Bookbuilding[*]

Archishman Chakraborty[†], Michael S. Pagano[‡], Robert A. Schwartz[§]

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Abstract

As participants reveal their orders to a market, prices are discovered and trading volume (quantity) is "found" (i.e. books get deeper). We refer to this as "bookbuilding," a non-trivial process that depends on a market's rules of order handling and order information disclosure. The paper shows that, for large participants, uncertainty about the profile of other traders in the market distorts trading volume and reduces the gains from trading. These inefficiencies are mitigated when participants can place multiple (scaled) orders and when submitted orders are displayed in an open book. In effect, the open book permits participants to engage in non-binding communication, thus facilitating efficient bookbuilding. These findings are robust to the inclusion of order submission costs and the ability to costlessly withdraw previously placed orders.

Key words: bookbuilding, call auctions, market microstructure

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[†] Department of Economics and Finance, Baruch College, CUNY, arch_chakraborty@baruch.cuny.edu.

[‡] Department of Finance, College of Commerce and Finance, Villanova University, Michael.Pagano@villanova.edu.

[§] Department of Economics and Finance, Baruch College, CUNY, robert_schwartz@baruch.cuny.edu.

1 Introduction

A trade cannot be made in a securities market until at least two participants step forth and reveal that they are willing to buy or to sell. The revelation may be to an information system (such as Autex), to an intermediary (broker or dealer), or to a marketplace. The marketplace may be a transparent limit order book facility (order-driven), a closed book (non-transparent) order-driven facility, or a dealer market (quote-driven). The revelation may or may not be anonymous, and it may or may not contain information about the size of the order or the price at which the participant is willing to trade. To get a market started, order-driven systems typically batch orders together during the pre-opening period, and begin trading with a single-price call auction. In a quote-driven environment, market makers who are responsible for posting two-sided quotes typically open with values that reflect the pre-opening activity that they have observed.[1]

Two major market centers that had previously been predominantly quote-driven (Nasdaq and the London Stock Exchange) now open (and also close) their markets with fully electronic call auctions. But regardless of the trading environment, as orders are revealed, books get deeper, volume builds, and prices are discovered. We refer to this forming of a market as bookbuilding. Bookbuilding is widely recognized as a primary market activity in the underwriting of securities (such as IPOs) but bookbuilding is also important on a daily basis to trading securities on a secondary market. In this paper, we examine the dynamics of bookbuilding within a secondary market and show that, in many cases, the book typically does not easily build to the most efficient outcome (in terms of total welfare, price discovery, and trading volume).

Bookbuilding is the process by which a market attempts to discover price and quantity. By "price discovery," we are referring to how a share value might be found that best reflects a broad market's desire to hold shares of a stock. By "quantity discovery" (a relatively unrecognized function of a marketplace), we are referring to how orders on both sides of the market may be revealed and turned into trades (see Paroush, Schwartz and Wolf, 2005). Both price and quantity discovery can start with a pre-opening, order accumulation procedure that culminates with a call auction, and then continues through the early stages of the continuous market (e.g., the first half-hour of the trading day). Price discovery is a complex process in an environment characterized by either asymmetric information and/or investor expectations that are divergent. Quantity discovery, on the other hand, is a complex process that operates in an environment where investors come to the market to buy and sell shares because their security valuations differ (i.e., they have divergent expectations) but where participants do not fully reveal their trading desires because they fear their

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[1] In the U.S., prior to the introduction of Nasdaq's Opening Cross in Fall 2004, the appreciable price volatility that had attended Nasdaq openings (at the time, a predominantly quote driven market) attracted the attention of the Securities and Exchange Commission. See the letter from Arthur Levitt, then Chairman of the SEC, to Frank Zarb, then CEO of Nasdaq (2000).

trades will be executed at inferior prices due to adverse market impact (including that attributable to front running) and/or poor market timing. Thus, the complexity of quantity discovery is an inherent and critical component of the bookbuilding problem.

We analyze bookbuilding within the context of an opening call auction. The call is both analytically convenient for our purposes and institutionally realistic. As noted, calls are widely used to open markets at the start of a trading day, and both Nasdaq and the major European bourses all open and close their markets with fully electronic calls. Symptomatic of the complexities of bookbuilding, the opening calls attract predominantly retail order flow, while institutional participation is appreciably larger in the closing calls.[2] We attribute this difference to the greater ease of bookbuilding at the closing call (prices have already been established and the average trade price for the day is known). If a substantial book is built, the call auction should be a highly efficient venue for institutional customers. The procedure, because it focuses liquidity at specific points in time, mitigates the market impact of relatively large orders. Additionally, the auction is attractive because, by basing price discovery on multiple orders, calls enable participants to trade at 'validated' prices.[3] However, encouraging the submission of the first orders to open an exchange market can be difficult if traders risk having their order quantities and prices revealed to other participants without their orders executing fully at reasonable prices. Adverse price changes due to market impact can result.

In light of the special difficulties involved in opening a market, alternative opening procedures have been considered in the microstructure literature.[4] Recognizing the desires of informed traders to masquerade as 'sheep', order bunching at and near the open has been analyzed.[5] These studies have not focused on bookbuilding per se, however.

Getting the first few institutional customers to step forward and disclose their intentions is not a trivial matter, and bookbuilding is not a simple process. Revelation of a trading intention can be costly to a participant. The very existence of an order is information that can be used to the detriment of the trader who revealed it. A posted order to buy or to sell is a 'free' option extended to others. Orders large enough to move market prices are commonly traded ahead of (also referred to as front running). And orders submitted in the early stages of bookbuilding are particularly

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[2] Pagano and Schwartz (2003) found that introduction of a closing call in the Paris market increased the accuracy of price determination at the close, attracted more institutional-sized orders than the opening call, and had a positive spillover effect on the quality of price determination at next day openings.

[3] By validation, we mean that enough other trades have occurred at, or in the near vicinity, of a price to establish the credibility of that price as a reasonable reflection of the broader market's demand to hold shares of a stock. A stock's closing price and volume weighted average price for a trading day (VWAP) are commonly used for this purpose. The price established in a call auction would be superior to VWAP as a benchmark provided that the book is sufficiently built by the time of the call.

[4] See, for instance, Domowitz (1993) and Madhavan and Domowitz (2001).

[5] See, for instance, Admati and Pfleiderer (1988).

prone to being mispriced in light of underlying market conditions.

The problems involved in bookbuilding may not be apparent in a predominantly retail environment because relatively atomistic participants typically think and act as price takers. This is not the case with institutional players. Large traders know that their actions will impact market prices. And, given the velocity of intra-day price movements that characterize the equity markets, traders are also afraid that, ex post, it could become apparent that they have traded at inappropriate, 'non-validated' prices.

For these reasons, unrevealed institutional propensities to trade may represent a substantial latent demand, even when books appear to have reasonable depth for retail customers (for further discussion and empirical analysis see Sarkar, Schwartz and Wolf, 2005). The especially heavy first half-hour volume immediately following the open that characterizes most major markets suggests that institutions hold their orders back from a market's opening, waiting for prices to be established at the open before stepping forward in the first half-hour or so of the continuous market. In the heavily institutional London market, volume is relatively low through the first half-hour after the call openings, and it builds monotonically as the day progresses; this is further indication of institutional reluctance to participate actively in price discovery.

We analyze bookbuilding with regard to institutional (large trader) order flow. We assume an environment characterized by many atomistic retail customers and a small number of large customers. For purposes of analytic tractability, we model an environment where customer orders are submitted to a call auction. The large participants, when formulating their own trading strategies, use the small orders (which are cumulated into buy and sell order functions) as a reference point. If no large player is present in the market, the intersection of the aggregated retail order functions establishes the clearing price and trading volume. A large trader's order is big enough to impact the clearing price if it is not offset sufficiently by another big, contra-side order, and it may also fail to execute fully (if at all). Consequently, large traders may have ex post regret, not necessarily because new information possessed by an informed trader has subsequently been revealed, but because their own trading strategies have resulted in their orders executing at poor and inappropriate prices.

Institutional traders' fear of trading at inappropriate prices may lead to the book not getting fully built. However, the value of the call auction depends on its ability to 'get the book built.' Consequently, suboptimal participation in bookbuilding can result in inefficient price and quantity (volume) discovery for the broader market. We model this market inefficiency as the outcome of a strategic game played by market participants. Our market structure is characterized by the following.

We examine an environment where institutional participants have different private valuations of the assets. This could arise if they have idiosyncratic motives for trading related to portfolio rebalancing or they have different models of the world. With private values, a large buyer and a large seller may wish to trade with each other even if they each know each other's motive for trading.[6] In our model, there may be either zero, one or two large traders in the market. Neither large trader knows if the other is in the market and, if so, the side of the market the other trader is on. Consequently, each faces uncertainty about the price at which his or her own order will execute, and whether it will execute at all. We show that even in the absence of asymmetric information about asset value, this uncertainty has serious repercussions for the bookbuilding process. Traders submit orders that are inefficient — too small when the market is two–sided (with the large traders on opposite sides of the market), and too large when the market is one–sided (with both large traders on the same side of the market and in direct competition with each other for the retail order flow).

To solve the problem of inefficient order submission, we further consider the bookbuilding problem when each large trader can submit multiple orders. We show that even a closed book auction (where no one observes anything about the other orders submitted but where multiple limit orders can be submitted) helps in building the book and reducing inefficiency. Such a system allows the large traders to hedge against uncertainty by breaking up their orders and submitting different parts at different limit prices. This is known as 'scaling' an order. When the market is two–sided and balanced, each trader executes all of his/her orders without adverse price impact. Otherwise, each trader executes only the smaller tranches submitted at more aggressive limit prices, without undue price impact. While submitting multiple orders in a closed book auction is always better from the perspective of the large traders, the magnitude of efficiency gains is quite sensitive to the existence of arbitrarily small order submission costs. These costs could be thought of as ticket charges or any other friction that makes it more expensive to submit two or more smaller orders rather than submit the same number of shares with one large order. In the presence of such small costs, we show that traders will end up trading inefficiently. Thus, total volume and trading gains will also be smaller, compared to a world without such costs. This occurs because, in a closed book environment, each trader cannot observe other traders's actions and therefore even a small order submission charge leads each trader to seek to economize on the number of orders (e.g., by submitting two larger orders rather than three smaller orders), giving rise to a strategic incentive for each trader to "shade" the total size of the order, leading ultimately to an unravelling of the efficient outcome.

Compared to a closed book auction, an open book auction is an environment where traders submit multiple orders sequentially upon observing orders that have already been submitted and may yet be cancelled. We show that such a format is at least as efficient as a closed book from

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[6] See Paroush, Schwartz and Wolf (2005) for an analysis of price determination in a market characterized by divergent expectations, and Sarkar, Schwartz and Wolf (2005) for evidence that the NYSE and Nasdaq markets are commonly two-sided.

the perspective of the large traders. In fact, in the presence of small order submission costs, an open book is strictly better than a closed book. In the open book auction, all traders initially submit small orders, building up their orders only if they see that contra-side orders have been placed. In effect, the first round of orders serves as a non-binding communication stage that allows, in equilibrium, each large trader to infer the market's composition. Thus, the open book builds gradually. However, it builds to an efficient level. In each state of the world, the quantities and prices that the large traders obtain are identical to the best they could hope to obtain in a world where the market configuration is common knowledge. Interestingly, these results for an open book call auction obtain even though no one is committed to their earlier orders (i.e., even when traders are allowed to 'bluff' with orders that they first place and then subsequently withdraw at no cost). Thus, one of our main findings is that greater disclosure can lead to more efficient call auctions from the perspective of volume, trading profits, liquidity provision, price discovery, and overall welfare.

The remainder of the paper is organized as follows. In Section 1.1 we review the relevant literature. Section 2 introduces the basic model. In Section 3 we consider the benchmark case of a two–sided market under uncertainty and show that efficiency is by no means guaranteed in such an environment, and that in fact it cannot obtain under even slight uncertainty about market composition. In Section 4 we consider the bookbuilding process in greater detail, with a closed book auction in Section 4.1 and an open book auction in Section 4.2. Section 5 discusses our results, possible extensions, and concludes while the Appendix contains all the proofs.

1.1 Review of the Literature

The discussion above suggests that market openings present a special challenge, and that a properly designed call auction system can be an efficient response to this challenge. Our model's findings are consistent with a number of related studies that have focused on different aspects of market transparency and trader anonymity.[7] As noted in O'Hara (1999), the issues of pre-trade market transparency and trader anonymity (or, alternatively, trader identity) are important factors that determine the overall performance of a financial market.

Our analysis is related to work pertaining to pre-announced commitments to trade (commonly referred to as 'sunshine trading'), as detailed in Grossman (1988) and Admati and Pfleiderer (1991). In a model of trading under asymmetric information, Grossman (1988) suggests that uninformed traders can benefit from pre-announcing their intentions to trade (in terms of a pre-committed price and quantity) because such action reduces the adverse selection problem by providing all traders with more information. Admati and Pfleiderer find that uninformed traders who pre-

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[7] This section draws upon the excellent discussion of market transparency, trader anonymity, and pre-announced trading found in O'Hara (1999), as well as the papers cited below.

announce their trades face lower expected trading costs due to a reduction in the variance of liquidity (or 'noise') trading and the increased informativeness of observed prices. Interestingly, in the adverse selection literature, uninformed traders who do not pre-announce face higher expected trading costs, thus encouraging all uninformed traders to engage in sunshine trading. Similarly, Roell (1990) finds that the uninformed trader has the incentive to pre-announce his or her intentions because a more-informed dual-capacity broker has the ability (and incentive) to quote more aggressively than the market maker and provide a better price to the uninformed trader.[8] All of these models focus on the adverse selection problem. Our paper relates to the above literature by showing that 'sunshine trades' can be sub-optimal even when no adverse selection problem exists. That is, even when there are no "informed traders," information about a large order can itself be used against the large order submitter so that other traders (both large and small) might profit.

In contrast to the above models, other related work does not provide as clear-cut an answer as to how market transparency can affect the performance of a financial market. For example, Madhavan (1992) examines the transparency of the size and direction of order flow in continuous, quote-driven versus batch-oriented (call auction) markets. He finds that the call market is a viable market design in equilibrium but that the aggregation of orders in a batch-trading system "reduces the transparency of the market, and hence the viability of the batch-trading market is purchased at the expense of market transparency." In addition, Pagano and Roell (1996) examine a microstructure model under adverse selection and show that uninformed traders might not be better off with a call auction when compared to a quote-driven dealer market or a continuous auction system.

Although there are no direct empirical tests of bookbuilding and the effects of the relative openness of call auction markets, the experimental results of McCabe, Rassenti, and Smith (1992) are broadly supportive of the idea that open call auctions can be an efficient market design. The authors find that even though a sealed bid, single-price auction is less efficient than a continuous double auction system, the call auction can be superior to the continuous market once the 'sealed bid' feature is replaced with a real-time, open display of the call's bids and offers. As McCabe et al. (1992) note, the improvement in the call market's efficiency is due in part to these open call markets giving traders real-time information that reduces transaction uncertainty and allows them to adjust their decisions. In other words, they found that transparency facilitates bookbuilding. This is consistent with our results and suggest that the degree of openness of the call auction is an important determinant of the relative efficiency of call versus continuous market designs (in terms of total surplus as well as reduced price volatility).

In addition to the experimental results of McCabe et. al. (1992), Boehmer, Saar, and Yu

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[8] Dual-capacity brokers are brokers that can execute trades for their own account as well as for their clients' accounts.

(2005) perform an empirical analysis of the introduction of the NYSE's OpenBook service (which provides increased pre-trade transparency of the limit order book for NYSE-listed stocks). They find that traders attempt to control their exposure to the limit order book by submitting smaller orders and canceling orders faster. In addition, the participation rates of NYSE specialists and their contribution to the depth of the book decreases after OpenBook was introduced. Despite these changes in trading behavior, the authors find that liquidity increased somewhat via an improvement in informational efficiency (i.e., better price discovery) and reduced market impact costs. The empirical results of Boehmer et al. (2005) suggest that the degree of openness of a limit order book influences trader activity and can lead to greater liquidity for all market participants.

Our model is also related to the literature on bargaining under two-sided asymmetric information (Myerson and Satterthwaite, 1982) and to the related literature on double auctions. Myerson and Satterthwaite (1982) demonstrate the impossibility of ex–post efficient trading when the possible gains from trade are not common knowledge. In our setting, due to the presence of retail customers, the gains from trade are common knowledge among the large traders. But achieving efficient levels of trading is still not trivial within the constraints of institutionally realistic call auction formats. We show, when the call auction has an open book format, that efficiency can be robustly achieved by using the earlier stages of the bookbuilding process as a non–binding communication stage. Starting with the work of Chatterjee and Samuelson (1983) and Wilson (1985), the literature on double auctions also focuses on the efficiency properties of call auctions (see, e.g., Klemperer (2000) and the references therein). The present paper differs from this literature due to the presence of non–strategic retail investors, and in allowing for multiple divisible orders to be submitted by each strategic player.

2 The Basic Model

Our basic model is constructed as follows. We assume a market that consists of a large number of small retail traders and two large institutional traders. The retail traders are atomistic and their orders are aggregated into retail trader buy and sell order functions. The aggregated retail buy orders at a price p is given by

$$d(p) = a(1 - p) \tag{1}$$

For ease of exposition, we refer to this as retail demand. The aggregated retail sell orders at a price p is given by

$$s(p) = ap, \tag{2}$$

which we refer to as retail supply. Note that retail demand intersects retail supply at a price equal to $\frac{1}{2}$ and a volume equal to $\frac{a}{2}$. The parameter $a \in (0,1)$ measures total retail activity.[9] As will be clear below, inclusion of retail customers is important to seed bookbuilding. Retail customers, being atomistic and unconcerned that their small orders will have adverse price impacts, are willing to submit their orders relatively early in the bookbuilding period. In a call auction environment, such early order placers are important catalysts for liquidity formation. Furthermore, their presence assures gains from trade for each large trader.

Each large trader may be a buyer (with a value, or reservation price, for the asset equal to 1) or a seller (with a value, or reservation price, for the asset equal to 0).[10] We also allow for the possibility that a large trader may not be present in the market. Let $t_i \in \{t_b, t_n, t_s\}$ denote the type of trader $i = 1, 2$. Thus, $t_i = t_b$ denotes that trader i is a buyer, $t_i = t_s$ denotes that he or she is a seller, while $t_i = t_n$ denotes that the trader is not present in the market. Let $t = (t_1, t_2)$ denote a type profile or market configuration and let $\phi(t) \geq 0$ denote the probability with which type profile t occurs, with $\sum_t \phi(t) = 1$. We assume that ϕ is symmetric, i.e., $\phi(t_1, t_2) = \phi(t_2, t_1)$ for all $t_1, t_2 \in \{t_b, t_n, t_s\}$. Each trader knows his or her own type but is uncertain about the type of the other trader. Notice that the valuation of each trader depends only on his/her own type. Thus, we consider a private value environment. However, we allow arbitrary correlation patterns in the joint distribution of types ϕ. It can be independent across trader types, positively correlated (implying that the market is likely to be one–sided with two large traders on the same side), or negatively correlated (implying that the market is likely to be two–sided with large traders on opposite sides).[11] Let $\phi(t_i|t_j)$ denote the probability that trader j assigns to trader i being of type t_i given that trader j is of type t_j.

The large traders both know the retail trader order flow, and each wants to trade at most one unit of the asset (each retail trader trades an infinitesimally small amount). They have to decide on their order–submission strategies. In the basic model, each trader i will simultaneously place either one limit buy order (b_i, p_i^b) stating a willingness to buy up to $b_i \in [0,1]$ units of the asset at a price no more than p_i^b; or one limit sell order (s_i, p_i^s) stating a willingness to sell up to $s_i \in [0,1]$ units of the asset at a price no less than p_i^s. In Section 4 we allow each trader to submit multiple limit orders.

The aggregated (total) buy order function at any price p, denoted by $D(p)$, is equal to the sum of retail buy orders $d(p)$ and the buy orders submitted by the large traders (if any). Similarly, the

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[9] In what follows we will exploit the linearity and symmetry of the retail buy and sell functions, but nothing depends on their precise form.

[10] All insights are robust to allowing more than two traders and to assigning reservation values v_b to the buyer and v_s to the seller with $1 \geq v_b > v_s \geq 0$.

[11] Positively correlated trader types would be consistent with the observed "herding" behavior of institutional investors while negatively correlated trader types could be consistent with "contrarian" investment behavior.

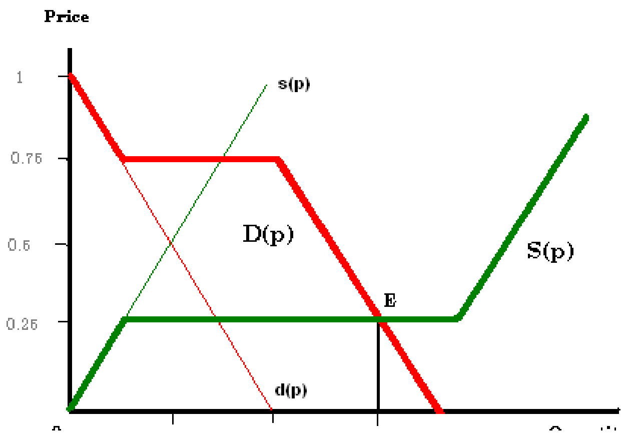

Figure 1: A Call Auction

total sell order function at any price p, denoted by $S(p)$, is the sum of retail sell orders $s(p)$ and sell orders from the large traders. While retail buys and sells vary continuously with p, the large traders submit discontinuous buy and sell order functions (i.e., step functions). As a result, there might not exist a market–clearing price at which the aggregated desire to buy *exactly* equals the aggregated desire to sell. Define the market–clearing price p^* as the highest price at which there is (weak) excess demand:

$$p^* = \sup_p \{p \in [0,1] \mid D(p) \geq S(p)\} \tag{3}$$

Figure 1 depicts a hypothetical market with orders submitted by a large buyer and a large seller that are not necessarily equilibrium orders. Total quantity is measured on the horizontal axis and price on the vertical axis, with the downward sloping curve denoting total buy orders, $D(p)$, and the upward sloping curve denoting total sell orders, $S(p)$.[12] Each horizontal shift in the total buy/sell order curve is equal to the size of an order submitted by one of the large traders and the shift occurs at the limit price of the large order. The intersection of the total buy and sell orders determines the market clearing price p^*. In the situation depicted in Figure 1, the large buyer has submitted an order of size $\frac{3}{8}$ at a limit price of $\frac{3}{4}$ whereas the large seller has submitted an order of size 1 at a limit price of $\frac{1}{4}$. The auction clears at a price of $p^* = \frac{1}{4}$ for a total volume of $\frac{3}{4}$ and the sell side has to be rationed (i.e., the large seller's order of size 1 is not completely filled).

In general, when $D(p^*) \neq S(p^*)$, the larger side of the market gets rationed. We assume that orders have price priority, i.e., all traders who have submitted buy orders at limit prices strictly greater than p^*, and all traders who have submitted sell orders at limit prices strictly less than p^*

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[12] In all our figures, we set $a = \frac{1}{2}$.

get their entire orders executed. However, if at p^*, $D(p^*) < S(p^*)$, then each large trader who has submitted a sell order at a limit price equal to p^* gets rationed. Such rationing is pro rata, i.e., each such trader gets an equal percentage or absolute share of the supply that is remaining after all sell orders submitted at limit prices strictly lower than p^* have been filled from the available supply. Symmetric remarks apply for the case where $D(p^*) > S(p^*)$. Notice that retail traders, being infinitesimal, never get rationed.[13]

Let b_i^* (respectively, s_i^*) be the actual amount that large trader i gets to trade when he or she has submitted a buy (sell) order. The gains from submitting a buy order for trader i of type t_b is $(1 - p^*)b_i^*$, which is the reservation value less the price paid times the number of shares actually traded. Similarly, the gains from submitting a sell order for trader i of type t_s is $p^* s_i^*$. Let the volume that the two sides of the market trade be denoted by

$$V = \min[D(p^*), S(p^*)]. \tag{4}$$

Further, define the retail buyer surplus as equal to the area under the retail buy order function that is above the market clearing price p^*, and similarly define the retail seller surplus as the area above the retail sell order function that is below p^*. Our notion of total surplus (or social welfare, W) is equal to the sum of the retail buyer and seller surplus as well as the profits or trading gains of the large traders. It is straightforward to verify that

$$W = \frac{1}{2}[V + \pi_1 + \pi_2] \tag{5}$$

where π_i is the payoff of large trader $i = 1, 2$.

In submitting their orders, each trader faces uncertainty about the type (and hence the order) of the other trader. A buyer knows that, if the other trader is a seller, then each trader can potentially execute a large order without adverse market (price) impact. On the other hand, if the other trader is on the same side (or is absent) then there will be adverse price impact and the entire submitted order might not get executed. Our objective is to identify the set of submission strategies where all traders have no incentive to deviate from their preferred order quantities and prices. Such a Nash equilibrium pair of orders submitted in this game of large order submission will reflect the trade-offs associated with order execution uncertainty and the benefits of trading.

One of our main contributions is to show that even a small amount of such uncertainty can lead to a dramatic reduction in equilibrium trading volume (and thus welfare), unless the call auction is suitably designed. This is so even though we abstract from adverse selection issues that can arise when each trader, being uncertain about his/her own valuation, is influenced by the other trader's reservation price. It is well known that such adverse selection can lead to inefficient amounts

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[13] Price priority followed by pro-rata rationing is not the only possible rule. Among other possible rules are uniform rationing or random allocations.

of trading, especially when the existence of gains from trade are not common knowledge (see, Myerson and Satterthwaite, 1982). In contrast, due to the presence of retail traders, the existence of gains from trade are common knowledge in our model, and the valuations of the asset by the two traders are private.[14] We show that, in such a situation, 'strategic' uncertainty about the size of orders on the other side of the market may lead each trader to trade inefficiently small amounts (also referred to as 'order shading'), unless the auction format is such that it allows traders to credibly communicate and accurately ascertain the composition of the market.

3 The Two–Sided Market: A Benchmark

To illustrate the role of strategic uncertainty and order shading even when there is no intrinsic uncertainty about market composition, we begin the analysis by considering the case where it is common knowledge that the market is two–sided, i.e., $\phi(t_b|t_s) = \phi(t_s|t_b) = 1$. In other words, each large trader knows with complete certainty that there is another large trader on the other side of the market who is willing, in principle, to trade with him or her without either trader having an adverse impact on prices. We show that even in this scenario where there is no intrinsic uncertainty regarding the presence of a counter–party willing to trade on the other side, strategic uncertainty regarding the actual order the other trader will submit may lead each trader to submit an inefficiently small order size in equilibrium.

To see the strategic issues at play here, consider the case where the buyer submits an order at a limit price of 1 while the seller submits an offsetting order at a limit price of zero. The market clearing price then equals $\frac{1}{2}$. However, this is not an equilibrium. For such a high order size from the buyer, the elasticity of the net buy order function that the seller faces is less than 1. Consequently, if the seller anticipates the buyer's order, then by submitting a smaller order he or she can obtain a better limit price for higher trading gains. Such order shading hurts the buyer and will lead to the submission of a smaller buy order as well. This order shading, in turn, causes trading profits and volume to be less than optimal.

Note, however, that if the buyer submitted the same order at a lower limit price of $\frac{1}{2}$, the seller could not have gained by shading the sell order. By doing so the seller would only have ended up trading a smaller amount at the same price. This underlines the crucial strategic role that limit prices will play in our model, especially by creating the possibility of multiple equilibria. Our first result fully characterizes the Nash equilibria of the single-order, two–sided market.

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[14] Our model can also be applied to non-financial markets which have large traders who can influence the price of the good or service that is being traded. For example, our model might be useful in understanding the market for trading pollution rights between large industrial firms.

Proposition 1 *In the two-sided market, a pair of orders* (b, p^b) *and* (s, p^s) *is a Nash equilibrium if and only if both trader's execute their entire orders, and prices satisfy*

$$p^b \geq p^* = \frac{1}{2} + \frac{1}{2a}(b - s) \geq p^s, \tag{6}$$

and orders satisfy,

$$b \geq \frac{a}{2} + \frac{s}{2}, \tag{7}$$

with equality if $p^s < p^*$; *and*

$$s \geq \frac{a}{2} + \frac{b}{2}, \tag{8}$$

with equality if $p^b > p^*$.

Proof. See the Appendix.

Proposition 1 states that both the buyer and the seller will manage to get their entire order executed in any equilibrium so that, from (4), it immediately follows that the market-clearing price is

$$p^* = \frac{1}{2} + \frac{1}{2a}(b - s).$$

Inequalities (7) and (8) state that, in any equilibrium, the order sizes are large enough so that the marginal profit from submitting an even larger order is not positive — the buyer does not want to 'move up' the supply function by submitting a larger order at a higher limit price, and the seller does not want to 'move down' the demand function by submitting a larger order at a lower limit price. In other words, in any equilibrium, each trader must be somewhere within the inelastic part of the (net) demand/supply function that he/she faces.

Proposition 1 does not further pin down the possible equilibrium outcomes, even if one restricts attention to symmetric equilibria. To obtain sharper predictions among the equilibria, one needs to analyze further the limit prices used by the two traders. As one example, consider the case where each trader posts a limit price away from the market clearing price, i.e., $p^s < p^* < p^b$. From Proposition 1, we see that, in such a case, (7) and (8) must hold with equality, i.e., the marginal gain from submitting an order of a slightly different size must equal zero. This implies that $b = s = \frac{1}{2}$ and, from (6), $p^* = \frac{1}{2}$. Such an equilibrium is the *most inefficient symmetric* equilibrium in terms of total welfare.[15] Figure 2 depicts such an equilibrium where $p^b = \frac{3}{4}$ and $p^s = \frac{1}{4}$.

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[15] Throughout what follows we will use the term symmetric equilibrium to denote the case where the two traders submit offsetting orders (i.e., one where $b = s$) whenever the market is two–sided. We refer to the above result depicted in Figure 2 as the most inefficient symmetric outcome because both large traderrs shade their orders to the maximum extent possible and force V and W to their lowest feasible levels consistent with equilibrium.

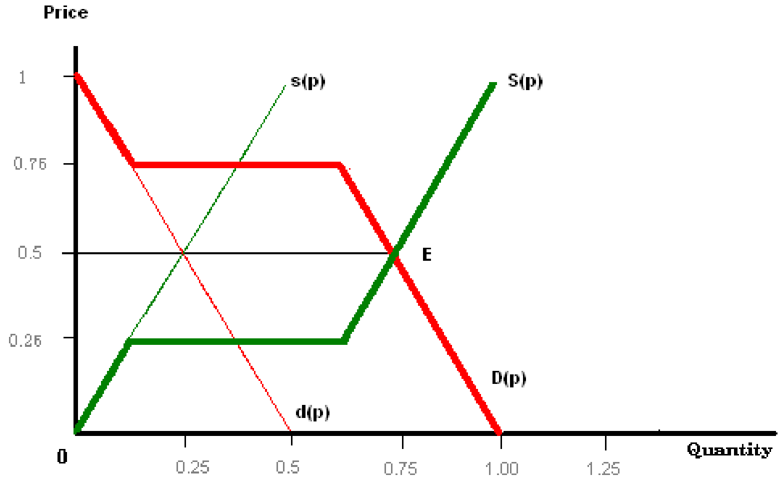

Figure 2: Inefficient symmetric equilibrium

At the other extreme is the *most efficient symmetric* equilibrium where $p^b = p^s = p^* = \frac{1}{2}$ and $b = s = 1$. In this equilibrium, each trader gets to trade his/her entire desired amount without any market impact. The traders achieve this by choosing limit prices in such a manner that neither party can gain by shading the order size — for if either does so he/she only ends up trading a smaller amount at the same price of $\frac{1}{2}$. Figure 3 depicts such an equilibrium. The difference in the outcome of the most inefficient and the most efficient symmetric equilibria, depicted in Figures 2 and 3 respectively, are stark and economically significant in terms of trading volume, total surplus, and the level of participation by large traders in the marketplace. For example, when $a = 1/2$, trading volume, V, is 40% less in the most inefficient outcome (relative to the most efficient equilibrium), at 0.75 versus 1.25, and total surplus W is 44% lower (at 5/8 versus 9/8). In addition, the large traders' share of total trading volume is nearly 17% lower (at 67% versus 80% of total trading volume).

Observe that, for the efficient outcome to be achieved, it is necessary that each trader chooses a limit price that equals the ultimate market clearing price. We now argue that such a strategically aggressive use of limit prices is unlikely to occur when each trader faces even a slight uncertainty about the existence of a counter–party.

Notice from Figure 3 that if the buyer attaches a slight probability to the seller not being present in the market, since the buyer's own limit price $p^b = \frac{1}{2}$, he or she does not expect to execute any part of his or her order when the seller is absent and the total supply is equal to the retail supply. Can the buyer do better when facing such an uncertainty? Given the seller's order $(1, \frac{1}{2})$, the buyer would do strictly better to submit an order $(1, p^b)$ for some $p^b > \frac{1}{2}$. The effect of

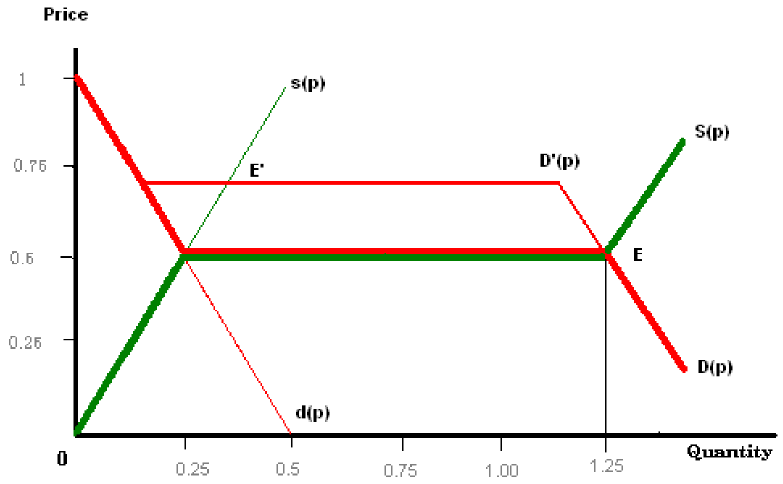

Figure 3: The efficient equilibrium & uncertainty

doing so on aggregate demand is depicted by the curve $D'(p)$ in Figure 3. By doing so, the buyer will still get to trade 1 unit when the seller is present, at a price of $\frac{1}{2}$. However, when the seller is absent, the buyer will execute some part of the order at a price at most equal to p^b (depicted by the point E′ in the figure). As a result, the pair of orders depicted in Figure 3 cannot be an equilibrium for any slight uncertainty about the type of the other trader — the buyer would like to set a higher limit price, and the seller a lower one. But, following the logic of Proposition 1, this in turn can create incentives for each trader to shade his or her order so that they both end up trading smaller amounts for lower trading gains. In other words, the most inefficient symmetric equilibrium outcome depicted in Figure 2 will be the one that will obtain under such uncertainty.

We conclude that a call auction is unlikely to achieve efficient amounts of trading even when large traders are only slightly uncertain that the market is balanced and two–sided. This conclusion, however, depends crucially on the restriction that each large trader can submit only one order. As we show in the next section, efficiency can be regained when each large trader is allowed to submit multiple orders at potentially different limit prices, even under arbitrary degrees of uncertainty.

4 Building the Book: Multiple Orders

We now consider a model where each trader faces uncertainty about market composition and can submit multiple orders so as to ‘hedge’ his/her bets against this uncertainty. Denote by (b_k, p_k^b) the k^{th} buy order and by (s_k, p_k^s) the k^{th} sell order submitted by a large trader. The basic idea is that each large trader chooses the limit prices such that not all the orders trade in unfavorable states of the world where the market is either one–sided (i.e., both large traders are on the same side of the market) or there is only one large trader in the market. At the same time, order submission strategies are such that, when the market is two–sided, each large trader successfully executes the entire desired size without adverse market impact and without creating incentives for the other trader to engage in order shading.

The transparency of the order submission process and the costs associated with submitting an order are of critical importance in determining whether or not multiple orders allow the large traders to reach an efficient amount of trading. In analyzing this, we first consider a closed–book auction where no trader observes the book (i.e., the orders submitted by the other trader) when submitting their orders. This is identical to the case where both traders submit all their orders simultaneously. We show that it is possible to trade efficient levels under such a format, but only if submitting any single order is not costly. In the presence of arbitrarily small costs of submitting an additional order, the closed book format cannot achieve an efficient level of trade in equilibrium because the large traders will reduce their number of orders so as to economize on these costs.

Next, we consider an open book auction where, at each point, all traders can simultaneously submit orders, having observed all orders submitted up to that point. Earlier orders can be subsequently cancelled at no cost, and orders may or may not have ticket charges. We show that in such a situation, efficient levels of trading can be achieved in equilibrium in a robust manner. The essential difference between the two formats arises out of the openness of the book— since each trader can observe the other trader's early order, he or she can condition the later orders on this order information, even though the other trader can subsequently cancel his or her early order. In short, earlier stages of the order submission process serve a communication function that allows, in equilibrium, each large trader to infer the composition of the market and the intentions of the other side.

4.1 The Closed Book Auction

An efficient outcome can be achieved in a closed book auction with costless order submission. To see this, consider the three orders $\{(b_k, p_k^b)\}_{k=1}^3$ and $\{(s_k, p_k^s)\}_{k=1}^3$ of each type, respectively, such that

$$b_1 = s_1 = \frac{a}{3};\ b_2 = s_2 = \frac{a}{6};\ \text{and } b_3 = s_3 = 1 - \frac{a}{2} \tag{9}$$

while

$$p_1^b \geq \frac{3}{4} + \frac{1}{12}\sqrt{5},\ p_2^b \geq \frac{3}{4},\ p_3^b = \frac{1}{2} \tag{10}$$

and

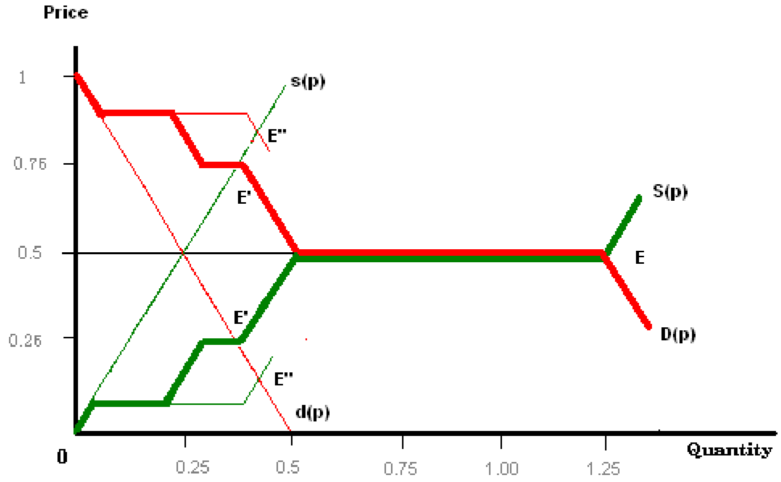

Figure 4: Efficiency in the closed book

$$p_1^s \leq \frac{1}{4} - \frac{1}{12}\sqrt{5},\ p_2^s \leq \frac{1}{4},\ p_3^s = \frac{1}{2}. \tag{11}$$

Figure 4 depicts such an order submission strategy for each trader. Given such strategies, when the market is two–sided, both the buyer and the seller execute all three orders, for a total size of $\frac{a}{3} + \frac{a}{6} + 1 - \frac{a}{2} = 1$ at a price of $\frac{1}{2}$ (point E in the figure). When only one large buyer (respectively, seller) is in the market, he or she executes only the first two orders for a total size of $\frac{a}{3} + \frac{a}{6} = \frac{a}{2}$ at a price of $\frac{3}{4}$ (respectively, $\frac{1}{4}$; see points E'). And when there are two buyers (respectively, sellers) in the market, each executes only the first order of size $\frac{a}{3}$ at a price of $\frac{5}{6}$ (respectively, $\frac{1}{6}$; see points E"). In Proposition 2 below, we show that this profile of strategies constitutes an efficient equilibrium of the closed book auction when order submission is costless.

Observe, however, that the efficiency gains in the closed book auction are sensitive to the introduction of small order submission costs or ticket charges. If each trader has to pay a small cost $c > 0$ for each order, then the outcome of the closed book auction that is depicted in Figure 4 is no longer an equilibrium. Furthermore, this is true regardless of how small the cost c is as long as it is positive.[16] Such costs may arise out of ticket charges that apply to each individual order or for other reasons (e.g., it is more difficult to design and implement a strategy with many orders submitted at limit prices, versus one aggregate order submitted at one limit price).

The intuition is as follows. Given the strategy of the large seller, notice first that by submitting

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[16] While we assume in what follows that the cost c is independent of the size and other details of the order, the arguments rely only on the assumption that the cost of submitting one order is less than the cost of submitting two orders of the same total size.

the order (b_1, p_1^b) together with one large second order $(b, p^b) = (1 - \frac{a}{3}, \frac{3}{4})$, the large buyer achieves an outcome that is identical to what he/she would obtain from submitting the three orders (b_1, p_1^b), (b_2, p_2^b) and (b_3, p_3^b) described above, in every state of the world. But since submitting two orders is, ceteris paribus, less costly than submitting 3, the buyer has a strict incentive to submit only two orders of the same total size. However, this leads to an unravelling of the efficient outcome. Since all of the large buyer's orders are at a limit prices away from 1/2, the large seller has an incentive to reduce the total quantity submitted so as to obtain a better price. This in turn leads the buyer to also submit a lower total size, and so on, implying that the efficient outcome cannot be an equilibrium in the presence of small costs.

Proposition 2
1. *In the closed book auction without order submission costs, there exist equilibria where efficient amounts of trading can be achieved when the market is two sided. Any pair of orders* $\{(b_k, p_k^b)\}_{k=1}^3$ *and* $\{(s_k, p_k^s)\}_{k=1}^3$ *satisfying (9)—(11) is such an equilibrium.*
2. *When there are arbitrarily small order submission costs, no such efficient equilibrium exists.*

Proof. See the Appendix.

The previous result shows that while allowing the traders to submit multiple orders does create the possibility of efficient trade, such a possibility is not robust to the presence of arbitrarily small order submission costs. In contrast, we show in the next section that the efficient equilibrium of the open book format is robust to such costs.

4.2 The Open Book Auction

The open book auction is modeled as follows. There are two stages of order submission indexed by $k = 1, 2$. In stage 1, traders simultaneously submit orders. Before stage 2, each trader observes all submitted stage 1 orders (i.e., the book) and can cancel the stage 1 order and/or submit a second order. We can think of this as an environment where the open book updates periodically (say, every 5 minutes), with each stage corresponding to such a time interval. A trader is not allowed to submit an order in stage 2 if he/she has not submitted an order in stage 1. Furthermore, a trader is not allowed to submit a sell (respectively, buy) order in stage 2 if he/she has submitted a buy (sell) order in stage 1, even if the stage 1 order is cancelled.[17] In contrast to the closed book auction, the open book auction implements the most efficient outcome depicted in Figure 4, in a perfect Bayesian equilibrium, even in the presence of small order submission costs.[18]

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[17] Some of these rules are not necessary but simplify the proofs to follow. The restriction to two stages is without loss of generality. With more stages, one can construct outcome–equivalent equilibria.

[18] This result also holds in the absence of order submission costs. In such a case, however, the open book is outcome–equivalent to a closed book.

Proposition 3 *In the open book auction with (or without) small order submission costs, there exist perfect Bayesian equilibria where each participant trades an efficient amount when the market is two–sided.*

Proof. See the Appendix.

In the equilibrium of the open book auction, the traders will gradually build up their orders, not submitting a large order too quickly. Unduly large early orders will be misinterpreted and so will be unprofitable given the equilibrium response of the other trader. The open book *can* build to an efficient outcome, but it cannot build too fast. In the real world there is considerable anecdotal evidence that such communication may occur through the common practice of 'testing the waters' or 'pinging.'[19]

While the open book has one efficient equilibrium, inefficient equilibria also exist. Such an inefficient equilibrium has nothing to do with the extrinsic uncertainty regarding market composition, but is a self–fulfilling coordination failure — each trader may submit and end up trading only small amounts in a two–sided market, only because each expects the other side to do likewise. This suggests that an animator as an intermediary may play an important role in preventing such inefficiencies from occurring.[20] Specifically, the animator, after observing from the early large orders that the market is two-sided, may submit a large sell order at a limit price slightly greater than 1/2, and a large buy order at a price slightly less than 1/2. The guarantee that a large order will execute against the animator's order, with minimal adverse price impact, will induce each large trader to submit large second orders, whether or not the trader fears order shading on the part of the other large trader. As a result, both large traders will profit from submitting large orders. Such orders will then offset each other and, given this, the animator's orders will not execute and the intermediary will not, in equilibrium, have to take a significant net position in the asset.

5 Discussion and Concluding Remarks

Our analysis suggests that attracting order flow as a market forms is a daily challenge. We refer to the process as bookbuilding. We have analyzed bookbuilding with a particular focus on the tactical trading decisions of large (institutional) participants. Bookbuilding is non-trivial in an environment where the optimal order timing and placement decisions of a large trader takes into account the decisions that a relatively small set of other large traders may make. For model

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[19] Note that this concept of 'testing the waters' is similar to the observed behavior of institutional traders who 'slice and dice' large orders into smaller ones so that these orders can interact with the retail order flow and reduce the large traders' overall market impact. See Cushing and Madhavan (2000) for an empirical analysis of intraday institutional equity trading behavior.

[20] Two major European markets, Euronext and Deutsche Börse, have incorporated designated market makers to this end for their less liquid issues. At Euronext Paris, the initial term for these intermediaries was "animateur".

simplicity, we have assumed two large traders, each of whom is uncertain about whether the other will be in the market at any given time. Further, if both are present, each is uncertain about the side of the market that the other will be on (i.e., whether the market will be one-sided or two-sided). We additionally assume a substantial number of atomistic (retail) participants each of whom, not expecting to have any significant impact on price, do not make tactical trading decisions. The retail order flow provides a background against which the two large participants play their trading game.

We have differentiated the two large participants according to their private share valuations rather than by any non-public information that they may possess (i.e., we abstract from adverse selection). A participant with a relatively high valuation is a potential buyer, and one with a relatively low valuation is a potential seller. Each would like to know the other's assessment, not to change his or her own valuation, but because knowing whether or not a trading partner or competitor exists enables a large trader to better assess the impact his or her own large order may have on the price. We show that, in the absence of adverse selection, bookbuilding may be insufficient due to the gaming strategies involving order shading that both large participants may have an incentive to employ. However, we also show that bookbuilding can achieve an efficient outcome if the structure of the financial market is designed appropriately to resolve these gaming strategies.

Our model assumes a call auction trading environment. We do so for model simplicity, but also because it is institutionally realistic — order-driven markets typically open their daily trading sessions with a call auction. Our analysis shows that each trader is worse off when not knowing the existence of the other trader or the side of the market that the other trader might be on. We also find that the most efficient outcome in a market with certainty can be achieved, even if trader types are not known (1) if multiple orders are permitted without a minimum ticket charge or other costs, and/or (2) if the call auction's book is open (i.e., when all traders can observe each other's order quantities and prices) and they submit multiple (scaled) orders. The second finding is robust to the inclusion of order submission costs, whereas the performance of a closed book auction is found to be uniformly inferior to an open book call.

How should a call best be structured so as to facilitate bookbuilding? As shown above, while a call should provide appropriate transparency, it might also allow orders to be hidden for customers who so desire. The combined use of an open display book and a hidden reserve book would accomplish this.[21] The two-book system gives participants the choice of displaying or of not displaying an order, while giving displayed orders priority and applying lower commission rates to

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[21] The Arizona Stock Exchange, a facility that has suspended operations, has incorporated this two-book procedure. Price is determined by orders in the open book, and open book orders have priority over reserve book orders. The reserve book operates as a crossing network (all reserve orders that cross at the single price determined in the open book execute at that price).

orders that are displayed to encourage participants to use the open book.

Inclusion of retail customers in a market is important to seed bookbuilding. As noted, retail customers, being rightfully less concerned that their small orders will have adverse price impacts, are more willing to submit their orders relatively early in the bookbuilding period. In a call auction, early order placers are important catalysts for liquidity formation, and their commissions should be lower to compensate them for this public benefit that they provide. Use of strict time priorities further encourages early submission of orders whenever partial execution is a risk.

As mentioned above, our focus on bookbuilding suggests that broker/dealer intermediaries can offer an important service by animating bookbuilding. In this respect, the retail order flow that is incorporated in our model may be viewed as an implicit animation procedure. When making their tactical decisions, the large participants take account of the retail order flow as captured by the parameter a. The heavier the retail order flow (the higher the value of a), the more efficient is the outcome of the strategic game played by the large participants. An intermediary, as animator, would in essence operate through the parameter a.[22] In reality, the animator may face some risk of participating in trades established at the time of the call if insufficient order flow is attracted to any particular auction, and the animator must be appropriately compensated for accepting this risk (presumably through commission income). We leave a full analysis of the bookbuilding role of such an animator for future research.

6 Appendix

Proof of Proposition 1 Consider a Nash equilibrium pair of orders (b, p^b) and (s, p^s). Note first that $p^b \geq p^*$. If not, $b^* = 0$ so that the buyer makes zero profits in equilibrium. Submitting an order $(b, 1)$ for some $b > 0$ gets him strictly positive profits, a contradiction. A similar argument for the seller establishes that $p^s \leq p^*$. It follows that $b^*, s^* > 0$.

If $b^* < b$, then $D(p^*) > S(p^*)$ and $p^* = p^b$. The seller can submit a slightly larger order $(s + \varepsilon, p^s)$ with $\varepsilon > 0$, and sell it without raising the price, thus strictly increasing his/her profits, a contradiction with equilibrium. Thus $b^* = b$. A symmetric argument establishes that $s^* = s$. It follows that $p^* = \frac{1}{2} + \frac{1}{2a}(b - s)$, in equilibrium. The equilibrium profit of the buyer and the seller are respectively equal to

$$\Pi_b = (\frac{1}{2} - b + s)b \tag{12}$$

and

$$\Pi_s = (\frac{1}{2} + b - s)s \tag{13}$$

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[22] NYSE Floor brokers recognize their role as "facilitators." Facilitating a trade (getting orders out of traders' pockets and turning them into trades) is comparable to the animation function.

Since the buyer can always submit a larger order with a higher reservation price, and get both orders executed, it follows that the the equilibrium marginal profit for the buyer (the first–derivative of the expression in (12)) cannot be strictly positive. If in fact $p^s < p^*$, then the buyer can also submit a lower order and get both orders executed, so that the marginal profit for the buyer must be zero in such a case, so that (7) follows. Similar remarks establish (8). Finally, it is straightforward to check that any pair of orders satisfying the conditions of the Proposition is a Nash equilibrium.■

In order to prove Propositions 2 and 3 we first establish a Lemma that fully characterizes equilibrium behavior in a market where it is known for sure that there is one large trader (parts 1 and 2) and in a market where it is known for sure that there are two large traders on the same side of the market (parts 3 and 4). In both the closed and open book auctions that we consider subsequently, the ability of each trader to submit multiple orders will imply that, in equilibrium, they can effectively condition their orders on the precise state of the world, i.e., whether or not there is another large trader in the market and which side of the market he/she is on.

Lemma 1 *1. In a market with the single large buyer, the buyer will submit and execute an order* $b = b^* = \frac{a}{2}$ *at a limit price* $p^b \geq p^* = \frac{3}{4}$.

2. In a market with the single large seller, the seller will submit and execute an order $s = s^* = \frac{a}{2}$ *at a limit price* $p^s \leq p^* = \frac{1}{4}$.

3. In a one-sided market with two buyers, a pair of buy orders (b_1, p_1^b) *and* (b_2, p_2^b) *with* $\min\{b_1, b_2\} < a$ *is a Nash equilibrium if and only if*

$$b_1 = b_1^* = b_2 = b_2^* = \frac{a}{3}$$

with each limit price being at least as high as $\frac{3}{4} + \frac{\sqrt{5}}{12}$. *The market clearing price* $p^* = \frac{5}{6}$ *and each large trader's payoff is* $\frac{a}{18}$. *Any pair of orders* $(b_1, 1)$ *and* $(b_2, 1)$ *with* $\min\{b_1, b_2\} \geq a$ *is a Nash equilibrium with* $p^* = 1$ *and zero profits for each large trader.*

4. In a one-sided market with two sellers, a pair of buy orders (s_1, p_1^s) *and* (s_2, p_2^s) *with* $\min\{s_1, s_2\} < a$ *is a Nash equilibrium if and only if*

$$s_1 = s_1^* = s_2 = s_2^* = \frac{a}{3}$$

with each limit price being at most as high as $\frac{1}{4} - \frac{\sqrt{5}}{12}$. *The market clearing price* $p^* = \frac{1}{6}$ *and each large trader's payoff is* $\frac{a}{18}$. *Any pair of orders* $(s_1, 0)$ *and* $(s_2, 0)$ *with* $\min\{s_1, s_2\} \geq a$ *is a Nash equilibrium with* $p^* = 0$ *and zero profits for each large trader.*

Proof of Lemma. Parts 1 and 2 are entirely immediate and we omit the details. We provide a proof for part 3 below. Part 4 then follows from symmetric arguments.

First, it is straightforward to check that a pair of orders $(b_1, 1)$ and $(b_2, 1)$ with $\min\{b_1, b_2\} \geq a$ is a Nash equilibrium. Given each trader's order, the price will equal 1, regardless of the other order, and so neither can do better by submitting a different order.

So consider the pair of orders (b_1, p_1^b) and (b_2, p_2^b) with $\min\{b_1, b_2\} < a$ and suppose that it is a Nash equilibrium. Without loss of generality, assume that $p_1^b \leq p_2^b$. Note first that $p^* < 1$. If not, neither trader earns positive profits. Since $\min\{b_1, b_2\} < a$, a trader can submit an order smaller than $a - \min_i b_i$ for a market–clearing price less than 1 and do better. It follows that both traders trade strictly positive quantities, or $b_i^* > 0$, and $p_i^b \geq p^*$ for all i. If not, i.e., $b_1^* = 0$, trader 1 can do better by submitting a sufficiently small order at a high enough limit price.

Note next that $p_1^b > p^*$. For, if $p_1^b = p^*$, since $b_1^* > 0$, trader 2 can do strictly better by submitting a slightly larger order at a limit price slightly higher than p_1^b as, by doing so, he/she will execute a larger quantity at the same price of $p_1^b = p^*$. Since $p_1^b > p^*$, $b_i = b_i^*$ for each trader. Furthermore, both orders will still get executed for small changes in either trader's order size. For any two such slightly different orders b_1' and b_2' the market clearing price equals

$$p^*(b_1', b_2') = \frac{1}{2} + \frac{1}{2a}(b_1' + b_2').$$

Since the orders (b_1, p_1^b) and (b_2, p_2^b) are a Nash equilibrium, the order size must be locally optimal for each trader so that we obtain

$$b_i \in \arg\max_{b_i'} (1 - p^*(b_i', b_j'))b_i' \text{ for all } i, j$$

The first–order necessary conditions for a maximum yield $b_1 = b_2 = \frac{a}{3}$ so that $p^* = \frac{5}{6}$. Each trader's payoff is $\frac{a}{18}$.

It remains to establish the bounds on the limit prices. If trader 1 submits the order at a limit price $p_1^b < 1$, then trader 2 can choose a strictly higher limit price than trader 1 and an order size that is large enough to knock out 1's order entirely, enabling trader 2 to trade at a price $p \geq p_1^b$ and a quantity equal to the net retail supply $ap - a(1-p)$. Since such a strategy is not profitable, we obtain a lower bound on p_1^b via the following inequality

$$\max_{p \geq p_1^b} (1-p)(ap - a(1-p)) \leq \frac{a}{18}$$

yielding

$$p_1^b \geq \frac{3}{4} + \frac{\sqrt{5}}{12}.$$

This establishes necessity. It is straightforward to verify that a pair of orders satisfying the conditions of the part 3 is a Nash equilibrium.■

Proof of Proposition 2 For part 1, consider, without loss of generality, trader 1 and suppose that he/she is a buyer. Note from Lemma 1.3 that given the strategy of the other trader, the order (b_1, p_1^b) is optimal and the other two orders are irrelevant, when trader 2 is also a buyer. Second, when trader 1 is the only trader in the market, from Lemma 1.1 we observe that the orders (b_1, p_1^b) and (b_2, p_2^b) are together optimal for trader 1 when he/she is a large buyer trading against the retail flow as, in such cases, he/she trades a quantity $\frac{a}{2}$ at a price $\frac{3}{4}$. Finally, consider the case where the market is two–sided. Given the strategy of trader 2, the best that trader 1 can do is trade a total amount 1 at a price of $\frac{1}{2}$. If he/she tries to trade a lower amount by lowering the size of his/her third order (b_3, p_3^b) then he/she simply trades a lower amount without improving the price, whereas if he/she submits a larger order at a higher limit price, then he/she still executes only a total size of 1 at the same price. Symmetric arguments apply in all other cases.

For part 2, notice first that, in any Nash equilibrium, any trader will submit at most three different orders, since there are three possible realizations of the uncertain state of the world, i.e., the type of the other trader. Any outcome achieved by more orders can be achieved by submitting three orders and saving on order costs. Note next that, due to arguments contained in the proof of Proposition 1, to achieve efficiency, it must be that the third order is at a limit price of $\frac{1}{2}$ for each type of trader, as otherwise the other trader has an incentive to shade his/her size and so obtain a more favorable price and higher profits, without changing the total number of orders. However, given that trader 2 behaves like this, trader 1 can save on submission costs by submitting only two orders, e.g., equal to $(\frac{a}{3}, \frac{3}{4} + \frac{1}{12}\sqrt{5})$ and $(1 - \frac{a}{3}, \frac{3}{4})$ that achieves the same outcome. It follows that there cannot be an equilibrium that achieves efficient trading, given arbitrarily small order submission costs.■

Proof of Proposition 3

Consider the following symmetric profile of strategies. A buyer initially submits an order $(b_1, p_1^b) = (\frac{a}{3}, 1)$ and a seller initially submits an order $(s_1, p_1^s) = (\frac{a}{3}, 0)$. In stage 2, if a buyer (respectively, seller) observes that the other large trader has also submitted a buy order $(b_1, p_1^b) = (\frac{a}{3}, 1)$ (respectively, sell order $(s_1, p_1^s) = (\frac{a}{3}, 0)$) in stage 1, then he/she does not submit another order. If a buyer (respectively, seller) observes that the other large trader has not submitted an order in stage 1 then he/she submits a second buy order $(b_2, p_2^b) = (\frac{a}{6}, \frac{3}{4})$ (respectively, sell order $(s_2, p_2^s) = (\frac{a}{6}, \frac{1}{4})$) in stage 2. If a buyer (respectively, seller) observes that the other large trader has submitted a sell order $(s_1, p_1^s) = (\frac{a}{3}, 0)$ (respectively, buy order $(b_1, p_1^b) = (\frac{a}{3}, 1)$) in stage 1, then he/she submits a second buy order $(b_2, p_2^b) = (1 - \frac{a}{3}, \frac{1}{2})$ (respectively, sell order $(s_2, p_2^s) = (1 - \frac{a}{3}, \frac{1}{2})$) in stage 2. This describes strategies that will occur on the path of play. We now turn to off–the–path–of–play behavior.

Whenever a buyer (seller) sees that the first period play does not conform to the strategies described above, although the other side has placed a buy (respectively, sell) order, he/she cancels his/her first order and places a buy order $(b_2, p_2^b) = (\frac{a}{3}, 1)$ (respectively, sell order $(s_2, p_2^s) = (\frac{a}{3}, 0)$ in stage 2. Whenever a buyer (respectively, seller) sees that the first period play does not conform to the strategies described above, and the other side has not placed an order, he/she cancels his/her first order and places a buy order $(b_2, p_2^b) = (\frac{a}{2}, \frac{3}{4})$ (respectively, sell order $(s_2, p_2^s) = (\frac{a}{2}, \frac{1}{4})$ in stage 2. Whenever a buyer (respectively, seller) sees that the first period play does not conform to the strategies described above, although the other side has placed a sell (respectively, buy) order, he/she cancels his/her first order and places a buy order $(b_2, p_2^b) = (a, \frac{3}{4})$ (respectively, sell order $(s_2, p_2^s) = (a, \frac{1}{4})$) in stage 2.

Using arguments contained in the proofs Proposition 1 and Lemma 1, it is straightforward to check that the profile described above is a perfect Bayesian equilibrium. We do so now.

Suppose first that in the first stage, two buy orders have been placed, each equal to the prescribed order of $(\frac{a}{3}, 1)$. Each trader then concludes that it is a one–sided market with two buyers. From Lemma *1*, this is an equilibrium of the one–sided market, and each trader will find it optimal to behave as specified, i.e., submit no further orders. Suppose next that in the first stage, two buy orders have been placed, with at least one not equal to $(\frac{a}{3}, 1)$. Each trader then concludes that it is a one–sided market with two buyers, but one buyer has deviated in stage 1. The specified stage 2 behavior of each cancelling the first stage order and submitting a buy order $(\frac{a}{3}, 1)$ is a mutual best–response after such an event. Symmetric considerations apply to the case where two sell orders have been placed in stage 1.

Suppose next that in the first stage one buy order equal to $(\frac{a}{3}, 1)$ and one sell order equal to $(\frac{a}{3}, 0)$ have been placed. Each trader then concludes that it is a two–sided market with one buyer and one seller. Arguments analogous to those in the proof of Lemma *1*, establish that the specified stage 2 behavior of placing a second order (buy or sell) of size $1 - \frac{a}{3}$ at a limit price of $\frac{1}{2}$ is optimal in stage 2 for each trader, given the anticipated behavior of the other. Suppose next that in the first stage, one buy order and one sell order have been placed, with either the buy order not equal to $(\frac{a}{3}, 1)$ or the sell order not equal to $(\frac{a}{3}, 0)$. Each trader then concludes that it is a two–sided market with one buyer and one seller, but one trader has deviated in stage 1. The specified stage 2 behavior of each cancelling the first stage order and submitting a buy order $(a, \frac{1}{2})$ is a mutual best–response after such an event, due to arguments similar to those in the proof of Lemma *1*.

Consider next the case where only one buy order $(\frac{a}{3}, 1)$ has been submitted in stage 1. The buyer who has submitted the order concludes that he/she is the single large trader, and due to arguments previously discussed for this case, it is optimal for him/her to behave as specified, i.e., submit a second order of size $(\frac{a}{6}, \frac{3}{4})$ so that he/she executes a total size of $\frac{a}{2}$ at a price $\frac{3}{4}$. Symmetric considerations apply to the case where one sell order has been placed in stage 1.

Finally, consider the optimality of stage 1 behavior. Consider the position of a large buyer and suppose that he/she deviates from his/her prescribed order in stage 1 and submits a different buy order. If the other trader is also a buyer, then given the specified stage 2 behavior, he/she will end up trading an amount $\frac{a}{3}$ at a price $\frac{5}{6}$, that is identical to what he/she would have done if he/she had not deviated. Similarly, if he/she observes no order from the other trader, then he/she expects to end up trading exactly the amount he/she would have traded had he/she not deviated. Finally, if the other trader submits the equilibrium stage 1 sell order $(\frac{a}{3}, 0)$, then because of his/her deviation he/she will end up trading a total amount a at a price of $\frac{1}{2}$ profitable. It follows that the stage 1 behavior of the buyer is optimal. Symmetric arguments establish the optimality of the stage 1 behavior of a seller.■

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