal, and in the aggregate there is a continuum of followers with total mass Δ.[2] We assume that $\Delta > 1$ so that followers may be able to submit in the aggregate an order that is larger than the maximum possible order that may come from noise. Followers know whether the leader is an informed trader (signal $s = I$) or a noise trader ($s = N$), although they do not know the nature of his information. Finally, there are competitive market makers who know only the priors.

There are two periods of trading $t \in \{1,2\}$ and each period all traders simultaneously submit market orders after observing the previous net order flow and prices. Subsequently, market makers set the price for that period after observing that period's net order flow. We denote by $x_t^I(v) \in \mathbb{R}$ the order of the informed leader in period t as a function of the realized value v; by $x_t^F(s) \in \mathbb{R}$ the aggregate order of followers as a function of their information s; and by $x_t^N \in \mathbb{R}$ that from noise.[3] If $x_t^i > 0$ (resp., $x_t^i < 0$) we say that the order by $i = I, F, N$ is a *buy* (resp., *sell*), with $x_t^i = 0$ a *no-trade*. The net order flow x_t in any period is the sum of the orders from the different groups of traders, with h_t the history of net order flows up to and including period t.

The price in period t is denoted by $p(h_t)$ and is set by competitive market makers to equal the expected value of the asset conditional on the observed order flow. Let $Q(x|v)$ and $R(x|v, h_1)$ be the cumulative distribution functions, for periods 1 and 2, respectively, that represent the informed leader's (behavior) strategy as a function of v and the prior history of net order flows. When Q and R admit densities, we will denote them by q and r, respectively.

Since $v \in \{-1, 1\}$, we must have $-1 \leq p(h_t) \leq 1$ for all h_t, t in any equilibrium. Since an informed trader will never undertake an expected loss-making trade in the final period, we must have $R(x|1, h_1) = 0$ for all $x < 0$ and $1 - R(x|-1, h_1) = 0$ for all $x > 0$, for all h_1 and in any equilibrium. We say that the informed leader's strategy involves bluffing if he makes a loss in period 1, presumably to obtain better prices later.

DEFINITION 1: *The insider is bluffing if* $Q(x|1) > 0$ *for some* $x < 0$ *or* $Q(x|-1) < 1$ *for some* $x > 0$. *Otherwise he is not bluffing.*

[2] The assumption of a continuum of followers implies that each follower's trades have no impact on prices, although in the aggregate followers will have an impact, as we show later. The followers are rational in the sense that they maximize the sum of trading profits over all periods.

[3] Excluding noise, the other orders will also depend in general on the observed history of previous orders from all market participants, but we suppress that dependence for the sake of notational ease.

The definition above captures the notion of bluffing in the precise sense of the quote from Kyle (1985) presented earlier. For instance, when $v = 1$ and $Q(x|1) > 0$ for some $x < 0$, the insider is undertaking loss-inducing early sells that can be profitable overall only if it interferes with the inferences of the other market participants and allows him to buy at favorable prices in the future. In what follows we use the term nonbluffing equilibrium to mean an equilibrium in which the insider is not bluffing.

II. Analysis

In this section we demonstrate our main result: that the leader will bluff in every equilibrium.[4] To this end, we begin by stating some necessary conditions of follower behavior in any (candidate) equilibrium where the leader is not bluffing.

LEMMA 1: *In any candidate nonbluffing equilibrium,*

(i) following any first-period buy (resp., sell) by the informed leader, the second-period price $p(h_2) = 1$ *(resp.,* $p(h_2) = -1$*) and the followers trade* $x_2^F(I) \geq 1 + x_2^F(N)$ *(resp.,* $x_2^F(I) \leq -1 + x_2^F(N)$*);*

(ii) the first-period order of the followers does not depend on their signal s and, without loss of generality, we may take $x_1^F(s) = 0$ *for all s.*

Part (i) follows from the fact that followers infer the value of v from the first-period order flow. Since a continuum of infinitesimal followers with large overall mass Δ will trade aggressively on this information, the information of the followers will be revealed to the market in period 2 and the price will equal v, eliminating all second-period profits for the insider. Part (ii) shows that the only way followers can earn nonnegative expected profits is by not revealing their signal s to the market makers via their first-period trades. In particular, whenever there exists a nonbluffing equilibrium where followers trade nonzero amounts in the first period, there exists another nonbluffing equilibrium where followers do not trade at all in the first period.

We turn now to characterizing the informed leader's behavior in the first period in a candidate nonbluffing equilibrium. From Lemma 1(ii), the first-period net order flow is equal to the informed leader's order, without loss of generality. Furthermore, since noise is distributed without a mass, the informed trader must play a mixed strategy without any mass in such a candidate equilibrium, putting weight on trade sizes that are bounded away from zero and earning strictly positive profits. Lemma 2 makes this precise.

LEMMA 2: *In any candidate nonbluffing equilibrium, the informed leader plays an atomless mixed strategy in the first period, summarized by the densities*

$$q(x|1) = 2\,\frac{1-\alpha}{\alpha}\,g(x)\left[\frac{x}{k} - 1\right], x \in (k, 1]$$

and

$$q(x|-1) = 2\,\frac{1-\alpha}{\alpha}\,g(x)\left[-\frac{x}{k} - 1\right],$$

$$x \in [-1, -k)$$

where $k \in (0,1)$ *satisfies the identity* $\int_k^1 q(x|1)\,dx = 1$ *and equals the informed leader's total trading profit.*

Using Lemmas 1 and 2, we show now that a nonbluffing equilibrium cannot exist. Suppose, to the contrary, that it does and consider the case where $v = 1$ and $s = I$. We construct a profitable deviation strategy for the leader.

Suppose that he first sells, i.e., trades $x_1^I = -k - \varepsilon$, where $\varepsilon > 0$ and small. Since for ε small $q(-k - \varepsilon|-1)$ must be small from Lemma 2, the market makers assign a small likelihood to the state of the world $s = I$ and $v = -1$ given such an order. Since in a nonbluffing equilibrium they attach no likelihood to the state that such a sell order comes from the informed leader who knows $v = 1$, the price $p(-k - \varepsilon)$ must then be close to zero, the expected value of the asset

[4] The proofs of all lemmas are posted on the AER Web site (http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.2.280).

given that there is no informed trading in the market. Therefore, the leader's first-period loss from the sell order is approximately k.

Since followers know that such a sell order is coming from an informed leader, upon observing the first-period sell order they will conclude that $v = -1$ in this nonbluffing equilibrium. They will then trade $x_2^F(I) \leq -1 + x_2^F(N)$ in the second period, by Lemma 1(i). If the leader submits an order $x_2^I = 1 + x_2^F(N) - x_2^F(I) > 0$ in the second period, the second period net order flow will be $x_2^I + x_2^F(I) = 1 + x_2^F(N)$.

From the perspective of the market makers, a second-period net order flow of $1 + x_2^F(N)$ following a first-period sell is consistent with $s = N$ since x_2^N may equal one. However, it is inconsistent with the state $s = I$ and $v = -1$ that the market makers attach positive probability to after observing the first-period sell. Indeed, since the informed leader does not buy in the second period when $v = -1$, the maximum second-period net order flow consistent with the state $s = I$ and $v = -1$ is at most $x_2^F(I) \leq -1 + x_2^F(N) < 1 + x_2^F(N)$. As a result, the market makers will conclude that $s = N$ and all second-period orders will execute at a price equal to zero, the expected value of the asset when $s = N$.

The overall profit for the informed leader from this deviation strategy is approximately $1 + x_2^F(N) - x_2^F(I) - k$, strictly greater than the nonbluffing profit k, since $x_2^F(N) - x_2^F(I) \geq 1 > k$ from Lemmas 1(i) and 2. Since the leader has a strictly profitable deviation whenever the market makers and followers expect him not to bluff, every equilibrium must then involve bluffing, yielding our main result.[5]

PROPOSITION 1: *For any $\alpha \in (0, 1)$, the insider must bluff in every equilibrium.*

The existence of followers creates incentives for the leader to bluff. Indeed, as Lemma 1 shows, if the leader does not bluff, then followers compete away all profits in the second period. By bluffing, the leader creates noise in the inference problem for followers (and the market), ensuring that he trades at favorable prices.

In order to see the impact of followers on bluffing, it is useful to consider the model in the absence of followers. In this case, as α becomes small, the insider's expected profit in a nonbluffing (candidate) equilibrium is close to two, since he is able to trade one unit each period at a price close to zero. On the other hand, the insider's expected profit converges to zero as α becomes large, and the market makers accurately infer the insider's information from the order flow. Since bluffing involves a loss in one period, the expected profits due to any bluffing strategy is bounded above by one. For bluffing to occur in every equilibrium in the absence of followers, we then need α to be sufficiently large. In such cases, the market makers attach a high probability to trades being driven by information resulting in low profits for the insider. By bluffing, the insider adds noise to the price formation process, along the lines of Chakraborty and Yılmaz (2004a, b). In this paper we show that the incentives to bluff are heightened when there is a large number of rational traders with information that is nested between that of the insider and the market. Bluffing necessarily occurs for any α.

III. Conclusion

We consider a simple dynamic model of trading by an informed leader in a model where a large number of rational traders compete and reduce the ability of the leader to hide the information content of his trades. For precisely this reason, the leader makes unprofitable bluffing trades early on and later profitably unwinds his position.

This illustrative model may be helpful in analyzing related environments. For instance, a nested information structure of the sort considered in this paper may also give rise to bluffing by a strategic trader who does not have any information about fundamentals. Such an uninformed trader may find it optimal to mimic an informed trader and move prices away from fundamentals, profiting later by reversing his initial position at more favorable prices.

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[5] The existence of an equilibrium in our game follows from Taesung Kim and Nicholas Yannelis (1997).

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