Auctions with Ceilings[*]

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Abstract

In symmetric common value auctions where bidders differ ex–post in information quality, a seller may benefit from imposing a ceiling on allowable bids. By reducing the winner's curse facing poorly informed bidders, a ceiling encourages them to bid aggressively. This may reduce information rents earned by better informed bidders, yielding the seller higher expected revenues compared to selling the object in a standard ascending auction or at a fixed posted price. Such a ceiling may be explicit (a firm commitment not to accept bids above the ceiling) or implicit (a credible threat not to honor the outcome if anyone bids higher than the ceiling). Either situation can be interpreted as one where the object is offered for sale at a fixed price or the best offer.

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1 Introduction

When faced with many potential buyers, should a seller simply allow unrestricted competitive bidding or should she specify an upper bound on allowable bids in order to increase expected revenues? When seeking to place a hitherto unknown but potentially talented player with a sports club, could the player and his agent gain by specifying a cap on the acceptable salary and other terms of trade? When outsourcing a public works contract, could a government agency lower expected costs by announcing a minimum price guarantee for the suppliers? In general, could one gain by putting limits on the share of the pie that one is willing to receive? We consider these questions in the context of a common value auction for a single object.

In such common value environments, a bidder fears overpaying precisely when her bid is the highest and she wins the auction — the act of winning is negative information. This winner's curse problem becomes acute when, at the bidding stage, some bidders have poorer information than others. In such cases, a poorly informed bidder can win only when not outbid by a better informed bidder. Anticipating such an acute winner's curse, rational but poorly informed bidders bid cautiously in equilibrium. This adversely affects the incentives to bid aggressively for all types of bidders and lowers the seller's expected revenues.

We show that in such a situation the seller can benefit by imposing an *explicit ceiling* on allowable bids. By imposing a ceiling the seller restrains competitive bidding by better informed bidders. As a result, any poorly informed bidder when winning at the ceiling attaches positive probability to the joint event that (i) the true–value of the good is higher than the ceiling and (ii) that he is tied with a bidder with superior information. This reduces the fear of being outbid for poorly informed bidders and encourages them to bid aggressively. Aggressive bidding by poorly informed bidders enables the seller to extract more information rents from better informed bidders. Consequently, compared to a standard auction without a ceiling, the seller earns higher revenues on average.

A ceiling is not costless — by imposing one the seller rules out the possibility of receiving a higher price. When considering the relative merits of an auction with a ceiling and an auction without one, the seller has to trade-off the expected 'benefit' of increased bidding below the ceiling with the expected 'cost' of not allowing bids higher than the ceiling. For a fixed number of bidders, we show that imposing a ceiling is profitable for the seller if and only if the expected proportion of better informed bidders is low enough. Moreover, imposing a ceiling is better for the seller if the total number of bidders is large enough.

In terms familiar from classical price theory, a standard auction allows the seller to obtain in every state of the world, a price for which demand equals supply. In fact, by exploiting competitive bidding among buyers, the seller is able to (imperfectly) price discriminate across different states of bidder valuations. With a price ceiling the seller commits to no price discrimination in states where the better informed bidders have high demand. This affects total demand — by alleviating the winner's curse it shifts outward the demand from poorly informed bidders. Consequently, the seller is able to generate more competition and obtain a higher market clearing price, in states where better informed demand is not so high. When most bidders are likely to be poorly informed, the seller's expected cost from ruling out prices higher than the ceiling is low while the expected benefit via a higher demand from poorly informed bidders is high. As a result, the seller does better overall from imposing a ceiling compared to a standard auction. Furthermore, when there are many bidders, each poorly informed bidder when bidding at the ceiling attaches a large probability to the presence of favorable information among at least some other bidders. As a result, the seller is able to set a relatively high ceiling but still generate high demand from poorly informed bidders, lowering the cost and raising the benefit from imposing a ceiling.[1]

In IPO auctions, explicit ceilings on allowable bids are often specified. The Google IPO is one example, where acceptable bids had to lie in the range \$85 to \$95. Such ceilings have been justified on the grounds of providing a 'level playing field' or a 'bidding guide' or an 'anchor price' for small investors.[2] In our symmetric auction of an indivisible object, all bidders are ex–ante identical. Nonetheless, the logic of our results extends to an IPO context and a bid ceiling acts as a bidding guide for bidders who are poorly informed at the bidding stage.

In 'reverse' auctions that are held in government procurement contexts, a bid ceiling takes the form of a minimum price guarantee. Although such practices are vulnerable to accusations of being 'sweetheart deals', our results provide one defense of why they may not be so — by reducing the winner's curse faced by potential suppliers, a minimum price guarantee may in fact lower the government's outsourcing costs.

The most important practical insight from our results is provided by a consideration of posted prices. With such mechanisms, the seller offers up the object at a fixed price, randomly allocating the object when faced with excess demand. That is, she chooses never to use a competitive bidding process in order to price discriminate across different states of buyer valuations. This has one virtue in common with an auction that has a ceiling — it also reduces the fear of being outbid for poorly informed bidders. As Bulow and Klemperer (2002) and Campbell and Levin (2002) show, a posted price may in fact dominate a standard auction in expected revenues.[3] In our setting however, a posted price introduces the bidders to a winner's curse problem on the 'downside', i.e., to winning the good and paying the posted price in states where better informed bidders do not bid. In

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[1] The idea that a price ceiling imposed by a seller may improve demand under conditions of incomplete/imperfect information is well–known in other non–auction contexts. Stiglitz and Weiss (1981) show that in credit markets with adverse selection an interest rate ceiling can benefit a lender by improving the quality of demand. Shapiro and Stiglitz (1984) show that in labor markets with moral hazard, a wage floor can improve productivity. In our model a price ceiling alleviates the winner's curse and so increases the quantity demanded.

[2] See http://www.hambrecht.com/, the website of the lead investment bank for the Google IPO.

[3] Wang (1993) makes the same point, but in an independent private values model where auctioning may be costly.

contrast, such a downside winner's curse is mitigated in an auction with a ceiling, since poorly informed bidders typically pay lower prices when better informed bidders do not bid at the ceiling. By holding an auction with a ceiling the seller combines the benefits of an auction and a posted price mechanism and so does better than either an auction or a posted price.

An auction (with or without a ceiling) involves commitment on the part of the seller to honor the rules of the auction. Although it is not unreasonable to expect that the seller has the ability to commit, one can imagine situations where such commitment is limited and it is feasible for the seller to cancel the auction after observing its outcome and offer the object to the winner (or any other bidder) at a take it or leave it price. We show that such limited commitment may, somewhat surprisingly, benefit the seller compared to the case where she fully commits to honor the rules of the standard auction. In fact, the possibility of cancellation may afford the seller opportunities for 'brinkmanship' that affects bidding behavior in the limited commitment game and generates an *implicit ceiling* on submitted bids. This benefits the seller for the same reasons that an explicit one does.

As an application of these ideas, consider the commonly observed phenomenon where a seller offers an object for sale at a negotiable posted price (equivalently, fixed price or best offer). In such situations, it is clear that she is willing to accept a lower market clearing price in the event that sufficient demand is not forthcoming at the fixed price, although it is less clear what she intends to do if anyone bids above the fixed price. If one interprets bidding in an auction as a description of a simple negotiation procedure between the buyers and the seller, an auction with an explicit ceiling can be thought of as a negotiable posted price mechanism, provided the seller is committed to not accepting a price higher than the pre–announced fixed price.[4]

Even when such commitment is absent however, our results on implicit ceilings imply that it may be an equilibrium for better informed bidders to interpret the fixed price as a bid ceiling and so not bid above it, anticipating cancellation if anyone actually does so. This encourages poorly informed bidders to bid up to the fixed price. The seller cancels the auction in the out of equilibrium event of someone actually bidding above the fixed price, as she then expects to obtain a higher price subsequently. The threat of cancellation is credible although cancellation never actually occurs. In equilibrium the seller indeed sells the object either at the pre–specified fixed price or, failing that, at the best offer below it.

In private value auctions a seller may benefit from undertaking inefficient amount of trades by imposing a reserve or floor price (i.e., a lowest allowable bid; see, e.g., Myerson (1981)). In common

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[4] A friend, the economist A. Citanna has informed us that when he bid higher than the fixed price in a fixed price or best offer mechanism that offered an apartment for rent in Paris, his bid was ignored by the owner's agent in allocating the apartment and, furthermore, it was evident that he had violated the Parisian code of proper bidding behavior. To the best of our knowledge, the owner was committed to not accepting the high bid because French anti–discrimination laws prevent accepting prices higher than the advertised one, although she was clearly open to accepting lower prices.

value contexts the gains from a reserve price are less pronounced, especially for a large number of bidders (see, e.g., Levin and Smith (1996)). We also provide an analysis of reserve prices in our model and show that the optimal reserve price is zero for a sufficiently large number of bidders. In common value environments such as ours, imposing a ceiling can benefit the seller more than imposing a floor. We do not consider the optimal direct mechanism in this paper. It is well known that with correlated signals an auction (with or without a ceiling) is not optimal.[5] We justify our focus on ceilings as a simple change in the rules of a standard auction, itself a commonly observed mechanism.

In a preliminary note, Chakraborty (2002) provides a numerical example of a sealed-bid auction with three bidders where imposition of an explicit ceiling can improve revenue. The present paper generalizes and extends the analysis of explicit ceilings and also provides novel results on limited commitment and implicit ceilings. In rather different contexts, Che and Gale (1998) as well as Gavious, Moldovanu and Sela (2002) show that imposition of bid ceilings may increase the seller's revenue in private value all–pay auctions, thus providing insights on the value of imposing caps on contributions by political lobbies. In Che and Gale (1998), the results are driven by ex ante asymmetry between publicly known bidder valuations, while Gavious et al. (2002) show that if bidders are ex ante symmetric a bid ceiling may be optimal but only if the total cost to a bidder is strictly convex in her bid.[6] In private value all–pay auctions, a ceiling encourages bidding by lowering a bidder's expected costs in the event that she *does not* win the auction. In contrast, in our common value winner–pays auction, a ceiling encourages bidding by lowering a bidder's expected winner's curse, in the event that she *does* win the auction.

Chen and Rosenthal (1996a, b) show that if buyers arrive randomly and sequentially, and incur inspection costs of discovering their own true valuations, a seller may benefit from imposing a price ceiling.[7] By imposing a ceiling the seller makes it worthwhile for buyers who turn up early to pay the cost, thereby avoiding inefficiencies arising out of too many bidders paying the cost. In contrast, we show that even when information acquisition is not costly and all prospective buyers can easily be attracted to the venue of the auction, a ceiling may still be beneficial since it mitigates the winner's curse.

Finally, with respect to our results on implicit ceilings and the value of the option to cancel, Horstmann and LaCasse (1997) analyze a model where an auction can be cancelled by an informed seller to convey information and also to allow bidders to independently obtain better information over the passage of time. In our setting however the seller is uninformed. The value of the option to cancel instead arises out of the fact that information may be revealed by the bidding process itself.

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[5] Cremer and Maclean (1988) and McAfee and Reny (1992) show that in such settings (near) full extraction of surplus is possible with interim incentive compatible and individually rational mechanisms. Such mechanisms are not often seen in practice.

[6] In a recent paper, Sahuguet (2004) extends these results to an asymmetric environment.

[7] See also Milgrom (2004) for a model with a similar flavor.

Taking this into account, better informed bidders bid less aggressively and forego some information rents, thereby encouraging overall participation and raising the seller's expected revenue. In our model, cancellation does not occur in equilibrium.

The rest of the paper proceeds as follows. In Section 2, we introduce our model, characterize the equilibrium of the standard auction and compare it with auctions that have a reserve price as well as with posted price mechanisms. In Section 3 we consider explicit ceilings and show when it dominates a standard auction and a posted price mechanism. In Section 4 we consider implicit ceilings. Section 5 concludes while the Appendix contains the majority of the proofs.

2 The Standard Auction

A seller wants to sell an indivisible object to one of $n \geq 2$ bidders in the set $N = \{1, ..., n\}$. The value of the object to the seller equals 0 whereas the common value of the object to all the buyers is denoted by a random variable V that is distributed according to a continuous increasing distribution function F with density f that is positive on $\mathbf{V} = [0, 1]$. We denote by v a realization of V while letting $\overline{v}$ be the expected value of V. Let $\phi(v) = \frac{f(v)}{1-F(v)}$ denote the hazard rate associated with F that we assume is monotone increasing. The payoff to a buyer from obtaining the object of value v at price p is $v - p$ and the payoff from not obtaining the object is zero. The payoff to the seller is the price that she receives in the event of making a sale and zero otherwise.

The realized value of V is not common knowledge, although each bidder may have private information about it. We impose the cleanest possible information structure on the bidders that allows us to make our points. Each bidder $i \in N$ knows the realization v of V with probability $\alpha \in (0, 1)$; otherwise, he receives no additional information and only knows the prior. The probability that any one bidder is informed is independent across bidders and independent of V. Each bidder knows whether or not he is informed but does not know how many other bidders are informed. For $0 \leq k \leq K \leq n$, let $\pi(k, K) = \binom{K}{k} \alpha^k (1 - \alpha)^{K-k}$ be the probability that k out of K bidders are informed. The seller is uninformed. We denote by $\theta_i \in \Theta = \mathbf{V} \cup \{u\}$ the type of bidder i, where $\theta_i = u$ denotes that bidder i is uninformed while $\theta_i = v$ denotes that bidder i is informed that $V = v$.

The information structure above is a simple formulation of the idea that bidders who are ex–ante identical may be ordered (in the sense of Blackwell), in the quality of their information at the interim stage since, with positive probability, the signal technology used by the bidders fails to provide any conclusive evidence. Nevertheless, the lack of conclusive evidence does not automatically preclude any bidder from participating in the auction. While each bidder privately knows whether or not he has precise information, he does not know how many other bidders possess such information. The fact that bidders are either perfectly informed or perfectly uninformed is not important for our results. It will become obvious that the case where each bidder either obtains the *same* (across bidders) noisy signal of true value and is uninformed otherwise is immediately covered by our results. Further, in Appendix C, we show that our qualitative results will also obtain when bidders may either be uninformed or may obtain *different* but correlated and noisy signals of the true value.[8] On the other hand, it is important for our results that, while each bidder knows whether or not he has an 'expert' evaluation of the object, he does not know how many others are experts, i.e., the degree of ex–post 'asymmetry' is not common knowledge.[9]

We suppose that the seller sells the object in a continuous ascending clock English (or Japanese) auction.[10] We allow the seller to set a reserve or floor price r (i.e., a minimum allowable bid) or a ceiling price c (i.e., a maximum allowable bid) with $0 \leq r \leq c \leq 1$.[11] Since the ascending clock is continuous, one must be mindful of knife–edge cases in defining the rules of the auction given a reserve and ceiling pair $\{r, c\}$. These rules are as follows. A clock (signifying the price p) rises continuously starting at r and continuing till c. At each point $p \in [r, c]$, each bidder has one of two actions: to remain active or to quit, upon observing bidding activity at all $p' < p$. Quits are final. Let $A(p)$ be the set of bidders who are active when the clock is at $p \in [r, c]$. For any set X, denote by $|X|$ its cardinality. Since quits are final, for $p > p'$, $A(p) \subset A(p')$, i.e., $|A(p)|$ is a non–increasing function of p. For $p > r$, let $Q(p) = \cap_{p':p'<p} A(p') - A(p)$ be the set of bidders who quit at p and let $Q(r) = N - A(r)$. Further, let $B(p) = \cup_{p':p'>p} A(p)$ be the set of bidders who are active beyond p with $B(c) = \varnothing$. Notice that since there are a finite number of bidders, the non-increasing function $|A(p)|$ can have only a finite number of discontinuities and further, $|B(p)| \leq |A(p)|$ for all p, with strict inequality only at their common points of discontinuity. Moreover, $|B(P)|$ is a non–increasing right-continuous function of p.

The auction assigns a winner and the price that he pays as follows. If $|A(r)| = 0$, then no bidder is ever active and the good is not sold. In all other cases, the auction stops at a price $P = \inf\{p \in [r, c] | \; |B(p)| \leq 1\}$ (the set is non–empty since $|B(c)| = 0$), equal to the price the winner pays. Since $|B(p)|$ is right-continuous in p we must have $|B(P)| \leq 1$. P is the lowest price

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[8] Similarly, the qualitative results also go through when poorly informed bidders have a noisy inferior signal of the true value (see Chakraborty, 2002).

[9] See Piccione and Tan (1996) for a model of an auction with a similar information structure. Engelbrecht–Wiggans et al. (1983) and Hendricks et al. (1994) are some of the other papers that consider bidders who differ in the quality of their information although such asymmetries are common knowledge in these papers. None of these papers consider bid ceilings.

[10] We focus on such an auction format only because it makes equilibrium strategies particularly easy to describe. All qualitative results also extend to the case where the seller instead holds a sealed-bid second price auction. Recall in this respect from Milgrom and Weber (1982) that an ascending auction dominates all other standard auction formats in general symmetric environments of which ours is a special case. Finally, in Appendix C we also show that the continuous auction and its equilibrium can be approximated by discrete ascending auctions where prices and values lie on a sufficiently fine discrete grid.

[11] Throughout we will assume that the seller can commit to any ceiling or floor price that she may impose and in fact to honoring the rules of the auction. In Section 4, we turn to the question of a lack of commitment and implicit ceilings.

beyond which at most one bidder is active.[12] If $|B(P)| = 1$ then the winner is the bidder in $B(P)$; while if $|B(P)| = 0$ but $|A(P)| > 0$ then the winner is decided by uniform randomization among the bidder(s) in $A(P)$. Finally, if $|B(P)| = |A(P)| = 0$ then it must be that $|Q(P)| > 0$ and the winner is decided by uniform randomization among the bidders in $Q(P)$. This completes the description of the auction with a reserve and ceiling pair $\{r, c\}$. The standard auction is a special case of the auction above where $r = 0$ and $c = 1$. A posted price mechanism is another special case where $r = c$.

In any auction defined by a pair $\{r, c\}$, a strategy for any bidder is to choose a probability of staying active at every point $p \in [r, c]$ upon observing the past history of activity $A(p')$ for $p' < p$, given the bidder's type. Formally, for any $p \in [r, c]$ let the collection $h_p = \{A(p')\}_{p'<p}$ denote the history of bidding activity prior to p. We say that h_p is a 'history of no quits' if $|A(p')| = n$ for all $p' < p$ and specify that the null history h_r is such a history. Further, let H_p be the set of possible histories prior to $p \in [r, c]$ with $H = \cup_{p\in[r,c]} H_p$ and let $H^n \subset H$ be the set of histories with no prior quits, of arbitrary length. A (behavior) strategy for bidder i is a function $\sigma_i : \Theta \times H \to [0, 1]$ that maps the type of the bidder θ_i and the history of bidding activity h_p into a probability $\sigma_i(\theta_i, h_p)$ of staying active at p, for each $p \in [r, c]$, conditional on having being active at all $p' < p$. Let $\sigma = (\sigma_1, ..., \sigma_n)$. For $i, j \in N$, $i \neq j$, let $\mu_{i,j} : \Theta \times H \to \Delta(\Theta)$ be the belief of bidder i about bidder j's types given his own type θ_i and the observed history h_p. Let $\mu_i = \{\mu_{i,j}\}_{j\neq i}$ and let $\mu = (\mu_1, ..., \mu_n)$. A perfect Bayesian equilibrium of the auction is a pair (σ, μ) such that σ is sequentially rational given μ and μ is derived from σ (and the priors) via Bayes' Rule if possible. Due to the symmetry of the model, it is natural to look for an equilibrium in symmetric bidding strategies, i.e., where σ_i does not depend on i. Our objective is to characterize the reserve and ceiling combination $\{r, c\}$ that maximizes the seller's ex–ante expected revenues, given that bidders play a symmetric equilibrium. For notational convenience, whenever we characterize a symmetric equilibrium in what follows, we will only specify the strategy profile σ and omit the belief profile μ, as it will cause no confusion.

We begin by deriving the symmetric equilibrium for the standard auction. In order to do so, it is convenient to define the function $S : [0, 1] \to \mathbb{R}_+$ as follows:

$$S(x) = (1 - F(x)) [E(V|V > x) - x] = \int_x^1 [1 - F(u)] \, du \tag{1}$$

It will often be useful to interpret $1 - F(x)$ as a "demand curve" written as a function of the price x. With such an interpretation, $S(x)$ is the "consumer's surplus," i.e., the area above the price x and under the demand curve $1 - F(x)$. Our first result shows that, in the symmetric equilibrium of the standard auction, any informed bidder stays active till the price reaches the value of the object regardless of the history, while an uninformed bidder stays active till the price reaches $\overline{v}$, the

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[12] If one lets $A(p) = \varnothing$ for $p > c$, P can also be taken to equal $\inf\{p \in [r, c] | \, |A(p)| \leq 1\}$.

ex-ante expected value of the object, but only if he observes no prior quits. Given such equilibrium behavior, $S(\overline{v})$ is the expected information rent an informed bidder earns when he is competing only against uninformed bidders and wins at a price $\overline{v}$. To state and prove the result concisely, let $\mathbf{1}_{\{\bullet\}}$ denote the indicator function and for $i = 1, ..., n$ and $p \in [0,1]$ define the sets $A_{-i}(p)$ and $Q_{-i}(p)$ as the set of bidders other than i who are active at and quit at p, respectively, in a manner identical to the sets $A(p)$ and $Q(p)$.

Proposition 1 *The following is a symmetric equilibrium of the standard auction: for all* $i = 1, ..., n$,

$$\sigma_i(\theta_i, h_p) = \begin{cases} \mathbf{1}_{\{p<v\}} & \text{all } h_p, \text{ if } \theta_i = v \in \mathbf{V} \\ \mathbf{1}_{\{p<\overline{v}\}}\mathbf{1}_{\{h_p \in H^n\}} & \text{if } \theta_i = u \end{cases} \tag{2}$$

The seller's equilibrium ex-ante expected revenue is given by:

$$R_s = \overline{v} - n\alpha(1-\alpha)^{n-1}S(\overline{v}). \tag{3}$$

Proof. We prove a stronger result. In Step 1 below we show that the strategy $\sigma_i(v, \bullet)$ is sequentially rational for bidder $i = 1, ..., n$ when $\theta_i = v \in \mathbf{V}$, for *every* strategy profile σ'_{-i} of the other bidders. In step 2 we show that, given informed bidders use the profile $\{\sigma_{-i}(v, \bullet)\}_{v \in \mathbf{V}}$, the strategy profile $\sigma_i(u, \bullet)$ is sequentially rational for bidder $i = 1, ..., n$ when $\theta_i = u$, for *every* symmetric pure strategy profile $\sigma'_{-i}(u, \bullet)$ used by other uninformed bidders.

Step 1: For $p \in [0,1]$ and an arbitrary history of bidding activity h_p with $A_{-i}(p') \neq \varnothing$ for all $p' < p$, consider bidder i when $\theta_i = v \in \mathbf{V}$ who has been active at all $p' < p$. Fix σ'_{-i}. Let $\mu_i[A_{-i}(p) = \varnothing | v, h_p]$ be the probability attached by i to all other previously active bidders quiting simultaneously at p given his type $\theta_i = v$ and the history h_p. Then

$$\mu_i[A_{-i}(p) = \varnothing | v, h_p] = \prod_{j \in \cap_{p':p'<p} A_{-i}(p')} \left[(1 - \sigma'_j(v, h_p))(1 - \mu_{i,j}(u|v, h_p)) + (1 - \sigma'_j(u, h_p))\mu_{i,j}(u|v, h_p)\right]$$

where $\mu_{i,j}(u|v, h_p)$ is the probability i attaches to j being uninformed given $\theta_i = v$ and h_p.

Suppose first that $p \geq v$. If bidder i quits at p, he wins the object only when (i) all other bidders also quit at p i.e., when $A_{-i}(p) = \varnothing = A(p) = B(p)$ and so $Q_{-i}(p)$ equals $\cap_{p'<p} A_{-i}(p')$ if $p > 0$ and equals $N - \{i\}$ otherwise; and (ii) i is chosen as the winner after uniform randomization among the $|Q_{-i}(p)| + 1$ bidders who quit, for an expected payoff equal to $\frac{\mu_i[A_{-i}(p)=\varnothing|v,h_p]}{|Q_{-i}(p)|+1}[v - p] \leq 0$. On the other hand, if bidder i continues at p, he wins with probability 1 and pays p when all other bidders quit at p since in this case $B(p) \subseteq A(p) = \{i\}$. Furthermore, if he subsequently wins the auction at a price $p' \geq p \geq v$, he earns non–positive payoffs conditional on winning. Therefore, the expected payoff from continuing at p is at most as high as $\mu_i[A_{-i}(p) = \varnothing | v, h_p][v-p] \leq \frac{\mu_i[A_{-i}(p)=\varnothing|v,h_p]}{|Q_{-i}(p)|+1}[v-p]$, since $p \geq v$. Therefore it is a weak best response of bidder i of type $\theta_i = v$ to quit at all $p \geq v$ regardless of h_p.

Next, suppose that $p < v$. Once again, if bidder i quits at p, he wins the object only if all other bidders quit at p and he wins the randomization for an expected payoff of $\frac{\mu_i[A_{-i}(p)=\varnothing|v,h_p]}{|Q_{-i}(p)|+1}[v-p] \geq 0$. However, if he continues at p and quits only when the price reaches v regardless of the history (i.e., plays according to $\sigma_i(v,\bullet)$), he wins with probability 1 when all other bidders quit at p and earns $v - p$ conditional on winning. Furthermore, if he subsequently wins the auction at a price $p' \geq p$ with $p' \leq v$, he makes non–negative expected profits. Finally, he cannot win the auction at any price $p'' > v$. It follows that his expected payoff from playing according to $\sigma_i(v,\bullet)$ is at least $\mu_i[A_{-i}(p) = \varnothing|v, h_p][v - p]$. Comparing with the expected payoff from quitting at p, we see that it is a weak best response for bidder i of type $\theta_i = v$ to continue at $p < v$ and quit only at v, regardless of h_p. This completes Step 1.

Step 2: Fix a symmetric pure strategy profile $\sigma'_{-i}(u,\bullet)$ for uninformed bidders and the profile $\{\sigma_{-i}(v,\bullet)\}_{v\in\mathbf{V}}$ for informed bidders and with $p \in [0,1]$ and an arbitrary history of bidding activity h_p with $A_{-i}(p') \neq \varnothing$ for all $p' < p$, consider bidder i with $\theta_i = u$ who has been active at all $p' < p$.

Consider first the case $h_p \in H^n$ and notice first that such a history is always consistent with the strategy profile used by the other bidders since, in particular, all other bidder may be informed and playing according to $\sigma_{-i}(v,\bullet)$ with $v \geq p$.

Suppose first that $p \geq \overline{v}$. If bidder i quits at p then he wins the object only when all other bidders also quit and i is chosen as the winner after uniform randomization among the bidders. Conditional on winning and conditional on every other bidder being uninformed, the expected payoff to i from quitting is $\overline{v} - p \leq 0$. The same is true if he continues at p and he wins the auction with probability 1 when all other bidders quit at p. Furthermore, if all other bidders do not quit at p and he subsequently wins the auction at a price $p' \geq p \geq \overline{v}$, he earns non-positive payoffs conditional on winning and conditional on every other bidder being uninformed. Thus, behaving according to $\sigma_i(u,\bullet)$ and quitting at $p \geq \overline{v}$ is a weak best–response to $\sigma'_{-i}(u,\bullet)$ conditional on every other bidder being uninformed. Further, given the behavior of informed bidders, uninformed bidder i can earn at most zero profits when every other bidder is not uninformed, i.e., there is a bidder $j \in A_{-i}(p')$ all $p' < p$, with $\theta_j = v \geq p$. In this case however bidder i also earns zero profits by quitting at p. Hence, behaving according to $\sigma_i(u,\bullet)$ is a weak best response conditional on the state where all other bidders are uninformed as well as on the state where at least one other bidder is informed, and so a best response overall, for any $h_p \in H^n$, $p \geq \overline{v}$.

Suppose next $h_p \in H^n$ but $p < \overline{v}$. As before, if bidder i quits at p then he wins the object only when all other bidders also quit and i wins the subsequent uniform randomization. Conditional on winning and conditional on every other bidder being uninformed, the expected payoff to i from quitting is $\overline{v} - p$. However, if he continues at p and plays according to $\sigma_i(u,\bullet)$, he wins with probability 1 when all other bidders quit at p and earns $\overline{v} - p$ conditional on winning and on every other bidder being uninformed. Furthermore, if he subsequently wins the auction at a price $p' \geq p$ with $p' \leq \overline{v}$, he makes non–negative expected profits conditional winning and on every other bidder being uninformed. Finally, he cannot win the auction at any price $p'' > \overline{v}$. Thus, behaving according to $\sigma_i(u, \bullet)$ and continuing at $p < v$ is a weak best–response conditional on every other bidder being uninformed. Further, recall that given the behavior of informed bidders and given $h_p \in H^n$ uninformed bidder i can earn at most zero profits when there is a bidder $j \neq i$, active at all $p' < p$ with $\theta_j = v \geq p$. In such a case however, by continuing at p and playing according to $\sigma_i(u, \bullet)$ bidder i will also earn zero profits. Thus, behaving according to $\sigma_i(u, \bullet)$ is a weak best response, whether or not all other bidders are uninformed, for any $h_p \in H^n$, $p < \overline{v}$. This establishes the result for for all $h_p \in H^n$, $p \in [0, 1]$.

To complete the proof, consider first the case where $h_p \notin H^n$ but quits have occurred at exactly one $p' < p$. Since $\sigma'_{-i}(u, \bullet)$ is a symmetric pure strategy profile and since $A_{-i}(p') \neq \varnothing$ for all $p' < p$, uninformed bidder i must conclude at h_p that all bidders who quit at p' must be informed and that $v = p'$, provided subsequent bidding behavior for $p'' \in (p', p)$ in the history h_p is consistent with $\sigma'_{-i}(u, \bullet)$. In such a case, regardless of the behavior of the other bidders, who i must believe are all uninformed, it is a weak best response for this bidder to quit at all $p'' > p$, since otherwise he only ends up paying a higher price for an object worth p' conditional on winning. In the case the history h_p is inconsistent with $\sigma'_{-i}(u, \bullet)$ (or if there have been multiple quits before p, a history necessarily inconsistent with the symmetric pure strategy profile $\sigma'_{-i}(u, \bullet)$ and $\{\sigma_{-i}(v, \bullet)\}_{v \in \mathbf{V}}$), the beliefs of bidder i cannot be derived by Bayes' rule. We suppose that after such histories bidder i attaches probability 1 to the event that $v = p'$ where p' is the lowest price at which quits occurred, so that it is a best response for bidder i to quit at all $p'' > p'$ given his beliefs. This shows that $\sigma_i(u, \bullet)$ is sequentially rational when informed bidders behave according to $\{\sigma_{-i}(v, \bullet)\}_{v \in \mathbf{V}}$ and all other uninformed bidders behave according to the arbitrary symmetric pure strategy profile $\sigma'_{-i}(u, \bullet)$ and so completes step 2.

Observe finally that since the object is always sold in equilibrium, the sum of the ex-ante expected payoffs of all bidders and the seller must equal the ex-ante expected value of the object $\overline{v}$. Since a bidder makes positive profits in equilibrium only when he is the only informed bidder and $v > \overline{v}$, and since there are n bidders, we immediately obtain (3) as the expression for expected revenues.■

We make the following remarks. The equilibrium of Proposition 1 is not the unique symmetric equilibrium, even though the steps in the proof are suggestive of an iterated dominance argument. There are other symmetric pure strategy equilibria that differ in behavior at knife–edge points, i.e., where a bidder of type v differs in his behavior at $p = v$, or a bidder of type u differs at $p = \overline{v}$. Such differences will not affect the seller's ex–ante expected revenues. We have been unable to find other symmetric equilibria, pure or mixed, where the seller obtains an ex–ante revenues that are different from (3). In Appendix C we consider a discrete approximation of the continuous model where the outcome of every symmetric equilibrium converges to that of the continuous case characterized above as the grid becomes fine. Notice also in this respect that the equilibrium characterized by

Proposition 1 is similar to separating symmetric equilibria characterized by Milgrom and Weber (1982) and Bikhchandani, Haile and Riley (2002), in more general models. The latter paper in particular demonstrates the existence of multiple symmetric equilibria that are identical from the perspective of seller revenues in continuous, irrevocable exit English auctions under common values.

With the standard auction in place as our benchmark mechanism we turn next to the question of imposing a reserve price $r > 0$ but no ceiling (i.e., $c = 1$). Levin and Smith (1996) show that the benefits from a reserve price asymptotically approach zero as the number of bidders increases. We confirm this finding in our context with our next result which shows that there is no benefit from imposing a reserve price for finite n. In order to do this, for each α we let n_α be the smallest value of n satisfying:

$$\frac{\overline{v} f(\overline{v})}{1 - F(\overline{v})} \geq \frac{\pi(1, n)}{1 - \pi(0, n)} \tag{4}$$

Proposition 2 *For each α, if $n \geq n_\alpha$, then the expected revenues of the seller from the standard auction is higher than the expected revenue from any auction with a positive reserve price.*

Proof. In the Appendix.□

For any positive reserve price r, informed bidder behavior will remain unchanged from the standard auction when $v \geq r$ (with no participation when $v < r$). However, for any positive reserve price, uninformed bidders will not participate with strictly positive probability, and will not participate at all for a sufficiently high reserve, e.g., when $r > \overline{v}$. In the Appendix we show that it does not pay the seller set such a high reserve price that excludes uninformed bidders completely and lowers the probability of making a sale, provided there are sufficiently many bidders (i.e., when (4) holds).[13]

For a sufficiently low reserve price, uninformed bidders will participate with positive probability. However, they will bid less aggressively than in the standard auction, typically quitting below $\overline{v}$. As a result, informed bidders will make higher profits in comparison to the standard auction. Since uninformed bidders make zero profits and the object is not sold with positive probability, the seller is strictly worse off from such a low reserve price relative to the standard auction, regardless of n.

Before we end this section we turn briefly to posted price mechanisms where the seller offers up the object at a fixed price. Bulow and Klemperer (2002) as well as Campbell and Levin (2002) demonstrate that posted price mechanisms can be attractive to a seller in comparison to the standard auction because the rent transferred to bidders when the true value of the object is higher than the posted price mitigates the winner's curse and encourages aggressive bidding. In our model, a posted price mechanism is not without its costs however as an uninformed bidder

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[13] Since $\overline{v} = E[\frac{1}{h(V)}]$, note from (4) that for all distributions where the inverse of the hazard rate $\frac{1}{h(V)}$ is a convex function such a high reserve price is not optimal for all $n \geq 2$. The uniform distribution is one example of such a distribution.

may end up winning the object with a greater chance when informed bidders know the value of the object is below the posted price and so do not bid. As we show next, the ‘upside’ benefit of reducing the winner’s curse that a posted price has may not be enough to swamp its ‘downside’ risk, so that the standard auction will yield the seller higher expected revenues compared to any posted price mechanism, at least for n large enough.

Proposition 3 *For each α, there exists n'_α such that if $n \geq n'_\alpha$, the expected revenues of the seller from the standard auction is higher than the expected revenue from any posted price mechanism.*

Proof. In the Appendix.□

The intuition for this result is similar to that for the last one. For any fixed price p, informed bidders participate if and only if $v > p$. For a high enough p (e.g., $p \geq \overline{v}$), uninformed bidders will not participate. In such a case, the seller can obviously do better by instead holding an auction with a reserve price equal to p. From Proposition 2 however, such an auction is dominated by the standard auction when n is large enough.

For a sufficiently low posted price, uninformed bidders will participate with positive probability. However, conditional on winning at the low posted price $p < \overline{v}$, informed bidders will make higher profits compared to winning at a price $\overline{v}$ in the standard auction. Furthermore, an informed bidder will make positive profits from winning not only when he is the only informed bidder (as in the standard auction) but also when there is more than one competing informed bidder present. Since the probability of being the only informed bidder becomes vanishingly small when there are many bidders, any posted price mechanism is dominated by the standard auction for n large enough.

3 Explicit Ceilings

The discussion of posted price mechanisms in the last section suggest that imposing a ceiling in an auction may be better for seller revenues than both a posted price as well as a standard auction. Like a posted price mechanism (but unlike a standard auction), a ceiling on allowable bids reduces the fear of uninformed bidders from being outbid precisely when the object is worth bidding for. Unlike a posted price mechanism (but like the standard auction), by holding an auction at prices below the ceiling the seller reduces the downside winner’s curse problem for uninformed bidders as they typically pay a price lower than the ceiling when the value of the good is low. In this section we consider a simple example of a ceiling and show that it dominates the standard auction when the number of bidders is large enough.

Let $c^* \in (\overline{v}, 1)$ be a candidate ceiling price that solves the following equation:

$$\pi(0, n-1)[\overline{v} - c^*] + (1 - \pi(0, n-1))S(c^*) = 0 \tag{5}$$

It is straightforward to verify the existence of a unique c^* that solves (5). In Proposition 4 we characterize the symmetric equilibrium of the auction with a ceiling c^* (and $r = 0$, henceforth referred to as the c^*–auction). In symmetric equilibrium, informed bidders stay in till the true value or the ceiling c^*, whichever is lower, while uninformed bidders bid till the ceiling, dropping out before as soon as any one else drops out. Expression (5) states that an uninformed bidder earns zero expected profits conditional on bidding at the ceiling, given that no quits have occurred before — the first term is his probability weighted expected payoff from winning in the event that all other bidders are uninformed and have bid till the ceiling, while the second term is his probability weighted expected payoff from winning in the event that at least one other bidder is informed and $V > c^*$.[14] Of course, the bidder earns zero profits when he does not win. Finally, if the true value of the good $V < c^*$ and at least one informed bidder is present, the latter drops out at V triggering drop outs by all uninformed bidders immediately after, so that uninformed bidders earn zero expected profits overall. Observe that c^* is increasing in n — when n becomes large an uninformed bidder attaches a large probability to at least one other bidder being informed and so is willing to bid till a high ceiling.[15]

With the ceiling c^* a bidder earns profits only when both the following conditions are satisfied: (i) he is informed, and (ii) the value of the good is greater than c^*. We therefore obtain expression (6) below for the seller's ex–ante expected revenues. Note than in contrast to the standard auction (but like a posted price mechanism), an informed bidder may earn strictly positive profits even when other bidders are informed.

Proposition 4 *Symmetric equilibrium strategies in the c^*–auction are as follows. Any informed bidder who knows $V = v$, quits only when $p > v$ if $v < c^*$, staying in till c^* otherwise, regardless of the history of quits; while any uninformed bidder quits whenever anyone else quits but otherwise does not quit at any $p \in [0, c^*]$. The ex–ante expected revenue for the seller equals*

$$R(c^*) = \overline{v} - \alpha S(c^*) \tag{6}$$

Proof. Fix the ceiling c^* defined by (5). It is straightforward to show, by using arguments identical to those used previously, that it is essentially a weakly dominant strategy for an informed bidder to stay in the auction till the true value, or the ceiling, whichever is lower.

Consider next uninformed bidders. Due to arguments identical to those contained in the proof of Proposition 1 it follows that uninformed bidders will not drop out at any price strictly less than $\overline{v}$ conditional on having observed no drop–outs, in a symmetric equilibrium.

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[14] Note that in both cases, the bidder will be tied with n other bidders when winning, and so will win with probability $\frac{1}{n}$, which thus cancels out from the zero–profit condition (5). Furthermore, the probabilities in expression (5) should both be conditional probabilities. But since this only has the effect of dividing the expression by a constant, it is harmless to drop it.

[15] We show in Appendix B that c^* is the highest possible ceiling consistent with the uninformed bidders bidding up to the ceiling with probability 1, and that with two bidders c^* is the optimal non-trivial ceiling.

Let $\sigma_{\overline{v}} \in [0,1]$ be the probability with which uninformed bidders continue at $p = \overline{v}$, given no prior drop–outs. Observe first that by dropping out at $\overline{v}$, an uninformed bidder earns zero expected payoffs, and this is true regardless of how other uninformed bidders behave. Observe next that $\sigma_{\overline{v}} > 0$ in a symmetric equilibrium. For if not, then by continuing at $\overline{v}$ till c^*, dropping out immediately after a drop–out, an uninformed bidder competes only against informed bidders and so makes strictly positive expected payoffs when he wins at the ceiling and zero payoffs otherwise.

We show next that, in fact, $\sigma_{\overline{v}} = 1$. Suppose not, so that an uninformed bidder is indifferent between continuing and dropping out at $\overline{v}$. Let $k' = 0, ..., n$ be the number of bidders who continue at $\overline{v}$. Then, conditional on observing $k' \geq 2$ bidders continuing at $\overline{v}$, any uninformed bidder who continues at $\overline{v}$ must continue with probability 1 at each $p \in (\overline{v}, c^*]$, given no further drop–outs. For suppose to the contrary that for some such k' uninformed bidders continue with probability $\sigma_p < 1$ at some $p > \overline{v}$. The expected payoffs from dropping out at p conditional on having reached it is equal to $(\overline{v} - p) < 0$, times the probability of winning the object upon dropping out at p, an event that occurs with positive probability only when all other bidders are uninformed. Observe that since $\sigma_p < 1$, this is equal to the expected payoffs overall from following this strategy of continuing at p with probability σ_p. However, for any such k' and p, by dropping out earlier, at some $p' = \overline{v} + \varepsilon < p$, any such uninformed bidder can earn strictly higher expected payoffs, for ε small enough, a contradiction establishing that uninformed bidders must continue till c^*, for each $k' \geq 2$, after continuing at $\overline{v}$, given no further drop–outs. It follows from this that, for each k', an uninformed bidder must make zero expected payoffs given that he continues at $\overline{v}$ and observes that k' bidders are continuing. For if from continuing at $\overline{v}$ and observing k', an uninformed bidder earns negative expected payoffs from continuing till c^*, for some k', then it is better for him to drop–out immediately and earn zero expected payoffs. Given this, if an uninformed bidders earns strictly positive payoffs after continuing at $\overline{v}$ and observing k', for some k', it is strictly better for him to continue at $\overline{v}$ rather than drop–out with positive probability, a contradiction with the supposition that he is indifferent between dropping out and continuing at $\overline{v}$. However, the expected payoffs from continuing at $\overline{v}$, observing k' and then continuing till c^*, in the case that $k' = n$, is equal to (ignoring multiplicative constants),

$$\pi(0, n-1)\xi_{\overline{v}}(n-1, n-1)[\overline{v} - c^*] + \sum_{k=1}^{n-1} \pi(k, n-1)\xi_{\overline{v}}(n-1-k, n-1-k)S(c^*) \tag{7}$$

where $\xi_{\overline{v}}(k, K) = \binom{K}{k}\sigma_{\overline{v}}^k(1-\sigma_{\overline{v}})^{K-k}$ is the probability that k out of K uninformed bidders continue at $\overline{v}$. Since $\sigma_{\overline{v}} \in (0,1)$, from the definition of c^* in (5) it is easily seen that the expression in (7) is strictly positive, a contradiction with the fact established above that the expected payoffs from continuing at $\overline{v}$ and observing $k' = n$ is zero. Thus, $\sigma_{\overline{v}} = 1$.

Given that uninformed bidders continue with probability 1 at $p = \overline{v}$, it follows that they must continue with probability 1 at any $p \in (\overline{v}, c^*]$, given no drop–outs— for if not, they make strictly negative profits from dropping out at any price $p > \overline{v}$ and zero profits from dropping out at $\overline{v}$. Finally, given these strategies of informed and uninformed bidders, it is a best–response for uninformed bidders to drop–out upon observing a drop–out at any $p < c^*$ since such a drop–out can come only from an informed bidder.

In equilibrium, from (5) it follows that uninformed bidders earn zero expected payoff, while informed bidders obtain a positive payoff equal to $S(c^*)$ only when the true value of the object is at least c^* and he wins with probability $1/n$. Since the good is always sold and there are n ex–ante identical bidders, the ex–ante expected revenue for the seller equals

$$R(c^*) = \overline{v} - \alpha S(c^*)$$

This concludes the proof.■

How does the seller do in the c^*–auction compared to the standard auction? Although by imposing a ceiling the seller gives up on high bids from better informed bidders, in Proposition 5 we show that the added participation that a ceiling generates from poorly informed bidders more than compensates the seller when there are many bidders or when most bidders are likely to be poorly informed.

Proposition 5 1. *For each $n \geq 2$, there exists $\alpha^*_n \in (0, 1]$ such that the c^*–auction earns the seller higher revenues than the standard auction iff $\alpha < \alpha^*_n$.*

2. *There exists n^* such that for all $n \geq n^*$, the c^*–auction earns the seller higher revenues than the standard auction for every $\alpha \in (0, 1)$.*

Proof. See the Appendix.□

In the standard auction, the seller leaves rents for bidders when only one of the bidders is informed and the true value of the object is at least the uninformed bidders' maximum possible bid $\overline{v}$. With a ceiling c^* however, since uninformed bidders bid till c^*, the seller gains in states where there is only one informed bidder and the value of the good $V > \overline{v}$, as she receives a price higher than $\overline{v}$ in these states. The seller also gains in states where all bidders present are uninformed, when she receives a price c^* that is higher than $\overline{v}$. However, a ceiling has some costs as the seller loses in states where multiple informed bidders are present and $V > c^*$. When $\alpha < \alpha^*_n$ the seller's expected loss in the last case is swamped by her gains from increased bidding by uninformed bidders. In such cases, the seller earns higher revenues by imposing a ceiling.

To gain intuition for the second part of the result note first that for a large number of bidders there are almost always two informed bidders present. Thus, the seller almost always loses from a ceiling whenever $V > c^*$. However, in order to guarantee participation from uninformed bidders till the ceiling, the expected loss to an uninformed bidder from winning at the ceiling when all other bidders are uninformed must balance out his expected gains from winning at the ceiling against at least one informed bidder. With a large number of bidders, each uninformed bidder knows that at least a few of the other bidders are likely to be informed and so is willing to bid till a high ceiling. In other words, as n grows, c^* grows as well so that the probability that $V > c^*$ times the expected loss to the seller from a ceiling in that event becomes small. However, the seller still gains below the ceiling, and hence overall, since uninformed bidders bid up to $c^* > \overline{v}$ in the c^*–auction. From Propositions 2 and 3 it then follows that the c^*–auction also dominates any auction with a reserve price as well as any posted price mechanism for n large enough.

A ceiling is beneficial in our model exactly because it encourages uninformed bidders to bid beyond $\overline{v}$, the ex–ante expected value of the object, in return for the positive probability of winning at the ceiling against an informed bidder and making positive profits when $V > c^*$. In contrast, in the standard auction, even though conditional on reaching $\overline{v}$ with no drop–outs an uninformed bidder may attach (for large n) a large probability to the event that $V > \overline{v}$, such a bidder will always be outbid by an informed bidder if he competes further and so is unwilling to do so. In a sense then, by banning price discrimination via competitive bidding above the ceiling, the seller provides a bidding guide or anchor price for uninformed bidders. This allows him to more effectively price discriminate below the ceiling.

Example Suppose that V follows the uniform distribution on $[0, 1]$ so that $\overline{v} = \frac{1}{2}$ and

$$S(x) = \frac{1}{2}(1 - x)^2$$

The expected revenue in a standard auction equals

$$R_s = \frac{1}{2} - n\alpha(1 - \alpha)^{n-1}\frac{1}{8}$$

Note that for the uniform distribution condition (4) holds for all $n \geq 2$, so that from Proposition 2, a standard auction dominates any auction with a positive reserve price. Furthermore, note from Proposition 3 that any posted price $p \geq \overline{v}$ is dominated by an auction with the same reserve price which, by the previous remark, is dominated by the standard auction for all n. Finally, it is easily seen that a posted price $p < \overline{v} < c^*$ is dominated by the c^*–auction for all n. To see this, observe that for such a posted price an informed bidder makes profits equal to $S(p)$ conditional on winning, and wins with probability at least $1/n$ (when all other bidders bid with him) for a total ex–ante expected profits for all bidders that is at least $\alpha S(p)$ (since uninformed bidders also make non–negative profits). But since $p < c^*$, $\alpha S(p) > \alpha S(c^*)$, the total ex–ante expected profits for all bidders in the c^*–auction.[16] Thus, either the standard auction or the c^*–auction dominate all other mechanisms that we have considered so far. Consequently, we will focus on these two mechanisms for the rest of this example.

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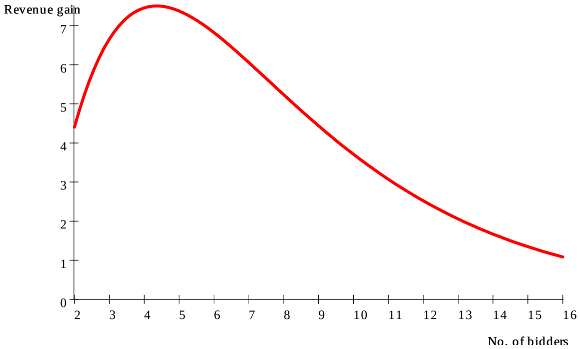
[16] While we do not state this as a proposition, this last argument clearly does not depend on the distributional assumptions made in this example.

For the uniform distribution, exploiting (5), it can be seen that

$$c^* = \frac{1}{1 + \sqrt{(1-\alpha)^{n-1}}}$$

For $n = 2$, the cut–off value $\alpha_2^* = 0.828$ below which the c^*–auction dominates the standard auction. Furthermore, for $n \geq n^* = 4$, the c^*–auction dominates the standard auction for all $\alpha \in (0, 1)$. The figure below depicts the percentage gain in revenue from the c^*–auction over the standard auction, as a function of n, when $\alpha = 0.25$. It is easy to verify that the percentage revenue–gain is the highest at $n = 4$, when $c^* = 0.60624$, $R(c^*) = 0.480\,62$ and $R_s = 0.447\,27$ for an expected gain in revenue of 7.46% and a reduction in rents by 63.25%.



Percentage gain in revenue

Bulow and Klemperer (1996) show that standard auctions, even when they are not optimal, frequently do quite well compared to the optimal mechanism. In their model an auction with a last minute reserve may be optimal, but the seller's revenues from such a mechanism with n bidders is lower than that from the standard auction with just one more bidder.[17] This is a compelling argument for devoting resources to expanding the size of the bidder pool rather than to performing the evaluations necessary for a last minute reserve, from the perspective of the seller. However, in our framework with hierarchically informed bidders, such a bound does not obtain and a simple ceiling yields significant gains over the standard auction. For instance, with $\alpha = 0.25$, the seller needs to have 11 bidders in the standard auction to have revenues higher than what she would earn in the c^*–auction with 4 bidders. Therefore, when expanding her market is not especially easy, the seller may be well–advised to impose a ceiling. While a ceiling imposes greater informational requirements on the seller in terms of estimating the market parameters compared to a standard auction, these requirements are no more onerous than those involving reserve prices or standard monopoly pricing.

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[17] In an auction with a last minute reserve, the seller holds an auction, but instead of transferring the good to the highest bidder at the going price, the seller offers the good to the highest bidder at a take-it-or-leave-it fixed price. It is important that the seller commits ex ante to such a menu of fixed prices, depending on the history of bidding (see footnote 22 in Bulow and Klemperer, 1996).

We do not consider the optimal mechanism in this paper (see, however, Appendix B for a characterization of the optimal ceiling and reserve combination). Mechanisms involving an auction followed by negotiation or bargaining between the seller and all or some of the bidders, of the sort considered by Bulow and Klemperer (1996) and Lopomo (2001), dominate an auction with or without a ceiling.[18] Instead, we justify our focus on auctions with ceilings as a simple change in the rules of a commonly observed mechanism that should be easy to understand for the bidders. Moreover, as we argue in the next section, an implicit ceiling may be focal for the bidders when the seller finds it difficult to commit to the rules of the auction.

4 Implicit Ceilings

So far we have considered a situation where the seller is able to commit to honoring the outcome of the auction, with or without a ceiling (or reserve) price. In many situations such an ex–ante commitment may not be feasible and the seller may be tempted to cancel the auction after it concludes and resell the object later. In this section, we consider a simple version of this lack of commitment problem and show that the lack of commitment changes the game fundamentally.[19]

While with commitment, we showed that there was a unique equilibrium in symmetric bidding strategies in the standard auction, there can be many such (perfect Bayesian) equilibria once commitment is absent. In particular, the strategies which constituted an equilibrium in the standard auction (call this the *standard outcome*) may continue to be an equilibrium in the game without commitment, but there can also be equilibria which display a ceiling feature, with ceiling equal to c^* (call this the c^**-outcome*). In the latter case, the lack of commitment creates an implicit ceiling and such an implicit ceiling benefits the seller for the same reason that an explicit one does.

In other words, the seller does not need to commit to the ceiling c^* in order to generate the c^*-outcome. Furthermore, the inability to commit to the outcome of a standard auction may actually make the seller *strictly* better off.[20] The intuition derives from the fact that the lack of commitment creates the possibility of a seller exploiting, after the fact, the information revealed

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[18] Bulow and Klemperer (1996) demonstrate optimality under independent signals or private values. Lopomo (2001) extends this result to correlated signals, but in a smaller class of mechanisms that satisfy a posterior implementability or no-regret condition. With interim implementability and correlated signals Lopomo shows the optimality of mechanisms with both auction and bargaining like features.

[19] While limited commitment may be most likely to occur in informal environments, the right to cancel is observed even in formal regulated environments. U.S. Government agencies such as the Department of the Interior and the Department of Agriculture reserve the right to cancel, respectively, offshore oil and gas lease auctions and procurement auctions even *after* announced reserve requirements have been met by the bidders. However, such a right is infrequently exercised. See Porter (1995) and MacDonald et al. (1998).

[20] That such a result is possible is entirely due to the fact that the standard auction is not the optimal mechanism. Since any equilibrium of a given extensive form game can be implemented as a direct mechanism by the revelation principle, the seller cannot be strictly better off from not being able to commit to the optimal direct mechanism.

by the bidding process. This may affect, differentially, the bidding incentives of different kinds of bidders and so end up earning the seller higher revenues on average.

To fix ideas, suppose that the seller is constrained to hold a standard auction but after the auction concludes according to its rules, she can cancel the auction and instead offer the object at a take-it-or-leave-it price. Call this price P_2. The price P_2 must be sequentially rational for the seller in equilibrium, i.e., it is not a pre–commitment. We consider two possible variants of the rules of the game after cancellation occurs. Under *Rule 1*, the seller can ask only those bidders who have not quit the auction prior to its conclusion to bid at the fixed price following cancellation. Under *Rule 2*, all bidders, including those who have quit prior to the conclusion of the auction, are allowed to submit a bid following cancellation.

We look for equilibria of this game with cancellation possibilities where cancellation does not occur on the path of play. We call such equilibrium outcomes cancellation–proof. Under some mild additional conditions, we show below that under Rule 1, the standard outcome is cancellation–proof. The seller never cancels the auction as she does not expect to obtain a higher price from the winning bidder if she does. However, under Rule 2, the standard outcome never emerges as an equilibrium — the seller will cancel the auction whenever the auction outcome reveals that $v > \overline{v}$ and only one bidder is informed, as she correctly expects to obtain a higher price from the other (uninformed) bidders upon cancellation.

In contrast, the c^*-outcome is always cancellation–proof under either rule. In such an equilibrium, no bidder bids above c^* fearing cancellation. The threat of cancellation is credible since the seller concludes that the true value of the object must be higher than c^* if any bidder actually bids higher than c^*, and so expects to obtain a price $P_2 > c^*$ from some bidder after cancellation. In equilibrium, the auction is never cancelled.[21]

To state and prove our result concisely, we first define some entities. Let

$$J(x, y) = [F(x) - F(y)][E(V|y < V < x) - x]$$

and

$$p^* = \arg\max_x \; [1 - F(x)]x$$

Recall that there is a unique interior p^*, by the monotone hazard rate assumption. We assume in what follows that $c^* \geq p^*$. While it is easy to find examples of distributions where this condition holds for all n (e.g., the uniform distribution), notice it will obtain for any distribution for n large enough, since c^* becomes large in such a case, while p^* is independent of n.

Proposition 6 *Suppose $c^* \geq p^*$.*

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[21] We ignore discounting or other direct costs of cancellation. Such costs will not affect our results. We do consider an endogenous indirect cost of cancellation however — by attempting to sell the object at a higher take it or leave it price following cancellation the seller may end up not selling the object at all.

1. *The c^*–outcome is cancellation–proof under either Rule 1 or 2.*
2. *The standard outcome is not cancellation–proof under Rule 2. It is cancellation proof under Rule 1 if $p^* \leq \overline{v}$.*

Proof. Part 1

Assume first that Rule 2 holds and consider the following strategy profile. Suppose that bidding strategies are as in the c^*–auction. Further, the seller does not cancel the auction whenever it concludes according to rules at some price $P \leq c^*$, expecting not to obtain a higher price if she does so. On the other hand, if the auction concludes with some bidder active at a price beyond c^*, the seller cancels the auction and sets a price $P_2 = 1$, given her (and uninformed bidders') beliefs that $v = 1$ in such an event. Whenever the seller cancels the auction and sets a price P_2, informed bidders bid at P_2 if and only if $v > P_2$, while uninformed bidders bid at P_2 if and only if the expected value of the object conditional on winning and given the observed history and conjectured behavior of other bidders is at least P_2. We show now that this constitutes an equilibrium of the cancellation game under Rule 2. Further, essentially the same arguments can be employed to arrive at this conclusion even under Rule 1.

It is immediate that bidding strategies are sequentially rational after the seller cancels the auction and sets a price P_2. We now investigate the sequential rationality of the seller's cancellation strategy. First, suppose that the auction concludes at some price $P \leq c^*$ with at least one quit at some $p < c^*$. Given the bidding behavior in the auction, if this occurs on the path of play, the seller's (and uninformed bidders') beliefs are pinned down and she must believe that $v = P$. We impose the same beliefs after any such history that is also off the path of play. Consequently, if the seller cancels and sets a price $P_2 > P$ she will not attract any bids and so it is optimal for her not to cancel.

Next suppose that at least one bidder has remained active beyond c^*. This is an off–the–path of play event and so we have freedom to specify the seller's beliefs about the value of the object. We consider the natural case where the seller believes that the winning bidder is informed and value of the object is at least c^* and, for simplicity, suppose that she believes that $v = 1$ and so sets $P_2 = 1$, expecting to sell the object at that price subsequently.

The critical case to consider is where the auction concludes at $P = c^*$ with no prior quits, and all bidders quit thereafter. If the seller does not cancel the auction she receives a price of c^* for sure. To get higher revenues after cancellation it is necessary that $P_2 \geq c^*$. We claim that it is an equilibrium for uninformed bidders not to bid at any such P_2, given that an informed bidder will bid at P_2 iff $v > P_2$. For if an uninformed bidder submits a bid at P_2, her expected payoff (omitting the conditional probability constant) is

$$\pi(0,n-1)(\overline{v}-P_2)+\sum_{k=1}^{n-1}\pi(k,n-1)\left[\frac{S(P_2)}{k+1}+J(P_2,c^*)\right] < \pi(0,n-1)(\overline{v}-c^*)+\sum_{k=1}^{n-1}\pi(k,n-1)S(c^*)=0$$

where the inequality obtains because $P_2 \geq c^*$, and because $\frac{S(P_2)}{k+1}+J(P_2,c^*) < S(c^*)$, as $k \geq 1$, $S(c^*) \geq S(P_2)$ and $J(P_2,\, c^*) \leq 0$; and the equality obtains from the definition of c^*. It follows that the seller may not receive a bid at the price P_2 with positive probability (if all bidders are uninformed), and conditional on receiving a bid, she should choose P_2 to maximize $[1-F(P_2)]P_2$ subject to $P_2 \geq c^*$, where we have ignored some constants in the objective function. Since $p^* \leq c^*$ by hypothesis, and since F satisfies the monotone hazard rate condition, we obtain $P_2 = c^*$ and so the seller is strictly better off from not cancelling the auction in the first place.

It remains to check that bidding behavior in the auction is optimal. But this follows from the fact that the prescribed strategy profile is an equilibrium of the c^*–auction and that the seller's cancellation strategy makes the cancellation game strategically identical to the c^*–auction from the perspective of the bidders. In particular, if any player contemplates staying in beyond c^*, given bidding has reached c^*, he then expects to face a take-it-or-leave-it price of 1, so that his expected payoff from continuing is 0. If he behaves as specified, then he expects to earn strictly positive profits from quitting after c^*, when $v > c^*$ and the bidder is informed, and non–negative profits if the bidder is uninformed. This concludes the proof of part 1.

Part 2.

Consider first Rule 2 and to obtain a contradiction suppose that the standard outcome is cancellation–proof. Consider the outcome of the auction when all bidders but one have quit after $\overline{v}$. Such an outcome is on the path of play. Thus, the seller (and all uninformed bidders) must infer that the remaining bidder is informed, and that $v > \overline{v}$. It follows that if the seller sets $P_2 = \overline{v} + \varepsilon$ then she will attract a bid at least from all the uninformed bidders who have quit at $\overline{v}$. Hence, she has a strict incentive to cancel the auction, yielding the desired contradiction.

Now suppose Rule 1 holds. We show that the standard outcome is cancellation proof provided $p^* \leq \overline{v}$, i.e., the following strategy profile is an equilibrium. Bidders behave as in the standard auction, the seller never cancels the auction. In the event the seller cancels the auction and sets a price P_2, informed bidders bid at P_2 if and only if $v > P_2$, while uninformed bidders bid at P_2 if and only if the expected value of the object conditional on winning and given the observed history and conjectured behavior of other bidders is at least P_2.

Once again, it is immediate that bidding strategies are sequentially rational after the seller cancels the auction and sets a price P_2. We now investigate the sequential rationality of the seller's cancellation strategy. First, if there is a quit at some $p < \overline{v}$, the seller and all uninformed bidders infer that the true value is p, and hence the seller honours the rules of the auction as she cannot attract bids by cancelling and setting a price P_2 that is higher than what she obtains in the auction. Second, if all bidders quit at $\overline{v}$, the seller infers they are all uninformed, and the seller again cannot gain by cancelling and setting $P_2 > \overline{v}$. Next, if at least two bidders remain in the auction at $\overline{v}$, the seller must infer they are both informed who will bid till the true value of the object, and so she has no incentive to cancel the auction when they both quit. We suppose that the seller believes that value of the object is equal to the price obtained in the auction after all other histories that are inconsistent with the standard outcome, and so does not cancel the auction.

Consider now the critical case where all bidders but one quit at $\overline{v}$. The seller knows then that the surviving bidder is informed, and the true value v is at least $\overline{v}$. If she does not cancel the auction, her revenue is $\overline{v}$. If she cancels the auction and asks for a higher price P_2 from this bidder, then she may fail to sell the object. That is, the optimal P_2 must maximize $[1 - F(P_2)]P_2$ subject to $P_2 \geq \overline{v}$, ignoring some constants. Once again, since $p^* \leq \overline{v}$ and F satisfies the monotone hazard rate property, it follows that she cannot do better by cancelling. To complete the proof, observe that given the cancellation strategy of the seller, the strategic considerations facing each bidder is the same as in the standard auction, so that bidding as in the standard auction must be optimal for each bidder given the behavior of others.■

Proposition 6 provides justification for our earlier claim that a fixed price or best offer mechanism may be interpreted as a negotiation mechanism that generates an implicit ceiling, when cancellation is a real possibility. To see this, suppose that at the beginning of the game the seller makes an announcement offering up the object at a "fixed-price P_0 or the best-offer", by which the seller claims he will either accept the best offer from the bidders, or sell it at the fixed price. This announcement is cheap talk, i.e., the seller cannot commit to implementing the content of his announcement. However, when $P_0 = c^*$ it can make the c^*–outcome focal. In fact, part 1 above shows that there is an equilibrium of this game in which the seller's announcement is self–fulfilling — no one bids above the fixed price, the auction is never cancelled and the seller indeed always sells the object at c^* or the best offer.[22]

A fixed price or best offer mechanism supported by an implicit threat of cancellation is by no means the only possible instance of an implicit ceiling. In many situations, a seller may not be able to affect the terms of sale at all, possibly because of established historical norms or because the auction format is decided upon by an intermediary. However, before the auction starts, the seller may still be able to modify the object that she sells by explicitly attaching a contractually specified discretionary right to buy it back from the winner. The price at which she will buy back the object may then act as an implicit ceiling. In a similar vein, governments frequently possess an implicit right to undo the privatization of formerly state–owned enterprises. Proposition 6 illustrates the possibilities of brinkmanship created by such a 'fog of uncertainty,' and we leave a

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[22] The standard outcome may also self–fulfilling in this sense, under the conditions of part 2 of Proposition 6, when $P_0 = 1$.

fuller investigation of such interesting issues for future research.

5 Conclusion

In common value environments with ex–post hierarchically informed bidders, a bid ceiling may significantly raise seller revenues. In particular, such a ceiling mechanism may dominate auctions with reserve prices as well as posted price mechanisms. Such price ceilings may be explicitly announced (and committed to) or implicit, arising in the latter case out of a credible threat to cancel the first auction after it is over and resell the object. While a bid ceiling prevents the seller from obtaining bids higher than the ceiling, it benefits the seller by alleviating the winner's curse problem for poorly informed bidders, and so, by increasing competition and demand below the ceiling. These results provide insight into the prevalence of fixed price or best offer mechanisms that make explicit the possibility of negotiations. More broadly interpreted, our results suggest that it may be beneficial to impose an upper limit on the share of the pie one is willing to receive in multi–agent bargaining environments. An investigation of such questions outside the context of auctions is therefore also of interest.

6 Appendix

6.1 Appendix A: Proofs

Proof of Proposition 2

We first define some basic objects in order to characterize the symmetric equilibrium of an auction with a reserve price $r > 0$. For $x \in [0, 1]$, define

$$G(x) = F(x)\,[E[V|V < x] - x] = -\int_0^x F(u)du \tag{8}$$

and observe that it is a negative, decreasing and concave function. Let r^* be defined as the solution to

$$\pi(0, n-1)[\overline{v} - r^*] + (1 - \pi(0, n-1))G(r^*) = 0 \tag{9}$$

Note from the definition of $G(.)$ that such an $r^* \in (0, \overline{v})$ exists and is unique. The next lemma characterizes symmetric equilibrium in the auction with a reserve price $r > 0$. We will say that a bidder is present at the open whenever he belongs to the set $A(r)$ and say that he participates in the auction (with positive probability) if he is present at the open (with positive probability).

Lemma 7 *Fix $r > 0$. Then symmetric equilibrium strategies are as follows. Any informed bidder who knows $V = v$ participates in the auction if and only if $v \geq r$, and given that he participates, quits only at $p > v$, staying in otherwise, regardless of the history of quits. Furthermore,*

1. *if $r \geq r^*$ then uninformed bidders participate in the auction with probability* 0*;*

2. *while if $r < r^*$, then uninformed bidders participate in the auction with probability $\sigma_r \in (0,1)$ given by the solution to*

$$\pi(0, n-1)\,[1-\sigma_r]^{n-1}\,[\overline{v} - r] + \sum_{k'=1}^{n-1} \pi(k', n-1)\,[1-\sigma_r]^{n-1-k'}\,G(r) = 0 \tag{10}$$

   and upon participation, an uninformed bidder quits as soon as anyone quits and otherwise quits only at $p > \overline{v}_k$, where $\overline{v}_k$ is the expected value of the object given all k bidders present at the open are uninformed, for $k \in \{1, ..., n\}$.

Proof of Lemma. The optimality of the behavior of informed bidders immediately follows using arguments analogous to the proof of Proposition 1. Given this consider the behavior of uninformed bidders in a symmetric equilibrium.

Fix $r > 0$ and let $\sigma_r \in [0,1]$ be the probability with which an uninformed bidder participates in the auction. Let $k \in \{0, 1, ..., n\}$ be the total number of bidders present at the open.

Observe first that given the behavior of informed bidders and conditional on participation and observing that $k > 1$, an uninformed bidder makes at most zero expected profits whenever at least one other bidder present at the open is informed. On the other hand, conditional on the event $k > 1$ and all other bidders present at the open are uninformed, an uninformed bidder must earn zero expected profits, in any symmetric equilibrium, at least whenever such an event has positive probability (i.e., $\sigma_r > 0$).

To see this, let $\overline{v}_k$ denote the expected value of the object given that all k bidders present at the open are uninformed (where we suppress the dependence of $\overline{v}_k$ on σ_r). If an uninformed bidder makes positive profits conditional on winning, given $k > 1$ and all other bidders are uninformed, then all uninformed bidders must quit with strictly positive probability at some price $p < \overline{v}_k$, conditional on a history of no quits. In such a case, the uninformed bidder wins the object with probability $\frac{1}{k}$ (for strictly positive profits $\overline{v}_k - p$) by quitting at p, when all other bidders are uninformed and also quit at p, and does not win the object in any other state (ignoring the zero probability case that $V = p$). If instead he continues at p and drops out immediately after he wins the object with higher probability whenever he wins by quitting at p, and earns the same expected profits in all other cases, a contradiction with equilibrium. Similarly, if an uninformed bidder earns negative profits conditional on $k > 1$ and all other bidders present are uninformed, all uninformed bidders must remain active with positive probability at some $p > \overline{v}_k$, conditional on a history with no quits. It is easy to see that by quitting with probability 1 at such a p an uninformed bidder strictly increases his expected profits. It follows that an uninformed bidder must earn zero expected profits conditional on being present at the open and observing $k > 1$, whenever $\sigma_r > 0$.

Consider next the profits of the uninformed bidder, conditional on being the only bidder present at the open (i.e., $k = 1$) and so winning at the reserve r. Letting $D = \sum_{k'=0}^{n-1} \pi(k', n-1)[1 - \sigma_r]^{n-1-k'}$, this is given by

$$\frac{1}{D}\left[\pi(0, n-1)[1-\sigma_r]^{n-1}[\overline{v} - r] + \sum_{k'=1}^{n-1} \pi(k', n-1)[1-\sigma_r]^{n-1-k'} G(r)\right]. \qquad (11)$$

The first term reflects the probability that all other bidders are uninformed and not present at the open times the expected profits from winning in this case. Each term in the sum reflects the probability that all k' informed bidders as well as all uninformed bidders are not present at the open, times the expected profits from winning in this case.

Observe first that $\sigma_r < 1$. For if $\sigma_r = 1$, then the expression in (11) reduces to $G(r) < 0$, so that an uninformed bidder will be strictly better off from not participating in the auction at all. Next notice that if $r \geq r^*$, then $\sigma_r = 0$. For if not the expression in brackets in (11) is strictly less than

$$\pi(0, n-1)[\overline{v} - r] + \sum_{k'=1}^{n-1} \pi(k', n-1) G(r) \leq 0$$

since $r \geq r^*$. Conversely, if $r < r^*$ then we must have $\sigma_r > 0$. For if not an uninformed bidder will strictly prefer to participate in the auction with probability 1 and quit immediately after r. It follows that when $r < r^*$, σ_r is chosen so that the expression in (11) exactly equals zero, i.e., σ_r is the solution to (10).

Furthermore, since conditional on participating and observing k, an uninformed bidder must always earn zero expected profits, he cannot do better than by quitting as soon as he observes a quit and quitting above $\overline{v}_k$ otherwise, where for $k \in \{1, ..., n\}$, $\overline{v}_k$ is defined as the solution to the zero expected profit condition conditional on winning at $\overline{v}_k$ and all k bidders are uninformed:

$$\pi(0, n-1)\xi_r(k-1, n-1)[\overline{v} - \overline{v}_k] + \sum_{k'=1}^{n-k} \pi(k', n-1)\xi_r(k-1, n-k'-1)F(r)[E[V|V < r] - \overline{v}_k] = 0$$

where for $0 \leq k \leq K \leq n$, $\xi_r(k, K) = \binom{K}{k}\sigma_r^k(1-\sigma_r)^{K-k}$ is the probability that k out of K uninformed bidders are present at the open. Observe that $\overline{v}(1) = r$ and $\overline{v}_k \leq \overline{v}_n = \overline{v}$. This concludes the proof of the lemma.■

For the proof of the Proposition, recall first that in the standard auction a bidder makes positive profits when he is the only informed bidder and $V > \overline{v}$, in which case he pays a price equal to $\overline{v}$. Using Lemma 7 observe next that when the seller imposes a reserve price $r \in (0, \overline{v}]$ such a bidder makes weakly higher expected profits as with positive probability he wins the object at a price weakly less than $\overline{v}$. Furthermore, an informed bidder with a realized value $v \in (r, \overline{v}]$ makes strictly positive profits when he is the only informed bidder, whereas in the standard auction such a bidder makes zero profits. Since uninformed bidders make zero profits in both auctions, it follows that each bidder makes weakly higher ex-ante expected profits in the auction with a reserve price $r \leq \overline{v}$. Since the object is not sold with strictly positive probability, it follows that the seller is strictly worse off from imposing a reserve price $r \in (0, \overline{v}]$, compared to the standard auction.

Consider next $r > \overline{v} > r^*$. Since uninformed bidders do not participate by Lemma 7, the seller's expected revenue is

$$\pi(1,n)(1-F(r))r + (1-\pi(0,n)-\pi(1,n))(1-F(r))E[V|V>r]$$

Using the monotone hazard rate condition, observe that for the optimal choice of r to be greater than $\overline{v}$, the derivative of the last expression above must be positive at $r = \overline{v}$. This yields

$$\frac{\overline{v}f(\overline{v})}{1-F(\overline{v})} < \frac{\pi(1,n)}{1-\pi(0,n)} = \frac{n\alpha(1-\alpha)^{n-1}}{1-(1-\alpha)^n}$$

For n large enough, this inequality is violated so that the optimal choice of a reserve r cannot be greater than $\overline{v}$. This completes the proof.■

Proof of Proposition 3

For any posted price $p \in [0,1]$, an informed bidder will participate only if $v \geq p$. Let $\sigma_p \in [0,1]$ be the probability with which an uninformed bidder participates in a symmetric equilibrium. For $0 \leq k \leq K \leq n$, let $\xi_p(k,K) = \binom{K}{k}\sigma_p^k(1-\sigma_p)^{K-k}$ be the probability that k out of K uninformed bidders participate. Consider the expected profits of an uninformed bidder from bidding at the posted price p. This is given by

$$\begin{array}{l}\pi(0,n-1)\sum_{k'=0}^{n-1}\xi_p(k',n-1)\frac{1}{k'+1}[\overline{v}-p] \\ +\sum_{k=1}^{n-1}\pi(k,n-1)\sum_{k'=0}^{n-1-k}\xi_p(k',n-1-k)[\frac{1}{k'+1}G(p)+\frac{1}{k+k'+1}S(p)]\end{array} \quad (12)$$

where the functions S and G have been defined in (1) and (8) respectively. The first term above is the probability that all other bidders are uninformed times the expected probability of winning in such a case times the expected payoff conditional on winning; the second term has the same interpretation, but when at least one bidder is informed. The bidder makes zero profits when he is not the winner.

A symmetric equilibrium exists for all p. To see this observe that if the expression in (12) is non–positive for $\sigma_p = 0$, then it is an equilibrium for uninformed bidders not to participate. On the other hand, if the expression in (12) is positive for $\sigma_p = 0$ but negative for $\sigma_p = 1$, then by continuity there exists $\sigma_p \in (0,1)$ for which the expression in (12) is exactly equal to 0 and uninformed bidders are willing to participate with positive probability. Finally, if the expression in (12) is non–negative for $\sigma_p = 1$, then it is an equilibrium for uninformed bidders to participate with probability 1.

Note next that from the definitions of S and G, for all $x \in [0,1]$

$$S(x) + G(x) = \overline{v} - x \quad (13)$$

Using this in (12), we can rewrite an uninformed bidders expected profits from participating as

$$\sum_{k=0}^{n-1} \pi(k, n-1) \sum_{k'=0}^{n-1-k} \xi_p(k', n-1-k) \left[\frac{1}{k'+1}(\overline{v} - p) + \frac{k}{(k+k'+1)(k'+1)} G(p) \right] \tag{14}$$

Suppose first that $p > \overline{v}$. Then the expression (14) is negative for any $\sigma_p > 0$ and so we must have $\sigma_p = 0$. From Lemma 7, if the seller holds an auction with a reserve price equal to p, then uninformed bidders will still not participate, but informed bidders will bid and pay a price above p whenever $v > p$ and at least two bidders are informed. As a result, the seller will earn higher revenues from such an auction with a reserve price p compared to offering up the object at a fixed price p. However, from Proposition 2, for $n \geq n_\alpha$ the seller earns even higher revenues from the standard auction compared to the revenues from the auction with a reserve price of p so that the standard auction dominates all such posted prices $p \geq \overline{v}$ for $n \geq n_\alpha$.

Next consider $p \leq \overline{v}$. Conditional on winning, an informed bidder's expected profit is given by $S(p)$. Furthermore, an informed bidder wins with probability at least $1/n$ when he bids at p. Since uninformed bidders earns non–negative profits, it follows that the sum of the expected profits of all bidders is at least $\alpha S(p)$. Recall that the analogous expression for the standard auction is $n\alpha(1-\alpha)^{n-1} S(\overline{v})$. Since S is a decreasing function and $p \leq \overline{v}$ it follows that the seller is strictly better off from the standard auction compared to any posted price $p \leq \overline{v}$, for any n large enough satisfying $n(1-\alpha)^{n-1} < 1$.■

Proof of Proposition 5

The ceiling dominates the standard auction iff

$$n(1-\alpha)^{n-1} S(\overline{v}) > S(c^*) \tag{15}$$

For $x \in [0, 1]$ define

$$U(x) \equiv \pi(0, n-1)[\overline{v} - x] + (1 - \pi(0, n-1)) S(x) \tag{16}$$

and observe from the definition of c^* in (5) that $U(c^*) = 0$. Using this last fact in (15), we obtain the following necessary and sufficient condition for the ceiling to dominate the standard auction:

$$c^* < \overline{v} + n\left(1 - (1-\alpha)^{n-1}\right) S(\overline{v})$$

Since $U(x)$ is easily checked to be a decreasing function, we conclude that a necessary and sufficient condition for this to hold is that

$$U(\overline{v} + n(1 - (1-\alpha)^{n-1}) S(\overline{v})) < 0,$$

which in turn yields,

$$-(1-\alpha)^{n-1} n S(\overline{v}) + S(\overline{v} + n\left(1 - (1-\alpha)^{n-1}\right) S(\overline{v})) < 0 \tag{17}$$

as our necessary and sufficient condition for the c^*–auction to dominate the standard auction in expected revenues.

For fixed n, using the fact that the derivative of $S(.)$ is bounded below by -1, it is easy to verify that the left–hand side of (17) is negative at $\alpha = 0$ and continuous and strictly increasing in $\alpha \in [0, 1)$. Furthermore, if n is such that $\overline{v} + nS(\overline{v}) < 1$, then this expression is positive at $\alpha = 1$, establishing the existence of $\alpha_n^* \in (0, 1)$ such that the c^*–auction dominates the standard auction iff $\alpha < \alpha_n^*$. On the other hand, if n is such that $\overline{v} + nS(\overline{v}) \geq 1$, then the left–hand side of (17) is equal to 0 at $\alpha = 1$. By the monotonicity of the expression in α it follows that for such n the c^*–auction dominates the standard auction for all $\alpha < \alpha_n^* = 1$, establishing part 1 of the Proposition and also part 2 with $n^* = \frac{1-\overline{v}}{S(\overline{v})}$.■[23,24]

6.2 Appendix B: Optimal ceiling and reserve prices

In this section we characterize the optimal explicit reserve and ceiling combination $\{r, c\} \in Z = \{r, c|0 \leq r \leq c \leq 1\}$ for the case of $n = 2$ bidders, under commitment. The restriction to 2 bidders is to minimize the number of cases to consider. We also assume that condition (4) holds for the case of 2 bidders. We then have the following result.

Proposition 8 *Suppose* $n = 2$ *and that (4) holds. Then the optimal reserve is* $r = 0$ *and the optimal ceiling* $c \in \{c^*, 1\}$*.*

Proof. We proceed in steps. In Step 1 we characterize the symmetric equilibrium with two bidders for every reserve and ceiling combination in the set Z. In Step 2, we show that a positive reserve price is not optimal. In Step 3, we show that either the simple ceiling c^* is optimal or a ceiling is not optimal at all.

Step 1

We begin by defining a few objects. For $\{r, c\} \in Z$, let

$$U_1(r, c) \equiv (1-\alpha)\frac{1}{2}[\overline{v} - c] + \alpha[G(r) + \frac{1}{2}S(c)]$$

and

$$U_0(r, c) \equiv (1-\alpha)[\overline{v} - r] + \alpha[G(r) + \frac{1}{2}S(c)]$$

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[23] With compact support for V one can also exploit the zero–profit condition (5), to show that the ratio of information rents in the c^*–auction to that in the standard auction converges to zero as n becomes large. This result does not extend to the unbounded support case and we omit presenting it formally.

[24] When V has support on $[0, \infty)$, all equilibria characterized for the compact support case continue to remain so, although the uniform lower bound on n in part 2 of the Proposition can no longer be obtained. However, a slightly weaker result obtains in such cases for distributions where $\phi^* = \lim_{v\to\infty} \phi(v) = \infty$. Specifically, for each $\alpha \in (0, 1)$ there exists a n_α^* (that possibly depends on α) such that the c^*–auction yields higher revenues than the standard auction for all $n \geq n_\alpha^*$. For distributions with $\phi^* < \infty$, the ceiling dominates for large n iff $\alpha < \alpha_\infty^* \equiv 1 - \exp[-S(\overline{v})\phi^*]$. Observe that part 1 of the Proposition does not need a compact support for V.

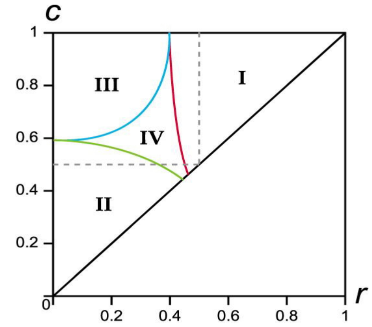

Figure 1: Equilibrium Zones

and

$$W(r,c) \equiv (1-\alpha)[1+\frac{\alpha G(r)}{(1-\alpha)[\overline{v}-r]}][\overline{v}-c]+\alpha S(c)$$

Using (13), notice that since $r \leq c$, whenever $U_1(r,c) \geq 0$ we have $U_0(r,c) > U_1(r,c)$. We will show below that symmetric equilibria have the following structure. Informed bidders participate iff $v \geq r$ and continue till $\min[v,c]$ regardless of drop–outs. Uninformed bidders participate with probability $\sigma_r \in [0,1]$, continuing till $\overline{v}$ with probability 1, continuing at $\overline{v}$ with probability $\sigma_{\overline{v}} \in [0,1]$, and continuing beyond till c with probability 1, quitting at any point immediately after observing a quit.

We show now that for $\{r,c\}$ such that $U_0(r,c) \leq 0$, $\sigma_r = 0$. This is represented by zone I in Figure 1. Further, for $\{r,c\}$ such that $U_1(r,c) \geq 0$, $\sigma_r = 1 = \sigma_{\overline{v}}$ (zone II in the figure); while for $\{r,c\}$ such that $U_1(r,c) < 0 < U_0(r,c)$, $\sigma_r \in (0,1]$ with $\sigma_r < 1$ if $r > 0$ (zones III and IV). In the last case, $\sigma_{\overline{v}} \in [0,1]$ with $\sigma_{\overline{v}} > 0$ iff $c < 1$ and $\sigma_{\overline{v}} < 1$ if $W(r,c) < 0$ (zone III) while $\sigma_{\overline{v}} = 1$ if $W(r,c) \geq 0$ (zone IV).

The behavior of informed bidders is immediate, for every $\{r,c\}$. As for uninformed bidders, conditional on participating and observing that both bidders are present, uninformed bidders must continue with probability 1 for all $p \in [r,c]$, with $p \neq \overline{v}$. For if they drop–out with positive probability at any $p < \overline{v}$, then by continuing at p and dropping out immediately after such a bidder makes strictly higher expected profits than from dropping out at p. On the other hand, if they drop–out with positive probability at $p > \overline{v}$, then the expected payoff is negative, whereas by dropping out at $\overline{v}$ such a bidder can guarantee zero expected profits. It follows that uninformed bidders must drop–out anytime they observe a drop–out at a $p \neq \overline{v}$.

Observe next from Propositions 1 and 2, that symmetric equilibrium behavior is as specified above in the case $c = 1$. As a result, in what follows we focus on the case where $c < 1$. Note that whenever $c < 1$ we must have $\sigma_{\overline{v}} > 0$— for if not, an uninformed bidder by continuing till c competes only against an informed bidder and makes strictly positive profits, whereas he makes zero profits from dropping out at $\overline{v}$.

Consider first a symmetric equilibrium where $\sigma_r = 0$. Since no uninformed bidder wants to participate we must have that the expected profits from participating be non–positive, or $U_0(r, c) \leq 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone.

Consider next a symmetric equilibrium where $\sigma_r = 1 = \sigma_{\overline{v}}$. Since uninformed bidders must earn non–negative expected profits, we must have $U_1(r, c) \geq 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone.

Next suppose that in a symmetric equilibrium $\sigma_r = 1$ but $\sigma_{\overline{v}} \in (0, 1)$. Conditional on reaching $\overline{v}$ and dropping out an uninformed bidder must make zero expected profits and since $\sigma_{\overline{v}} \in (0, 1)$ must also make zero expected profits from continuing till c. This implies that $\sigma_{\overline{v}}$ must solve:

$$(1-\alpha)\sigma_{\overline{v}}\frac{1}{2}[\overline{v}-c] + \alpha\frac{1}{2}S(c) = 0 \tag{18}$$

It follows that $c > c^*$. Notice next that the ex–ante expected profits of an uninformed bidder is equal to a weighted sum of his expected profits when he is the only bidder present at the open and his expected profits when both bidders are present at the open. Due to arguments used to establish (18), the second expected value is equal to zero, while since $\sigma_r = 1$, the first expected value is negative if $r > 0$ and zero otherwise. It follows that $r = 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone and furthermore that $U_1(r, c) < 0 < U_0(r, c)$.

Consider now a symmetric equilibrium where $\sigma_r \in (0, 1)$ and $\sigma_{\overline{v}} \in (0, 1)$. Using arguments similar to those used in establishing (18) we must have that uninformed bidders must earn zero profits so that (given σ_r), $\sigma_{\overline{v}}$ must solve

$$(1-\alpha)\sigma_r\sigma_{\overline{v}}\frac{1}{2}[\overline{v}-c] + \alpha\frac{1}{2}S(c) = 0 \tag{19}$$

It follows that $c > \overline{v}$. Furthermore, since $\sigma_r \in (0, 1)$, uninformed bidders must earn zero profits from participating, i.e., σ_r must solve

$$(1-\alpha)(1-\sigma_r)[\overline{v}-r] + \alpha G(r) = 0 \tag{20}$$

It follows that $r \in (0, \overline{v})$. Furthermore, using the solution for σ_r obtained in (20) in (19), we obtain

$$(1-\alpha)[1 + \frac{\alpha G(r)}{(1-\alpha)[\overline{v}-r]}]\sigma_{\overline{v}}[\overline{v}-c] + \alpha[S(c)] = 0$$

Since $c > \overline{v}$ and $r \in (0, \overline{v})$, we must then have $W(r, c) < 0$ and, using (19), $U_1(r, c) < 0$ and furthermore, using (20), $U_0(r, c) > 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone.

Finally, consider a symmetric equilibrium where $\sigma_r \in (0, 1)$ but $\sigma_{\overline{v}} = 1$. Since uninformed bidders randomize participation, we must have that the ex–ante expected profits of an uninformed bidder equals zero, i.e.,

$$(1-\alpha)[(1-\sigma_r)(\overline{v}-r) + \sigma_r\frac{1}{2}(\overline{v}-c)] + \alpha[G(r) + \frac{1}{2}S(c)] = 0 \tag{21}$$

It follows that $r < \overline{v}$. For if not, since $c \geq r$ and by (13), $G(r) + \frac{1}{2}S(c) \leq \frac{1}{2}G(r) + \frac{1}{2}(\overline{v} - r) < 0$, we obtain a contradiction with (21). Furthermore, the expression in (21) is monotonically decreasing in σ_r. It follows that it is strictly less than $U_0(r, c)$ and strictly greater than $U_1(r, c)$, yielding $U_1(r, c) < 0 < U_0(r, c)$. Furthermore, uninformed bidders must make non–negative profits conditional on reaching $\overline{v}$ and continuing, i.e.,

$$(1 - \alpha)\sigma_r[\overline{v} - c] + \alpha[S(c)] \geq 0$$

From (21) it follows that

$$\sigma_r \geq 1 + \frac{\alpha G(r)}{(1 - \alpha)[\overline{v} - r]} \tag{22}$$

so that, substituting the term on the right–hand side of the last expression for σ_r in the left–hand side of (21) and using the monotonicity of the expression in (21), we obtain $W(r, c) \geq 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone. This completes the description for symmetric equilibria for all $\{r, c\} \in Z$.

Step 2

We will show that each auction $\{r, c\} \in Z$ with a reserve price $r > 0$ is strictly dominated in expected revenues by some auction $\{0, c'\} \in Z$, that has no reserve.

Consider first the expected revenues of the seller for the auction with reserve and ceiling pair $\{r, c\} \in Z$ such that $r \geq \overline{v}$. From step 1, using (13) observe that then $U_0(r, c) < 0$ so that $\sigma_r = 0$. Given the behavior of informed bidders it follows immediately that the seller will do better by instead holding the auction $\{r, 1\}$, which by the assumed conditions of the proposition is dominated by the standard auction $\{0, 1\}$. The same logic also eliminates all auctions $\{r, c\}$ where $U_0(r, c) \leq 0$, allowing us to focus throughout what follows on the case where $U_0(r, c) > 0$ so that $\sigma_r > 0$.

Next consider the auction $\{r, c\}$ with $0 < r < \overline{v}$ and $c^* < c < 1$ and $U_0(r, c) > 0$. From step 1, we must have $\sigma_r \in (0, 1)$ and $\sigma_{\overline{v}} \in (0, 1]$, as in such a case $U_1(r, c) < 0$. We compare the expected revenues such an auction with that from the auction $\{0, c\}$ that has the same ceiling but no reserve. From step 1, in the auction $\{0, c\}$ uninformed bidders participate with probability 1 and continue at $\overline{v}$ with probability $\sigma'_{\overline{v}} \in (0, 1)$. In either auction an informed bidder makes positive profits only when he is the only informed bidder or when $V > c$ while uninformed bidders make zero expected profits. Furthermore, the object is sold with probability 1 in the auction $\{0, c\}$, while it is not sold with positive probability in the auction $\{r, c\}$. It thus suffices to show that informed bidders earn higher expected profits in the auction $\{r, c\}$ compared to the auction $\{0, c\}$. Suppose first that $\{r, c\}$ is such that $W(r, c) \geq 0$. Then $\sigma_{\overline{v}} = 1$ in the auction $\{r, c\}$. Furthermore, letting $\Pi(r, c|v)$ denote the expected profits of a bidder conditional on being informed and conditional on the realized value of $V = v$, it is seen that

$$\Pi(r, c|v) = \begin{cases} 0 & \text{when } v < r \\ (1-\alpha)(1-\sigma_r)[v-r] & \text{when } r \leq v < c \\ (1-\alpha)(1-\sigma_r)[v-r] + ((1-\alpha)\sigma_r + \alpha)\frac{1}{2}[v-c] & \text{when } v \geq c \end{cases} \tag{23}$$

In contrast, in the auction $\{0, c\}$, conditional on being an informed bidder and the realized value of v of V, the expected profits of an informed bidder is

$$\Pi(0, c|v) = \begin{cases} 0 & \text{when } v < \overline{v} \\ (1-\alpha)(1-\sigma'_{\overline{v}})[v-\overline{v}] & \text{when } \overline{v} \leq v < c \\ (1-\alpha)(1-\sigma'_{\overline{v}})[v-\overline{v}] + ((1-\alpha)\sigma'_{\overline{v}} + \alpha)\frac{1}{2}[v-c] & \text{when } v > c \end{cases} \tag{24}$$

Comparing state by state, we see that the $\{0, c\}$ auction dominates the $\{r, c\}$ auction if $\sigma_r \leq \sigma'_{\overline{v}}$. But, from step 1, $\sigma'_{\overline{v}}$ solves (18). Since $c > c^* > \overline{v}$, the expression in (18) is decreasing in $\sigma_{\overline{v}}$, so that if $\sigma_r > \sigma'_{\overline{v}}$, then the expression in (18) is negative if we replace $\sigma_{\overline{v}}$ with σ_r in it. Since, again from step 1, σ_r solves (21), it follows that σ_r must satisfy

$$(1-\alpha)(1-\sigma_r)(\overline{v} - r) + \alpha G(r) > 0$$

which, since $r < \overline{v}$, yields the desired contradiction with (22).

Suppose next that $\{r, c\}$ is such that $W(r, c) < 0$. Then $\sigma_{\overline{v}} \in (0, 1)$ in the auction $\{r, c\}$ and solves (19) whereas $\sigma'_{\overline{v}}$ (corresponding to the $\{0, c\}$ auction) solves (18) so that we obtain $\sigma_r\sigma_{\overline{v}} = \sigma'_{\overline{v}}$. Furthermore conditional on being an informed bidder and the realized value v of V, the expected profits of an informed bidder in the $\{r, c\}$ auction is equal to 0 when $v < r$, equal to $(1-\alpha)(1-\sigma_r)[v-r]$ when $r \leq v < \overline{v}$, equal to

$$(1-\alpha)[(1-\sigma_r)(v-r) + \sigma_r(1-\sigma_{\overline{v}})(v-\overline{v})]$$

when $\overline{v} \leq v < c$ and equal to

$$(1-\alpha)[(1-\sigma_r)(v-r) + \sigma_r(1-\sigma_{\overline{v}})(v-\overline{v})] + [(1-\alpha)\sigma_r\sigma_{\overline{v}} + \alpha]\frac{1}{2}S(c)$$

when $v \geq c$. Since $\sigma_r\sigma_{\overline{v}} = \sigma'_{\overline{v}}$, comparing state by state with the expression for $\Pi(0, c|v)$ it follows that the $\{0, c\}$ auction strictly dominates the $\{r, c\}$ auction.

Next, consider the auction $\{r, c\}$ with $0 < r < \overline{v}$ and $r \leq c \leq c^*$ and $U_0(r, c) > 0 > U_1(r, c)$. It is immediate from step 1 then that $W(r, c) \geq 0$ so that $\sigma_r \in (0, 1)$ and $\sigma_{\overline{v}} = 1$. We compare such an auction with the c^*–auction $\{0, c^*\}$. Conditional on being an informed bidder and the realized value v of V, the expected profits of an informed bidder in the $\{0, c^*\}$ auction is equal to $\frac{1}{2}(v - c^*)$ when $v \geq c^*$ and 0 otherwise. Since $c^* \geq c$, comparing with the expression for $\Pi(r, c|v)$ in (23), we see that informed bidders earn strictly higher profits in the $\{r, c\}$ auction compared to the $\{0, c^*\}$ auction. Since uninformed bidders earn zero expected profits in both auctions and the object is sold with probability 1 in the $\{0, c^*\}$ auction but not in the $\{r, c\}$ auction, it follows that the former strictly dominates the latter in expected revenues.

It remains to consider auctions with $\{r, c\}$ such that $0 < r < \overline{v}$ and $r \leq c \leq c^*$ and $U_1(r, c) \geq 0$. From step 1, $\sigma_r = \sigma_{\overline{v}} = 1$ so that conditional on being informed and the realized value v of V, the profits of an informed bidder equals $\frac{1}{2}[v - c]$ when $v \geq c$ and 0 otherwise. This is higher than the corresponding expression in the $\{0, c^*\}$ auction, state by state. Uninformed bidders earn non–negative expected profits in the $\{r, c\}$ auction and zero expected profits in the $\{0, c^*\}$ auction. Furthermore, since $r > 0$, the object is not sold with positive probability in the $\{r, c\}$ auction but sold with probability 1 in the $\{0, c^*\}$ auction. It follows that the latter strictly dominates the former in expected revenues. This completes step 2.

Step 3

From step 2, it follows that the revenue maximizing choice of a reserve and ceiling pair $\{r, c\}$ must satisfy $r = 0$. Then, the object is always sold with probability 1, for any c. Furthermore, it is easy to see that the optimal ceiling c is at least c^*— from step 1, compared to an auction $\{0, c\}$ with $c < c^*$, both informed and uninformed bidders earn lower profits in the auction $\{0, c^*\}$.

Let $\Pi(0, c)$ be the unconditional expected information rents that the buyers earn in any auction with $r = 0$ and $c \in [c^*, 1]$. Since all these auctions are efficient (from Step 1), in order to compare revenues across these auctions it suffices to compare $\Pi(0, c)$ for different values of c. Observe first that for $c \in [c^*, 1]$,

$$\Pi(0, c) = \alpha[(\alpha + (1 - \alpha)\sigma_{\overline{v}})\frac{1}{2}S(c) + (1 - \alpha)(1 - \sigma_{\overline{v}})S(\overline{v})]$$

Using the solution (18) for $\sigma_{\overline{v}}$, let

$$\Delta(c) \equiv \Pi(0, c) - \Pi(0, 1) = \frac{1}{2}\frac{\alpha S(c)}{c - \overline{v}}[c + S(c) - \overline{v} - 2S(\overline{v})] \tag{25}$$

Note that $\Delta(c) \leq 0$ iff

$$c + S(c) \leq \overline{v} + 2S(\overline{v}) \tag{26}$$

The left–hand side of (26) is monotone increasing in c, implying that if the standard auction dominates the c^*–auction (i.e., if (26) is violated for $c = c^*$), then it also dominates all ceilings $c < 1$. Conversely, for c such that (26) holds, the derivative with respect to c of $\Delta(c)$ is

$$\Delta'(c) = [c + S(c) - \overline{v} - 2S(\overline{v})]\frac{d}{dc}[\frac{\alpha S(c)}{c - \overline{v}}] + \frac{\alpha S(c)}{c - \overline{v}}[1 + S'(c)] > 0$$

implying that when the c^*–auction dominates the standard auction, it also dominates all ceilings $c > c^*$. This completes the proof.■

6.3 Appendix C: Alternative information structure (and private values)

In this section we perform a simple continuity exercise on the information structure of our model. Our intent is to show that the desirability of a ceiling does not depend on informed bidders obtaining perfectly correlated information. The check also allows us to clarify that a ceiling may be desirable in affiliated private value environments in which bidders may not possess precise information about their valuation. For simplicity we suppose that there are only two bidders and focus on demonstrating the robustness of the results obtained in Proposition 5.1. The robustness exercise also allows us to demonstrate the desirability of ceilings in environments with correlated private values.

To begin with suppose that we have pure common values and that, as in our baseline model, each bidder is uninformed with probability $1-\alpha$, independently across bidders. In contrast to our baseline model suppose that with probability α, bidder i obtains an informative signal $X_i \in [0,1]$, where X_1 and X_2 are correlated but not necessarily equal to each other. The common value of the object to the bidders is given by

$$V = \frac{X_1 + X_2}{2}$$

We construct a sequence of information structures for the signals $X = (X_1, X_2)$ that converge to the structure of our baseline model. This is done as follows. For each $\varepsilon \in (0,1)$ consider a random variable Ω that is uniformly distributed in the interval $[\frac{\varepsilon}{2}, 1-\frac{\varepsilon}{2}]$ and suppose that conditional on $\Omega = \omega$, X_1 and X_2 are independently and uniformly distributed in the interval $[\omega - \frac{\varepsilon}{2}, \omega + \frac{\varepsilon}{2}]$. Let f_X denote the joint density of X. Then f_X has support on the set $\{x \in [0,1]^2 || x_1 - x_2| \leq \varepsilon\}$, and is given by

$$f_X(x_1,x_2) = \begin{cases} \frac{\min[x_1,x_2]}{\varepsilon^2(1-\varepsilon)} & \text{if } \max[x_1,x_2] < \varepsilon \\ \frac{1-\max[x_1,x_2]}{\varepsilon^2(1-\varepsilon)} & \text{if } \min[x_1,x_2] > 1-\varepsilon \\ \frac{\min[x_1,x_2]-\max[x_1,x_2]+\varepsilon}{\varepsilon^2(1-\varepsilon)} & \text{otherwise} \end{cases}$$

As ε becomes small, this information structure converges to that for our baseline model. We note (and as should become apparent) that the uniformity assumptions for the distributions for Ω and the conditional distributions for X_i are not necessary but simplifies the analysis. Furthermore, the lack of full support for f_X on $[0,1]^2$ is also not necessary. One can always slightly perturb the model by allowing a density that is positive but bounded above by ε outside the support of f_X and obtain the same conclusions as we do below.

Using the expression for f_X, for all $i, j \in \{1,2\}$, $i \neq j$ and any $x \in [0,1]$ one obtains that

$$E[X_i | X_j = x] = \begin{cases} \frac{x+\varepsilon}{2} & \text{if } x < \varepsilon \\ x & \text{if } \varepsilon \leq x \leq 1-\varepsilon \\ \frac{1-\varepsilon+x}{2} & \text{if } x > 1-\varepsilon \end{cases}$$

Now suppose ε is small enough such that $\varepsilon < \overline{v} = E[V] < 1-\varepsilon$, and consider the standard auction. It is straightforward to verify that symmetric equilibrium strategies in the standard auction are as follows. Any bidder who is uninformed quits at $p > \overline{v}$, staying in otherwise. On the other hand, an informed bidder i with a signal $X_i = x$ quits at $p > x = E[V|X_1 = X_2 = x]$, staying in otherwise.

Let $Z_1 = \max[X_1, X_2]$ and $Z_2 = \min[X_1, X_2]$. The expected revenues of the seller in the standard auction equals

$$R_s(\varepsilon) = \overline{v} - \alpha^2 E[\frac{Z_1 - Z_2}{2}] - 2\alpha(1-\alpha)\left[\Pr[X_i < \varepsilon]E[\frac{\varepsilon - X_i}{4}|X_i < \varepsilon] + \Pr[X_i > \overline{v}](E[V|X_i > \overline{v}] - \overline{v})\right]$$

Notice that this is continuous in ε and further converges to the expression in (3) as ε converges to 0.

Consider next any candidate ceiling $c \in (\overline{v}, 1-\varepsilon)$ that is defined as follows. For each c, let $x^*(c)$ be defined by

$$(1-\alpha)(x^*(c) - c) + \alpha \Pr[X_j \geq x^*(c)|X_i = x^*(c)](E[V|X_i = x^*(c) \leq X_j] - c) = 0 \tag{27}$$

It is immediate that such an $x^*(c)$ exists. Note that $x^*(c) \in (c - \frac{\varepsilon}{2}, c)$, it is continuous in c and it converges to c as ε goes to 0. Consider now a ceiling $c^*_\varepsilon \in (\overline{v}, 1-\varepsilon)$ that satisfies

$$(1-\alpha)(\overline{v} - c^*_\varepsilon) + \alpha \Pr[X_j > x^*(c^*_\varepsilon)](E[V|X_i \geq x^*(c^*_\varepsilon)] - c^*_\varepsilon) = 0 \tag{28}$$

It is immediate that for ε small enough such a ceiling $c^*_\varepsilon \in (\overline{v}, 1-\varepsilon)$ exists. Furthermore, c^*_ε is continuous in ε and converges to c^* (defined by (5)) as ε goes to 0.

It is not difficult to verify that in an auction with a ceiling c^*_ε symmetric equilibrium strategies must be as follows. Any uninformed bidder stays in till c^*_ε while any informed bidder who observes $X_i = x$ stays in till x if $x < x^*(c^*_\varepsilon)$ and stays in till c^*_ε otherwise. Condition (27) guarantees that any informed bidder with a signal $x^*(c^*_\varepsilon)$ is exactly indifferent between quitting and staying on till the ceiling given that the other bidder behaves as specified. This implies that all bidders with signals greater than $x^*(c^*_\varepsilon)$ strictly prefer to continue beyond their signal to the ceiling and that all bidders with lower signals strictly prefer to quit after the clock reaches their signals (equal to the expected value of the object given that he has the same signal with the other bidder). Condition (28) is the analogue of condition (5) and guarantees that any uninformed bidder makes zero expected profits conditional on winning at the ceiling c^*_ε.

Suppressing some notation, the expected revenues of the seller in the c^*_ε–auction can be written as

$$\begin{aligned} R(c^*_\varepsilon, \varepsilon) \;\; = \;\; & \overline{v} - \alpha^2 \left[\Pr[Z_2 < x^*]E[\frac{Z_1 - Z_2}{2}|Z_2 < x^*] + \Pr[Z_2 \geq x^*](E[V|Z_2 \geq x^*] - c^*_\varepsilon)\right] \\ & -2\alpha(1-\alpha)\left[\Pr[X_i < \varepsilon]E[\frac{\varepsilon - X_i}{4}|X_i < \varepsilon] + \Pr[X_i > x^*](E[V|X_i > x^*] - c^*_\varepsilon)\right] \\ & -(1-\alpha)^2\left[\overline{v} - c^*_\varepsilon\right] \end{aligned}$$

Observe that $R(c^*_\varepsilon, \varepsilon)$ is continuous in c^*_ε and ε and converges to the expression in (6) as ε goes to zero.

Because of the continuity of $R(c^*_\varepsilon, \varepsilon)$ and $R_s(\varepsilon)$ in ε it follows from Proposition 5.1 that for each α less than (resp. greater than) the cut–off value α^*_2, the c^*_ε–auction dominates (resp., is dominated by) the standard auction in expected revenues, for ε small enough. This establishes the robustness of our results to the possibility that bidders either obtain imperfectly correlated and imperfect signals of the true value or are uninformed.

We conclude this section by using a slightly modified version of the example above to illustrate that a ceiling may be beneficial even in a setting of correlated *private values.* To see this suppose that the signal distribution f_X is as above but the value of the object to bidder i is equal to X_i. With probability α each bidder observes his value for the object while with probability $1-\alpha$ he does not. In the latter case, he assigns positive probability to the other bidder possessing information relevant for determining his own value for the object. It is not difficult to verify that in this model the bidding behavior in the standard auction is identical to that characterized above. Furthermore, the bidding behavior in the c^*_ε–auction is also essentially unchanged with the only difference being that now $x^*(c^*_\varepsilon) = c^*_\varepsilon$. Using Proposition 5.1, continuity arguments identical to those above then establish that for each $\alpha < \alpha^*_2$ (resp., $\alpha > \alpha^*_2$) , the c^*_ε–auction dominates (resp., is dominated by) the standard auction in expected revenues, for ε small enough.

7 Appendix D: A Discrete Approximation

Consider the following discrete approximation to the model with a continuum of prices and values. Suppose that the value of the object $v \in G_\delta = \{\delta, 2\delta, ..., 1-\delta, 1\}$ where $\delta > 0$ (and small). The prior distribution for v is given by $\Phi_\delta(v) = F(\delta \left\lceil \frac{v}{\delta} \right\rceil)$, $v \in [0,1]$, where $\lceil x \rceil$ is the integer part of x. Let $\overline{v}_\delta$ be the expected value of the object. Notice that Φ_δ and $\overline{v}_\delta$ converge to F and $\overline{v}$ respectively, as δ becomes small.

The discrete ascending auction consists of a price $p \in G_\delta$ that rises in increments of δ. Let $p_\delta = \max\{p \in G_\delta | p \leq \overline{v}_\delta\}$. At each price $p \in G_\delta$ each bidder decides whether to remain active ($a_i(p) = 1$) or quit ($a_i(p) = 0$), upon observing the number of bidders active at all $p' < p$. Let $a(p) = \#\{p | a_i(p) = 1\}$ denote the number of bidders who are active at p and let $q(p) = a(p) - a(p-\delta)$ be the number of bidders who quit at p. Set $a(0) = n$, $q(0) = 0$ and $a(1+\delta) = 0$.

Define $p^* = \inf\{p \geq 0 | a(p) \leq 1\}$, which exists since $a(1+\delta) = 0$. The price the winner pays is given by

$$P = \begin{cases} p^* - \delta & \text{if } a(p^*) = 0,\ q(p^* - \delta) > 0,\ p^* \leq p_\delta \\ p^* & \text{otherwise} \end{cases}$$

The winner is the bidder in $\{i | a_i(p^*) = 1\}$ (if one such exists), or is equally likely to be one of the bidders in $\{i | a_i(p^*) = 0\}$.

A history of bidding behavior at $p \in G_\delta$ is denoted by $h_p = \{a(p')\}_{p' < p} \in H_p$. Let $H = \cup_{p \in G_\delta} H_p$ be the set of all possible histories. A type for bidder i is denoted by $\theta_i \in \Theta = G_\delta \cup \{u\}$ and a behavior strategy for bidder i is a mapping $\sigma_i : \Theta \times H \to \Delta[\{0,1\}]$, where $\sigma_i(\theta_i, h_p)$ is the probability that bidder i stays active at p ($a_i(p) = 1$), given his type θ_i and given the observed history h_p.

We look for a symmetric (perfect Bayesian) equilibrium, where each bidder uses the same strategy σ^*. The following lemma characterizes the behavior of informed bidders (i.e., $\theta_i = v \in G_\delta$) at any $p \in G_\delta$ and history h_p, withe $\pi_p > 0$, where π_p is the probability that all other active bidders are informed given $\{v, h_p\}$.

Lemma 9 *For any $v \in G_\delta$ and $h_p \in H$ with $\pi_p > 0$ and all i, in any symmetric equilibrium, $\sigma_i^*(v, h_p) = 0$ for all $p \geq v$ and $\sigma_i^*(v, h_p) = 1$ if $p < v$, except when $p = v - \delta \leq p_\delta$, $a(p-\delta) = 2$ and $q(p-\delta) > 0$, in which case any $\sigma_i^*(v, h_p) \in [0,1]$ can be part of a symmetric equilibrium.*

Proof. Pick any informed bidder i with $\theta_i = v \in G_\delta$ and consider his behavior at $p \in G_\delta$ after a history h_p with $a(p-\delta) = k \geq 2$ bidders (including i) active till $p - \delta$. Let the probability that all other bidders quit at p given $\{h_p, v\}$ be Q_p and let the probability that all other bidders quit at p given that they are not all informed be R_p. Then

$$Q_p = (1 - \sigma^*(v, h_p))^{k-1}\pi_p + (1 - \pi_p)R_p. \tag{29}$$

Let U_p denote the expected profits to bidder i given that he and some other bidders do not quit at p. We proceed in steps.

<u>Step 1</u> $p \geq v$.

Consider first h_p such that $q(p-\delta) = 0$ or if $q(p-\delta) > 0$ then $p > p_\delta$. In this case if bidder i quits at p, he wins the auction with probability $\frac{1}{k}$ only if all the $k-1$ other bidders also quit at p. Since $q(p-\delta) = 0$, the price $P = p^* = p$ in this case. The expected payoff from quitting at p is thus

$$Q_p \frac{1}{k}(v - p)$$

On the other hand if bidder i stays active at p, he may win the auction and pay $P = p^* = p$ if all other bidders quit at p and earns an expected payoff U_p otherwise. The expected payoff from staying active at p is thus

$$Q_p(v-p) + (1 - Q_p)U_p;$$

Notice first that if i stays active at p and not all other bidders quit at p, he can win the auction at any price $p' \geq p \geq v$. Since he earns zero profits if he does not win the auction we must have $U_p \leq 0$.

We claim that in this case $\sigma^*(v, h_p) = 0$ in a symmetric equilibrium. For suppose not. Since $\pi_p > 0$, we must then have $Q_p < 1$. Furthermore, since with positive probability at least some other informed bidders are present and since $\sigma_i^*(v, h_p) > 0$, there is strictly positive probability that no bidder quits at p. In such a case, any informed bidder i must win at some $P > p \geq v$ with strictly positive probability and earn strictly negative payoffs, and earn non-positive payoffs in all other cases. It follows that $U_p < 0$. But then quitting at p is a strict best response for bidder i.

Consider next h_p such that $q(p-\delta) > 0$ and $p \leq p_\delta$. In such a case the expected payoff from quitting at p is

$$Q_p \frac{1}{k}(v - p + \delta)$$

since $p^* = p$ but $P = p^* - \delta$, whereas the expected payoff from staying active at p is

$$Q_p(v-p) + (1-Q_p)U_p;$$

If $\sigma^*(v, h_p) > 0$ in a symmetric equilibrium, since $\pi_p > 0$ we must have $Q_p < 1$ (indeed, $Q_p = 0$ and $\sigma^*(v, h_p) = 1$). As argued above, there is strictly positive probability that no bidder quits at p. In such a case, any informed bidder i must win at some $P > p \geq v$ with strictly positive probability and earn strictly negative payoffs, and earn non-positive payoffs in all other cases. It follows that $U_p < 0$. But then quitting at p is a strict best response for bidder i.

Step 2 $p < v$

Once again, begin by considering h_p with $q(p-\delta) = 0$ or if $q(p-\delta) > 0$ then $p > p_\delta$. The expected payoff to i from quitting at p is

$$Q_p \frac{1}{k}(v - p)$$

whereas the expected payoff from staying at p is

$$Q_p(v-p) + (1-Q_p)U_p$$

Notice that after staying at p, if bidder i quits at $p + \delta \leq v$, then he gauarantees himself non-negative profits. We conclude that $U_p \geq 0$. But then $\sigma^*(v, h_p) = 1$. For if $\sigma^*(v, h_p) < 1$, comparing the last two expressions and the fact that $U_p \geq 0$, we see that $Q_p = 0$. Since $\pi_p = 0$, we see from (29) that $\sigma^*(v, h_p) = 1$, a contradiction.

Next consider h_p with $q(p-\delta) > 0$ and $p \leq p_\delta$. In such a case, the expected payoff to i from quitting at p is

$$Q_p \frac{1}{k}(v - p + \delta)$$

whereas the expected payoff from staying at p is

$$Q_p(v-p) + (1-Q_p)U_p$$

where, as before, $U_p \geq 0$.

Notice first that $\frac{1}{k}(v - p + \delta) \leq v - p$, with strict inequality if either $k > 2$ or $p + \delta < v$. In such cases since $U_p \geq 0$, if $\sigma^*(v, h_p) < 1$, then we must have $Q_p = 0$ implying, since $\pi_p > 0$, $\sigma^*(v, h_p) < 1$ via (29), a contradiction.

So suppose $p + \delta = v$ and $k = 2$. From Step 1, if $\sigma^*(v, h_p) > 0$, and neither of the $k = 2$ active bidders quit at p, informed bidder i must quit with probability 1 at $p + \delta = v$, since $\pi_{p+\delta} > 0$. In such a event, if bidder i wins the auction, we must have $P = p^* = p + \delta = v$, since $q(p) = 0$. But then $U_p = 0$ and so any $\sigma^*(v, h_p) \in (0, 1]$ can be part of a symmetric equilibrium. Furthermore, $\sigma^*(v, h_p) = 0$ can also be part of a symmetric equilibrium, since in the off the path of play event neither bidder quits at p, the price bidder i will pay if he wins later will be at least v, so that it is a strict best response for bidder i to quit at p.We conclude that when $a(p - \delta) = k = 2$ and $p + \delta = v$ and $q(p - \delta) > 0$, any $\sigma^*(v, h_p) \in [0, 1]$ can be part of an equilibrium. This concludes the proof.■

The next lemma establishes some necessary conditions on the behavior of uninformed bidders behavior in any symmetric equilibrium. To state it concisely, lety μ_p be the probability that all other bidders active till $p - \delta$ are uninformed, given $\{u, h_p\}$.

Lemma 10 *In any symmetric equilibrium, for all $h_p \in H$ with $\mu_p > 0$, and all i,*
(i) if $q(p') = 0$ for all $p' < p$, then $\sigma^*(u, h_p) = 0$ if $p > p_\delta$, $\sigma^*(u, h_p) = 1$ if $p < p_\delta$ and $\sigma^*(u, h_p) \in [0, 1)$ if $p = p_\delta$ and strictly positive iff $p_\delta < \overline{v}_\delta$.
(ii) if $q(p - \delta) > 0$ and $q(p') = 0$ for all $p' < p - \delta$, then $\sigma_i^*(u, h_p) = 0$.

Proof. For any h_p with $a(p - \delta) = k \geq 2$ and any uninformed bidder i, let q_p the probability that all other bidders quit at p given that they are all uninformed and let r_p be the probability that all other bidders quit at p given that at least one is informed. Finally, let W_p be the expected payoff to an uninformed bidder if he does not quit at p and some other bidders also do not quit at p.

Part (i) $q(p') = 0$ for all $p' < p$

In this case $k = n$. If uninformed bidder i quits at p then he wins the auction only if all other bidders also quit at p. Since $q(p') = 0$ for all $p' < p$, we must then have $P = p$ and further, in the event that some informed bidder quits at p, bidder i must conclude that $p = v$. The expected payoff to bidder i from quitting at p is then

$$\mu_p q_p \frac{1}{n}(\overline{v}_\delta - p) + (1 - \mu_p) r_p \frac{1}{n}(p - p)$$

On the other hand, if bidder i stays active at p, his expected payoff is

$$\mu_p q_p(\overline{v}_\delta - p) + (1 - \mu_p) r_p (p - p) + (1 - \mu_p q_p - (1 - \mu_p) r_p) W_p$$

Consider first the case where $p > p_\delta$. Then $W_p \leq 0$, for if the uninformed wins with an informed bidder active at p, since the latter only quits when $p' = v$, bidder i must earn zero profits from winning in such an event, zero profits from not winning, and negative profits if he wins with only uninformed bidders active at p. If, $\sigma^*(u, h_p) > 0$, we must then have $q_p = 0$ and so $\sigma^*(u, h_p) = 1$.

Then, if i stays at p, in a symmetric equilibrium he must win at a price $p' > p$ against other uninformed bidders with strictly positive probability , implying $W_p < 0$. But then it is strictly better for i to quit at p.

Consider next the case where $p < p_\delta$. In such a case, if bidder i stays at p and not all bidders quit at p, and quits at $p + \delta$ and wins then he pays a price $P \leq p + \delta \leq \overline{v}_\delta$. He thus makes non-negative profits when all other bidders are uninformed. If some bidders are informed that $v = p$ quit at p, then $P = p$ and he makes zero profits upon winning. If some bidders are informed that $v = p + \delta$ and quit at $p + \delta$, then $P = p + \delta$ and he makes zero profits. Finally, in the case that only 2 bidders remain active at p and the other bidder is informed that $v = p + 2\delta$ and quits at $p + \delta$, then $P = p$ and he makes positive profits upon winning. We conclude that $W_p \geq 0$. Then, if $\sigma^*(u, h_p) < 1$ we must have $q_p = 0$ and so $\sigma^*(u, h_p) = 1$, a contradiction. We conclude $\sigma^*(u, h_p) = 1$.

Consider finally the case where $p = p_\delta$. Notice that if not all bidders quit at p_δ, then $P = p^* > p_\delta$. Suppose first $\overline{v}_\delta = p_\delta$. If $\sigma^*(u, h_p) > 0$, there is strictly positive probability in a symmetric eqm that all uninformed bidders stay at p_δ, in which case $P > p_\delta$ and an uninformed bidder makes negative profits from winning when all other bidders are uninformed and at most zero profits in all other cases. It follows that $\sigma^*(u, h_p) = 0$ in this case. Suppose next $\overline{v}_\delta > p_\delta$. In this case, if $\sigma^*(u, h_p) = 0$, then $q_p = 1$.If i stays at p and quits at $p + \delta$, with some other bidders also active at p, then they must be informed, so that if i wins at $p + \delta$ he earns zero profits. Since i wins with strictly higher probability at p, from staying at p, and since $\mu_p > 0$, we conclude that i's deviation is profitable. Hence we must have $\sigma^*(u, h_p) > 0$. On the other hand, if $\sigma^*(u, h_p) = 1$, then $q_p = 0$. By continuing at p in a symmetric equilibrium, i wins with strictly positive probability at some $p' > p_\delta$ with strictly positive probability, when all other bidders are uninformed. In such a case he earns negative profits. In all other cases (i.e., with informed bidders present) he earns zero profits. In other words, $W_p < 0$ and so it is strictly better for i to quit at p. We conclude $\sigma^*(u, h_p) < 1$. This concludes part (i).

Part (ii) $q(p - \delta) > 0$, $q(p') = 0$ for all $p' < p - \delta$.

Consider first the case where $p \leq p_\delta$. From part (i) and the previous lemma, any uninformed bidder must conclude that the bidders quitting at $p - \delta$ are informed and that $v = p - \delta$. If uninformed bidder i quits at p, and wins, then $p^* = p \leq p_\delta$, but since $a(p^*) = 0$ and $q(p^* - \delta) > 0$ he will pay a price $P = p^* - \delta = p - \delta = v$, for zero profits. On the other hand, if he stays at p, $a(p) \geq 1$ and so $P \geq p$. If $\sigma^*(u, h_p) > 0$, then bidder i has a strictly positive probability of winning at such p, for strictly negative profits. We conclude that $\sigma^*(u, h_p) = 0$ in such a case.

Next consider the case where $p > p_\delta$. Given the first quit at $p - \delta$, from the previous lemma, the uninformed bidder i must attach positive probability to the event that the quitting bidders (at $p - \delta$) are informed and $v = p - \delta < p$. Consider first the case where some quitting bidders are informed, and so all remaining bidders are uninformed. If i quits at p and wins, then $P = p - \delta$, yielding him zero profits. If he stays at p and $\sigma^*(u, h_p) > 0$, then there is strictly positive probability that all bidders stay and pay $p' \geq p$ when they win, for strictly negative profits. Thus, he is strictly better off from quitting in this case. In the other case (i.e., all quitting bidders at $p - \delta$ are uninformed), the expected payoff from quitting at $p > p_\delta$ is

$$\mu_p q_p \frac{1}{k}(\overline{v}_\delta - p) + (1 - \mu_p) r_p \frac{1}{k}(p - p)$$

whereas the expected payoff from staying at p is

$$\mu_p q_p (\overline{v}_\delta - p) + (1 - \mu_p) r_p (p - p) + (1 - \mu_p q_p - (1 - \mu_p) r_p) W_p$$

Notice first that $W_p \leq 0$, since in the event i and some other bidderstay at p and i wins he pays a price $p > \overline{v}_\delta$, for negative profits when all other bidders are uninformed, and zero profits by the previous lemma when at least one other bidder is informed. Since $\overline{v}_\delta < p$, it follows that if $\sigma^*(u, h_p) > 0$, we must have $q_p = 0$ and so $\sigma^*(u, h_p) = 1$. Then, if i stays at p, in a symmetric equilibrium he must win at a price $p' \geq p$ when all bidders are uninformed with strictly positive probability, i.e., $W_p < 0$. We conclude $\sigma^*(u, h_p) = 0$. This completes part (ii).■

From the two lemmas we conclude that uninformed bidders will start that auction with probability 1 (if $\delta < p_\delta$) and quit immediately after observing a quit. Otherwise they will continue till $p_\delta - \delta$ with probability 1, possibly randomize at p_δ and drop out at $p > p_\delta$. Informed bidder will continue till $v - \delta$ with probability 1 and quit thereafter. The case where informed bidders randomize at $p = v - \delta \leq p_\delta$ given $q(p - \delta) > 0$ and $a(p - \delta) = 2$ is off the path of play.

Events at the auction

Recall $a(0) = n$ and $q(0) = 0$ and $p_\delta > \delta$.Let $k \geq 0$ be the number of informed bidders present.

1. By case 3b, all uninformed bidders are active at $p = \delta$, and so are all informed bidders iff $v > \delta$ by cases 1a and 2a.

2. If $v = \delta$ and $k > 0$, by case 4b, all remaining (uninformed) bidders quit at 2δ, to win at $P = \delta$ and zero profits. Otherwise the auction continues.

3. If $v > \delta$ but $k = 0$, by case 3a,3b,3c, uninformed bidders stay till p_δ, randomize there, quitting at $p_\delta + \delta$, earning profits of the order $\overline{v}_\delta - p_\delta$.

4. If $v > \delta$ but $v < p_\delta$ and $k > 0$, then by case 1a informed bidders quit at v (cases 1a,2a) and uninformed bidders (if any) quit at $v + \delta$ (cases 3b, 4b).

5. If $v = p_\delta$ and $k > 0$, then by case 1a informed bidders quit at p_δ (cases 1a,2a), some uninformed bidders (if any) may also quit at p_δ, and the rest at $p_\delta + \delta$. (cases 3b,3c,4a,4b).

6. If $v = p_\delta + \delta$ and $k > 0$, then some uninformed bidders (may) quit at p_δ. All remaining bidders quit at $p_\delta + \delta$, by cases 1a, 1b, 3a, 4a. The price is at least p_δ for profits of order at most δ.

7. If $v = p_\delta + 2\delta$ and $k > 0$, then some uninformed bidders (may) quit at p_δ and the rest at $p_\delta + \delta$, by cases 3a,4a. If $a(p_\delta) = 2$, then all remaining informed bidders may also quit at $p_\delta + \delta$ for profits of order δ, or else quit after one more tick.

8. If $v > p_\delta + 2\delta$ and $k = 1$, then the auction concludes at $P = p_\delta$ or $p_\delta + \delta$. If $k > 1$, then it concludes at v.

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