Adverse Selection and Convertible Bonds*

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Abstract

Informational asymmetries between a firm and investors may lead to adverse selection in capital markets. This paper demonstrates that when the market obtains noisy information about a firm over time, this adverse selection problem can be costlessly solved by issuing callable convertible bonds with restrictive call provisions. Such securities can be designed to make the payoff to new claimholders independent of the private information of the manager. This eliminates the possibility of any dilution of equity or underinvestment and implements the symmetric information outcome in either a pooling or a separating equilibrium. The same first–best efficient outcome can also be implemented by issuing floating price and mandatory convertibles.

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1 Introduction

Consider a firm that needs to raise capital in order to finance a new project. Suppose that potential investors have less information about the value of the firm's assets in place and future prospects in comparison to the management. Since any security issued by the firm is priced in competitive markets at its (discounted) expected value conditional on the investors' information, the claims sold to outsiders may dilute the value of claims held by the existing owners of the firm when the manager's information is better than average. As in Akerlof (1970), this possibility of dilution gives rise to an adverse selection problem— the manager, acting in the interest of the existing owners, may prefer to forego a positive net present value project instead of selling undervalued claims to finance the investment. The usual unravelling then leads to a socially inefficient outcome where capital is raised only by low quality firms. In this paper, we reconsider this classic problem of inefficient underinvestment that arises due to adverse selection, which was first analyzed by Myers and Majluf (1984).

We start from the premise that the initial asymmetry of information about the firm's assets in place and investment opportunities is likely to be resolved over time even though at each date the manager's information is superior to that held in the market. Analyst announcements, future earnings, outcomes of research and development, announcements of mergers and acquisitions or decisions by regulators are some of the events that may reveal valuable information to the public over time. Our main goal is to use the future imperfect resolution of the initial asymmetry of information to design a security whose value is *independent of the initial private information* of the manager. In equilibrium, the price obtained for such a security in competitive markets will be a 'fair' price from the perspective of the manager regardless of his private information. As a result, the symmetric information outcome of no dissipation or dilution will be implemented, solving the adverse selection problem costlessly.

We show that the optimal security has all the features of commonly observed convertible debt or preferred stock contracts.[1] Such a contract gives the bondholder the option to convert the bond into another security, typically the common stock of the issuing firm at pre-specified terms summarized by a conversion price. It can be thought of as a combination of two different securities: a debt contract plus a long-lived 'call' option to convert the bond into equity that has greater upside potential. Alternatively, it can also be viewed as equity plus a 'put' option to convert to safer debt.

Convertible bonds are frequently callable. The callability feature gives the firm the ability to buy back the bond before maturity by paying a prespecified call price. If the convertible is called, however, the bondholders still have the right to convert the bond into common stock instead of tendering it to the firm. Such contracts also typically impose restrictions on the call provision that prevent the

[1] In order to focus on the inefficiencies arising out of asymmetric information, we abstract away from considerations of tax or clientele effects, as well as bankruptcy and financial distress costs. As a result, debt is equivalent to preferred stock in the context of our model. For simplicity, we will refer to the senior claim as debt.

manager from calling the bond unless the firm's prospects improve sufficiently over time and the stock price is higher than a threshold or trigger value.

We use these features of callable convertible bonds with restrictive call provisions in order to costlessly solve the adverse selection problem. The idea is as follows. Since the put option to convert to safer debt is valuable to investors, the equity-aligned manager seeks to force early conversion by calling the convertible. By doing so he is able to reduce the expected value of claims held by outsiders, thus increasing the value of the residual claims held by equity holders. However, the trigger price restriction on the call provision allows the manager to force conversion only when sufficiently favorable information about the firm arrives in the market and raises the stock price. Since the probability that favorable future information will arrive in the market is positively correlated with the initial information of the manager, he expects to force early conversion more often when he has good information. Consequently, the investors are less likely to end up holding a valuable option to convert when the firm is more valuable and more likely to end up holding such an option when the firm is less valuable. This allows the expected value of initial claims sold to be independent of the manager's private information.

The ability to extinguish the convertibility option via a conversion forcing call can be thought of as a 'bet' the manager lays with the market that good news will arrive in the future. The market overestimates the expected payoff from this bet whenever it underestimates the expected payoff from the claims on cash flows sold by the manager. For a suitably designed convertible bond, these two effects exactly offset each other. The value of such a security is then independent of the manager's private information, thereby eliminating the possibility of inefficient underinvestment arising out of adverse selection.

This symmetric information outcome can also be implemented with floating price convertibles, i.e., convertibles with conversion prices that depend on publicly observed market values, including mandatory convertibles that are automatically converted into equity. As with callable convertibles, the optimal mandatory convertible has the property that, in equilibrium, the market value of the claims held is lower when more favorable information is later revealed to the market. The higher the quality of the manager's initial information, the more likely it is that good information will later be revealed to the market, in effect keeping the initial expected value independent of the private information of the manager. Such a security exists as long as the resolution of the initial asymmetry of information occurs with enough fidelity, enabling the manager to avoid any inefficiencies.

Our results show that the underinvestment problem can be solved without any need for signaling. Indeed, we focus primarily on an efficient pooling equilibrium in which the manager issues the same optimal convertible bond regardless of his private information. We also show that there exist outcome-equivalent efficient separating equilibria that must involve similar securities, especially when the possibility of bankruptcy is a concern. Such multiplicity of equilibria is a common feature of games of incomplete information. Different equilibria and the associated optimal securities differ in terms of their observable implications, as we discuss in detail later in the paper.

The available evidence from the growing global convertible market suggests that managers do use call provisions to extinguish the convertibility option whenever they can.[2] However, the prevalence of call restrictions make reliance on such forcing calls risky. To take one case, on December 8, 1999, Human Genome Science (HGS) raised $200 million by issuing subordinated convertible notes that were due in 2006. Under the terms of the issue, HGS could not call the bond before December 2002 unless the stock price crossed a trigger price (equal to $107.44, or 150% of the conversion price) and stayed there for 20 out of 30 consecutive trading days prior to the call date. In retrospect, HGS could not have chosen a better time for the issue, as the market was energized by prospects of genomics-led medical discoveries. The genomics/biotech sector turned out to be the best performing equity sector for that period, gaining 60% during the first three months of 2000. As a result, HGS was able to satisfy its call restrictions and call the bonds to force conversion on March 2, 2000, just 85 days after the original issue, its own stock having gained 96.1% over the period. Buoyed by the enthusiastic response of the market to the potential of genomics research, HGS undertook another convertible issue on March 6, 2000. The bonds in the issue were due in 2007, and they also had a three year restrictive call provision, with a trigger price set at $164.25 (again, equal to 150% of the conversion price). However, HGS stock performed less favorably than originally anticipated and this second issue could not be forced into early conversion.

The experience of HGS is by no means unique. Another well-known case concerns MCI Communications Corporation during the years 1978-83, a period of dramatic growth for MCI that was financed by frequent infusions of external capital (see, e.g., Greenwald (1984)). Between December 1978 and July 1983, MCI undertook seven successive convertible issues, raising a total of $1.895 billion. As with HGS, all these convertibles were callable, but with restrictive call provisions— the bonds could be called only if the market price of MCI stock exceeded the conversion price by a pre-specified margin of around 25% for 30 consecutive trading days around the call date. MCI's stock price rose enough for it to be able to force conversion on the first five of these seven issues by February 1983. However, MCI fared poorly in product market competition with AT&T and its stock price went into sharp decline subsequently. The call restrictions made it impossible for MCI to force conversion on the last two issues and it was left with a debt burden that it had difficulty servicing.

The paper is structured as follows. In Section 2, we discuss the related theoretical literature. In Section 3, we set up our basic model. Section 4 contains our results on the the optimality of standard callable convertible securities. Section 4.1 discusses the benchmark case where the asymmetry of

[2] The total size of the convertible market was $600 billion in the early 2000s. In 2001, for example, there were around 400 new issues in the U.S. convertible market that raised a total of $106.8 billion (source: Securities Data Company, Inc.). See also www.convertbond.com, a division of Morgan Stanley Dean Witter, and Francis, Toy and Whittaker (2000).

information is perfectly resolved over time while Section 4.2 analyzes the case where the asymmetry of information is never perfectly resolved. Our results on mandatory convertibles are presented in Section 5. We discuss the empirical implications of our results in more detail in Section 6. Section 7 contains our concluding remarks including possible extensions, while the Appendix contains all of the longer proofs.

2 Related Literature

The seminal work of Myers and Majluf (1984) has been followed by a large literature attempting to identify securities that mitigate the dilution and associated underinvestment problem. Brennan (1986) is the closest in spirit to our work. Brennan points out that a floating-priced convertible security can avoid the adverse selection problem if the conversion price depends on the market price. Such a security is automatically converted into $1/p$ shares, where p is the market price at the time of conversion so that the total dollar value of the security is independent of the market price. If the private information of the manager is perfectly reflected in the market price at the time of conversion, then the adverse selection problem can be costlessly solved with such a security. When the manager's private information is imperfectly incorporated into the market price however, issuing such a security leads to dilution and may cause underinvestment. In contrast, we show that first–best efficiency can be achieved with commonly used securities, such as a callable convertible bond with a fixed conversion ratio and restrictive call provisions, even when the manager's private information is never perfectly known by the market. We show also that this can be done via floating–price convertibles, provided the market value of the security is decreasing in the share price.

A significant portion of the literature following Myers and Majluf (1984) focusses on modes of financing that allow the management to separate by signaling its type and thus solve the underinvestment problem. Since separation by signaling may be costly, it may create another source of inefficiency and dissipation in value that might even exceed the dissipation in value caused by dilution.[3] In fact, Nachman and Noe (1994) show that non-dissipative signaling is not possible if the firm is limited to issuing securities with payoffs that are weakly increasing in the underlying cash flows.[4] Our work shows that the first-best outcome can in fact be implemented in equilibrium, without any signaling or any dissipation in value. The difference with Nachman and Noe is that the manager is able to utilize call provisions and the effect of information revelation on future market prices in order to force investors to choose between different non-decreasing securities.

[3] The reader is referred to Harris and Raviv (1991) for a more thorough survey of the earlier signaling literature. In addition to costly signaling, there are papers analyzing how costly information acquisition might be used to mitigate adverse selection. See, e.g., Fulghieri and Lukin (2001).

[4] Innes (1990) points out that restricting attention to such non–decreasing securities prevents the creation of an agency problem whereby the manager temporarily inflates cash flows and so reduces the payout to investors.

A number of papers restrict their attention to securities with non-decreasing payoffs and yet manage to attain separation via signaling without dissipation in value. This is done either by introducing frictions that are absent in the Nachman and Noe (1994) set up or by expanding the strategy space. Such signaling devices have the special property that they that are costly to mimic for the bad type but not costly in equilibrium for the good type. Among these, Stein (1992) shows that callable convertible debt can be used by good firms to signal their types and separate from bad firms, in a model where the initial asymmetry of information is completely resolved by the time the security is called. The bad firm does not mimic the good firm, provided the expected cost of financial distress from doing so is high enough to overcome the benefits of selling an overvalued claim. Since the initial information asymmetry is perfectly resolved, good firms are able to call the bonds and force conversion, thereby avoiding the same costs of financial distress.[5] This ability to avoid the costs of financial distress is the "back door equity" value of convertibles in Stein's setting. In contrast, we consider a more general environment in which the initial asymmetry of information is never perfectly resolved and where the value of the optimal security is independent of the private information of the manager and the beliefs of the market. As a result, there is no scope for mispricing whether or not the bad firm mimics the good firm, and even though the manager cannot guarantee that he will be able to force conversion in the future. In our setting, the back-door equity value of a convertible security arises simply from the fact that the manager may be able to exploit market reaction to 'good news' in the future and extinguish a valuable long-lived option to convert via a forcing call. The probability of being able to do so is correlated with the manager's private information. The manager can take advantage of this correlation in designing a security that raises funds without dissipation or dilution at a fair price. As we discuss later, there is empirical evidence that managers are not always able to force conversion. Therefore, if bankruptcy and financial distress costs are a real concern, the manager should simply use convertible preferred stock or mandatorily convertible securities similar to those we analyze.

Constantinides and Grundy (1989) show that securities similar to (noncallable) convertible bonds can costlessly solve the adverse selection problem by signaling information, provided the firm is also allowed to buy back shares. In the absence of the possibility to buy back shares, there is no fully revealing equilibrium involving securities whose value is increasing in cash flows. On the other hand, Brennan and Kraus (1987) show that the good type may separate from the bad type by retiring existing debt, which is too costly for the bad type to mimic. Our model also allows the manager to buy back previously issued securities, but such strategies are not utilized in equilibrium.

Nyborg (1995) considers signaling with convertible debt in a model where new private information arrives at each date to a risk-averse manager. In his model, there is information in both the type of

[5] The perfect resolution of the informational asymmetry implies that a strategy of using short-term debt and refinancing later is also optimal in Stein's (1992) setting. This equivalence does not obtain in our model where there may be residual asymmetric information at each date. See Section 3 for a fuller discussion.

security initially issued and the decision to call a previously issued convertible. He shows that risk-averse managers signal their quality by not calling immediately, whereas bad managers whose equity is expected to decline in the near future are forced to call.[6] Therefore, forced conversion is accompanied by a negative stock price reaction. In our model, the manager may also have (residual) private information at the time of the call. Crucially however, with risk neutrality there is no information dependent cost from delaying the call, precluding a signaling role for delayed call decisions in our setting.

Other explanations have been offered for the use of convertible securities. While in our model the asymmetric information is ordered in the sense of first order stochastic dominance, Brennan and Schwartz (1987) show that convertible bonds can be designed to be independent of asymmetric information about volatility, and not the mean, of cash flows. This provides insight on the possible role of convertibles in mitigating the asset substitution and other incentive problems caused by conflicts of interest among senior and junior claim holders (see Jensen and Meckling, 1976). Green (1984) shows that such incentive problems can be mitigated by convertible debt rather than straight debt. Cornelli and Yosha (2003) analyze a problem in which a manager can manipulate the interim signal about the quality of a project. If the investment occurs in multiple stages, this possibility of "window dressing" results in a conflict of interest and thus inefficient investment. Convertible debt can be used to solve this problem. A growing literature also studies the use of convertible securities as a means of mitigating moral hazard problems within staged venture capital financing (see, e.g., Repullo and Suarez (1998)) while a large and earlier literature analyzes the pricing of convertible securities, the optimal exercise of call options and stock returns at the announcement of convertible debt calls (e.g., Ingersoll (1977a and 1977b) and Brennan and Schwartz (1977, 1980)).

The idea that an adverse selection problem may be alleviated via contract design without any need for signaling has applications beyond project financing. In a durable good context, Grossman (1981) investigates the role of warranties in solving the lemons problem. He shows that pooling with an optimally designed warranty contract is optimal, in a setting where the future performance of the good is public information that perfectly reveals its value.

One objection to such a warranty policy is that a large variance in its final value may be unpleasant for risk-averse buyers. In our context however, the availability of well–developed markets for hedging instruments makes such an objection less problematic. Lutz (1989) puts forward a different objection to such warranties. She argues that these warranty contracts may not be seen in practice, since buyers of the good may have an incentive to undetectably damage the durable good in order to obtain a large warranty payment. In our project financing context, such manipulation of the underlying assets by the buyers of the convertible security (and the associated moral hazard problem) is less likely to be an issue since the buyers do not actually own the assets but only claims written on those assets.

[6] This is similar to Harris and Raviv (1985), except that in Nyborg (1995) the decision to issue a convertible is also endogenous.

However, to the extent that they can manipulate the underlying stock price, this may give rise to an analogous agency problem in our setting. We discuss the relatively higher vulnerability of mandatory convertibles compared to standard callable convertibles to such stock price manipulation in Section 6.

Finally, the belief-independence property of our optimal security has some conceptual similarities with the informed principal problem under common values. As Maskin and Tirole (1992) show, the informed principal will offer a menu of contracts that is acceptable to the agent regardless of his beliefs about the principal's type since different types of principal will subsequently self-sort by choosing from the menu. The optimal contract will be second-best efficient. In our setting, the equilibrium value of the optimal security is also independent of the beliefs of the market about the type of the manager and the manager can achieve the first best outcome by offering a single contract regardless of his type. This is possible because the private information of the manager is later publicly revealed, allowing the different types of the manager to stochastically separate over time.

3 The Basic Model

The basic structure of our model is essentially identical to that of Myers and Majluf (1984). We consider a firm that has both assets in place and a new investment opportunity. The values of both the new investment opportunity and assets in place are uncertain. The uncertainty is captured by the "type" of the firm, θ_1 or θ_2, that has prior probabilities, λ_1 or λ_2.[7] The manager privately knows the type of his firm and the cash flows from both the assets in place and from the new investment opportunity depend on the type. Initially, the firm is all equity, with the number of shares outstanding given by $M = 1$.[8] The firm does not have sufficient internal funds to invest in the new project and has to raise capital by selling additional securities. The manager makes his decisions to maximize the welfare of the existing shareholders, the riskless rate is normalized to 0, and all agents are risk–neutral.

Let A_i stand for the expected value of the cash flows from the assets in place given type θ_i. The manager has to raise an amount $I > 0$ from outside investors in order to invest in a project. The new investment and assets in place *combined* produce a random cash flow of $X \geq 0$. Let $G(x|\,\theta_i)$ denote the cumulative distribution function of X given θ_i. We assume that project cash flows for type θ_2 first order stochastically dominate those for type θ_1:

$$\text{For all } x,\ G(x|\theta_2) \leq G(x|\theta_1). \tag{1}$$

Define the expected value of the total cash flows for type θ_i of the firm, given that it invests, to be $V_i = E[X|\theta_i]$. Let $\overline{V}$ denote the ex–ante expected value of V_i. From (1), we have $V_2 \geq V_1$. To focus on non-trivial cases, we assume henceforth that $V_2 > V_1$.

[7] In Section 5 we consider the N type case for $N > 2$. The restriction to finite types is for convenience only.

[8] The restriction to an all–equity firm simplifies the exposition. All results extend in a straightforward manner to the case where the firm has existing senior debt outstanding, provided we interpret all cash flows as net of prior obligations.

In line with Myers and Majluf (1984), we assume that projects have positive NPV regardless of the manager's type, i.e.,

$$V_i - A_i > I, \tag{2}$$

for all $i = 1, 2$. With symmetric information, and in competitive markets, all types of the manager will undertake the socially efficient outcome of investing in the project. The expected value of the claims to cash flows sold will equal I, the outlay for the project and expected payoff of the existing shareholders from investing will equal $V_i - I$, the NPV of the *total* cash flows to the firm of type θ_i. In what follows we will refer to such an outcome interchangeably as symmetric information or the first-best efficient outcome. In the presence of informational asymmetries between the manager and the market, the first–best efficient outcome may not be consistent with equilibrium behavior as shown by Myers and Majluf (1984).

If the cash flows from the project together with the assets in place of the firm are greater than or equal to the cost of the project with probability one, then the firm can always issue riskless secured debt at zero cost and the problem would be uninteresting. To rule out the possibility of riskless debt, we assume that, (at least) when the type of the manager is θ_1, with strictly positive probability the total cash flows X will fail to cover the required outlay of I, i.e.,

$$G(I|\theta_1) > 0. \tag{3}$$

We turn now to the timing structure. Our model has three dates, 0, 1, and 2, and two (groups of) players, the manager (who maximizes the welfare of old shareholders) and the potential investors. The manager knows θ when he makes his investment and financing decisions. In contrast, initially the investors are uninformed about θ, though later they will obtain information about the firm type. Furthermore, these investors are competitive and rational, so that at each date they value all securities at their expected value given publicly available information. We will refer to the set of potential investors collectively as the market.

At date 0, given his private information, the manager decides whether or not to invest and what securities to issue to finance the project. The market is uninformed about the manager's type at date 0 and competitively values the securities issued by the manager, taking into account any information revealed by the issue itself. The manager invests at date 0 if the issue succeeds, while at date 2 the total cash flows are realized and distributed.[9] At the intermediate date 1, some of the asymmetric information present at date 0 is resolved. Specifically, the market publicly observes a signal $m \in \{m_1, m_2\}$ of the type of the manager. We assume that the probability with which a signal m_i is observed, given the manager is of the type θ_i, is equal to $\beta \in (\frac{1}{2}, 1]$. The parameter β is a proxy

[9] Though this possibility never arises in equilibrium, we assume that if the manager fails to raise the required outlay for the project at date 0, he invests the amount raised in a riskless asset. On the other hand, if he raises more than the required outlay, he immediately distributes the excess as dividends.

for the degree to which the initial asymmetry of information between the manager and the market is resolved between the time the investment is undertaken but before cash flows are realized. The case $\beta = 1$ corresponds to the case of perfect resolution. On the other hand, the case $\beta = \frac{1}{2}$ corresponds to the case where none of the asymmetry is ever resolved before cash flows are realized. In general, date 1 can be thought of as the time necessary for information of quality measured by β to be disclosed to the market. Though our results do not depend on a specific interpretation of the signal m, it might help the reader to think of it as an analyst announcement or the outcome of a patent application, which may or may not be approved.

Let $S(x)$ be the final payoff from any security S as a function of realized date 2 cash flows x. We restrict attention to securities that (i) satisfy limited liability (i.e., $0 \leq S(x) \leq x$ for all x) and (ii) have final payoffs that are a non–decreasing function of x. An equity share $\alpha \in (0, 1)$ with payoff αx is an example of such a security, as is a debt contract with face value F and payoff $\min[F, x]$. A simple (i.e., non-callable) convertible bond with face value F, ownership share α, and payoff $\max[\alpha x, \min(F, x)]$ is also an example of such a security. While we focus on these common types of securities in the analysis to follow, we allow the manager to choose any security that satisfies (i) and (ii) above. In particular, property (ii) implies that $E[S(X)|\theta_i]$ must be non-decreasing in i, by (1). This implies that the good type of the manager will necessarily face dilution of the claims of existing owners when offering such a security to the market, giving rise to the possibility of inefficient underinvestment arising out of adverse selection.

We also allow the manager to issue securities whose payoffs depend on the date 1 endogenous market response to the realized public signal m. For example, the manager is allowed to issue a callable convertible bond that can be called only if the stock price exceeds a threshold value.[10] The manager may then manage to force conversion into equity for some public signals, with investors holding the convertible for others. Since the final payoff to investors satisfy conditions (i) and (ii) above, regardless of whether the manager forces conversion or not, such a security is also admissible. Similar remarks apply to the case of a floating price convertible whose conversion rate depends on the stock price.

This completes our description of a dynamic game of incomplete information between the manager and the market. Our notion of equilibrium will correspond to the perfect Bayesian equilibria of this game.

We end this section with brief comments about two assumptions of our model. First, since the dilution costs of adverse selection arise via transfers from the old claimholders to the new claimholders,

[10] We do not allow the manager to make payoffs *directly* contingent on the public signal m. Such securities are not common, presumably because signals such as analyst disclosure are quite amorphous and contracts directly contingent on them are not enforceable in a court. If such securities were allowed, one can establish the uniqueness of the efficient equilibrium outcome.

it is necessary for our results that the manager attach a 'welfare' weight to old claimholders that is strictly greater than what he attaches to the new group. For simplicity, we assume that the manager cares only about the old shareholders. Such an assumption is common in the literature and may be innocuous in our context, since in practice younger firms (which are more susceptible to adverse selection in the first place) do seem to display a higher degree of managerial ownership, presumably aligning managerial interests with those of the existing claimholders.[11] Second, we implicitly assume that the manager cannot postpone his investment decision to a later date, possibly because actions by competitors will erode the value of the project if he does so. Similarly, it is also not possible for the manager to anticipate a future need for cash and raise the required cash early, at a date before date 0, under conditions of symmetric information.[12]

4 The Optimality of Callable Convertible Securities

To gain intuition, we start by analyzing the benchmark case where the date 0 asymmetry of information is perfectly resolved at date 1. This corresponds to the case where $\beta = 1$. In Subsection 4.2, we will consider the case where $\beta < 1$, so that the date 0 asymmetry of information is never perfectly resolved.

4.1 Perfect Resolution of Asymmetric Information

Under perfect resolution of the asymmetry of information, a callable convertible security can be summarized by the variables F denoting the face value of the bond, α denoting the share of the firm the bondholders will have if they decide to convert into common stock, k denoting the call price that the bondholders will receive if the bond is called and they decide to surrender the bond, T_{call} the maturity date of the call provision, and T_{conv} the maturity date of the convertibility option. It will become clear later that nothing can be gained by calling the bond before date 1 (i.e., the first best cannot be achieved). Accordingly, we assume henceforth that the callability option cannot be exercised before date 1. In practice, convertibles frequently have call protection periods that prevent calls within a minimum initial period. As mentioned earlier, in the context of the model, date 1 can be thought of as the time required for information of quality β to be disclosed to the market.

[11] See, e.g., Graham and Harvey (2001). It is not clear, however, that this is an optimal provision of managerial incentives from the perspective of the shareholders. Ex–ante, uninformed shareholders may prefer that the manager maximize the total value of the firm and so not underinvest. See Dybvig and Zender (1991) for a similar point. However, such a contract may not be renegotiation proof at the interim stage (Persons, 1994). A complete characterization of the optimal provision of managerial incentives is beyond the scope of this paper.

[12] Perhaps because there is no such date— the firm may be 'born' under conditions of asymmetric information. Note in this respect that we allow the expected value of the assets in place A_i to depend on the manager's type. A strategy of raising a lot of cash early may also create agency problems between the manager and shareholders, arising out of inefficient use of 'free' cash, and so may be prevented by the latter group.

The expected value of a share α given a type θ_i is denoted by αV_i. Similarly, let $C_i(\alpha, F) = E[\max(\alpha X, \min(X, F)) \,|\, \theta_i]$ be the expected value of a non-callable convertible, conditional on θ_i. In line with common practice, we assume that if the firm calls the convertible security, the holders still retain the right to convert into equity and do not have to surrender the security as long as they convert.

We show in what follows that such a security exists with the property that all types of the manager can raise the required outlay by issuing it without any dissipation in the value of existing equity.[13] This is achieved by designing the security to ensure that a bad signal in the future allows investors to hold a convertible whereas a good signal allows the manager to extinguish the protective put option leaving investors with straight equity. For any type of the manager, a convertible is more valuable than just the equity part of the security. A properly designed callable convertible ensures that its value when prospects are unfavorable equals the value of the straight equity part when prospects appear more favorable. Consequently, the fair date 0 value of the security is equal to the required outlay of I regardless of the manager's information at that date.

Suppose that $T_{call} = 1$, so that the call provision expires on date 1 while $T_{conv} = 2$, so that the convertibility option is long-lived and expires on date 2.[14] Let the equity share α be a 'fair' share of the cash flows given that the manager's type is θ_2. That is, α satisfies:

$$\alpha V_2 = I. \tag{4}$$

Next, suppose that the face value F is such that, the value of the non-callable convertible is equal to the outlay of I, given that the manager has bad news, i.e., his type is θ_1,

$$C_1(\alpha, F) = I. \tag{5}$$

It is easily seen that such an F can always be found. At $F = 0$, the expression in (5) is equal to $\alpha V_1 < I$, using (4), while if F becomes large the expression in (5) approaches $V_1 > I$. By continuity there exists an intermediate value of F such that (5) holds. We show now that the call price k can be chosen suitably to finance the project regardless of θ, at zero cost for the existing shareholders. To do this we proceed backwards in time and analyze the optimality of the manager's decision to call the bonds and the optimality of the investors' decisions to convert.

Suppose that we are in date 1 and $m = m_2$, so that it is common knowledge that $\theta = \theta_2$. We want the call value k to be such that the manager wants to call the bonds in this case. The optimality of the call decision depends in turn on the optimal conversion decision of the bondholders, both in the case where the bonds are called and when they are not. If the bonds are not called, the bondholders

[13] We focus here on a pooling equilibrium here. In the next subsection, we discuss the possibility of other equilibria.

[14] The difference in the expiration dates of the two options is not essential but captures the commonly observed feature that the call price varies over time. Similarly we assume that the debt part of the security is a zero–coupon bond. These assumptions keep the analysis simple and do not affect any qualitative results.

will not want to convert as the convertibility option is more valuable alive than dead, i.e., since their payoff from not converting is at least as high as their payoff from converting:

$$C_2(\alpha, F) \geq C_1(\alpha, F) = I = \alpha V_2. \tag{6}$$

If the bond is called, then the bondholders will want to convert if their payoff is at least as high as that from holding the bond, i.e., if:

$$k \leq \alpha V_2 = I. \tag{7}$$

Suppose that k is such that (7) holds. Then, when $\theta = \theta_2$, the manager will want to call the bonds and force conversion, as the payoff for the old shareholders from doing so equals $(1-\alpha)V_2$, which is at least as high as $V_2 - C_2(\alpha, F)$, the payoff from not forcing conversion.

Suppose next that we are in date 1 and $m = m_1$, so that it is common knowledge that $\theta = \theta_1$. We want the call value k to be such that the manager does not want to call the bonds and the bondholders do not want to convert the bonds if they are not called. If the bonds are not called, the bondholders do not want to convert as:

$$\alpha V_1 < \alpha V_2 = I = C_1(\alpha, F). \tag{8}$$

Therefore, the manager will not want to call the bonds if:

$$k \geq C_1(\alpha, F) = I. \tag{9}$$

From (7) and (9), if:

$$k = I, \tag{10}$$

then the manager will call the bond to force conversion if $\theta = \theta_2$ and will not call the bond if $\theta = \theta_1$. In the latter case, bondholders will not convert. For such a bond and sequentially optimal call and conversion decisions, the payoff to the bondholders will be equal to I regardless of the private information of the manager, using (4) and (5). As a result, when the bond is issued at date 0, investors will not face any adverse selection and will be willing to provide I, the expected value of the issue. Furthermore, the expected payoff for the old shareholders in type θ_i of the firm at date 0 will equal $V_i - I$, the first best value given θ_i. As a result, the manager will always invest. Finally, we have to specify beliefs off the equilibrium path at date 0 to complete the characterization of this perfect Bayesian equilibrium. We suppose that the uniformed investors believe that $\theta = \theta_1$ whenever the manager issues any other security at date 0. Thus, neither type of the manager has an incentive to deviate.

Proposition 1 *Suppose $\beta = 1$. Then there exists a pooling equilibrium that implements the first-best efficient outcome in which the manager invests by issuing the callable convertible security with α and F given by (4) and (5), k given by (10) $T_{call} = 1$ and $T_{conv} = 2$. In this equilibrium, the manager will call to force conversion iff m_2 is observed.*

Proof. Follows from the discussion above. ■

The value of the optimal security that we characterize above is independent of the private information of the manager. Thus, the security is correctly valued even though the bad type mimics the good type. This property of the optimal security will be seen to carry over to the case where the resolution of the asymmetry of information is imperfect. Notice in this respect that we did not specify any soft call restrictions on the security. In the next section, we will see that such restrictions are needed only when the signal m is noisy, i.e., $\beta < 1$.

When $\beta = 1$, first–best efficiency can also be achieved simply by using short–term debt that matures in period 1 and then refinancing when there is no asymmetry of information. Specifically, at date 0, the manager can issue short–term risk–free debt, issuing any other security to retire the debt at zero cost in the symmetric information environment of date 1. Similarly, the efficient outcome could also be implemented by floating price convertibles of the sort considered by Brennan (1986). This equivalence breaks down when the date 0 asymmetry of information is never perfectly resolved. In such cases, convertibles of the type we characterize strictly dominate the short term debt-and-refinancing strategy, as well as Brennan-type securities. The simple scenario of this section nevertheless serves to bring out the intuition as to why callable convertible securities mitigate adverse selection problems.

4.2 Imperfect Resolution of Asymmetric Information

We now turn to the case where the date 0 asymmetry of information is only imperfectly resolved at date 1, i.e., $\beta \in (1/2, 1)$. As we show below, as long as β is high enough, there exists a callable convertible security that can achieve the first-best. The structure of such an optimal security will be very similar to the callable convertible bond characterized in Section 4.1. The manager will call to force conversion into equity only when good news is disclosed at date 1 and the security will not be called or converted when bad news is disclosed. The modification to the previous case is that in the case of bad news, the manager will be prevented by a restriction on the call provision from calling the bond and forcing conversion. This restriction on the call provision will take the form that the security can be called only when the date 1 share price of the firm exceeds a certain threshold or trigger value p. By specifying this restriction at date 0, the manager will be able to commit to not calling the bond and using his privileged information in the future at the expense of the new investors, ultimately benefitting the existing claimholders. Such a callable convertible bond with a restrictive call provision is determined by $(F, \alpha, k, p, T_{call}, T_{conv})$. We also let $\rho = \frac{F}{\alpha}$ denote the conversion price of this bond, i.e., the face value of debt the bondholder must give up per share acquired upon conversion.[15] We first characterize the callable convertible that implements the symmetric information in a pooling

[15] Since we have normalized the number of existing shares M of the firm to equal 1, a fraction α of equity ownership translates into a *number* of shares $\frac{\alpha}{1-\alpha}$, implying a conversion price of $\frac{(1-\alpha)F}{\alpha}$. Our notion of a conversion price adjusts for dilution effects.

equilibrium outcome similar to the one in the previous section. Subsequently, we consider efficient separating equilibria and identify some comparative static properties of the optimal security.

Let the maturity date for the call provision T_{call} be set for date 1 and the maturity date of the convertibility option T_{conv} be set for date 2, as before. In a candidate pooling equilibrium, both types of the manager issue the security at date 0 and expect the security to be converted (due to a forcing call) to equity at date 1 if and only if good news is disclosed (i.e., $m = m_2$) at that date. Suppose that, given this expectation, each type $\theta \in \{\theta_1, \theta_2\}$ of the manager estimates that the expected value of the claims sold equals I, the cost of the project. Since $\Pr[m_i|\theta_i] = \beta$, the face value F and equity share α must then satisfy the following two equations:

$$\beta C_1(\alpha, F) + (1-\beta)\alpha V_1 = I, \tag{11}$$

$$(1-\beta)C_2(\alpha, F) + \beta\alpha V_2 = I. \tag{12}$$

The first equation states that the required outlay of I is equal to the expected value of the security, conditional on θ_1 and the fact that the security will be converted to equity when $m = m_2$ (an event that occurs with probability $1-\beta$ conditional on θ_1) but not when $m = m_1$ (an event that occurs with probability β conditional on θ_1). The second equation has the same interpretation but conditional on θ_2, in which case the event $m = m_2$ (resp., $m = m_1$) has probability β (resp., $1-\beta$).

In Figure 1, the curve L_1 depicts the α and F pairs that satisfy (11) and L_2 the pairs that satisfy (12). For $F = 0$, the convertible is identical to equity and the equity share α must equal $\frac{I}{V_i}$ in order for type θ_i to raise I as shown by the vertical intercepts of L_1 and L_2 in the figure. As F rises the required equity share α must (weakly) fall. In fact, since type θ_2 forces conversion to equity with probability β, a rise in F does not lower α by as much compared to type θ_1 who manages to force conversion only with probability $1-\beta$. As we show in the Appendix, for β sufficiently high, L_1 must fall faster than L_2 as F rises, intersecting L_2 *from above* for some $\alpha \in (0, 1)$ and $F > 0$. Such an intersection point (point P in the figure) corresponds to a security that has expected value equal to the required outlay of I for each possible type of the manager. In competitive markets it will trade at a price of I at date 0 regardless of the beliefs of the market about the type of the manager. We will refer to such a security as a *belief-independent convertible*.

We now establish the sequential optimality of call and conversion decisions. If the security is converted at date 1, it must be the outcome of a conversion forcing call since by definition a convertibility option is worth more alive than dead. Indeed, using (11) and (12) we must have:

$$\alpha V_i < I < C_i(\alpha, F) \text{ for } i = 1, 2. \tag{13}$$

This is the "back-door equity" value of the convertible security in the context of our model— when converted, the equity share of the new claim-holders will be lower than what they would obtain under symmetric information. To compensate the new claim-holders, the face value of the debt part of the

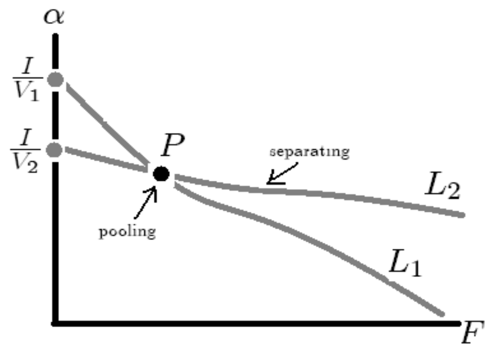

Figure 1: Efficient Equilibria

claim is raised and the convertible value is higher than what they would obtain under symmetric information.

Since the convertibility option is worth more alive than dead regardless of the manager's private information, the same relationship must hold given the market's information. This implies that the new claimholders will not convert unless forced to do so, while the manager would like to force conversion whenever he can. However, if the manager always forces conversion, the expected value of the security to the new claim-holders will fall below I and they will be unwilling to provide funding hurting the existing claim-holders. Consequently, a restriction on the call provision is necessary as it enables the manager to commit to not calling the bond unless good news is disclosed. In turn this will make the investors willing to pay I for the issue at date 0. We now determine the call price and the trigger price that are needed for this to hold.

Let $\mu_i(m)$ be the posterior probability at date 1 attached by the market to the event that $\theta = \theta_i$ after observing m. Note that since $\beta > \frac{1}{2}$, we must have $\mu_2(m_2) > \mu_2(m_1)$. Choose any call price k that is less than the expected value of the equity claim given $m = m_2$,

$$k < \sum_{i=1}^{2} \mu_i(m_2)\alpha V_i. \tag{14}$$

In equilibrium, when $m = m_2$, the right–hand side of (14) will be the date 1 market value of the equity part of the security. As a result, the bondholders will convert when it is called.[16] Choose the trigger price p to be in between the market value of old shareholders' claims when $m = m_1$ and when $m = m_2$, i.e.,

$$\sum_{i=1}^{2} \mu_i(m_1)[V_i - C_i(\alpha, F)] < p < \sum_{i=1}^{2} \mu_i(m_2)(1-\alpha)V_i. \tag{15}$$

[16] We suppose that the manager raises the required money by issuing equity, in the off–the–path of play event that bondholders surrender the bond upon a call.

Such an interval for p exists from (13). In equilibrium, the date 1 stock price will equal the right–hand side of (15) when $m = m_2$ so that the manager will be able to force conversion by calling, while when $m = m_1$ the stock price will equal the left–hand side of (15), so that the bond cannot be called and will not be converted. This completes our characterization of the optimal security. In the Appendix, we prove that it is an equilibrium for all types of the manager to issue this security at date 0 and that in this equilibrium there will be no dilution. Even though the asymmetry of information is never exactly resolved the adverse selection problem is exactly solved when β is high enough.

Proposition 2 *For β large enough there exists a pooling equilibrium that implements the first-best efficient outcome in which the manager invests by issuing a callable convertible security with a restrictive call provision where F and α satisfy (11) and (12), k and p satisfy (14) and (15), $T_{call} = 1$ and $T_{conv} = 2$. In this equilibrium, the manager satisfies the call restrictions and calls to force conversion if and only if $m = m_2$ is observed.*

Proof. See the Appendix. ■

The convertible bond characterized above has payoffs that are non–decreasing in underlying cash flows. Nevertheless, the call provision and attached restrictions make the expected equilibrium value of the security independent of the manager's private information. The manager will call the bond to force conversion to equity when good information is disclosed and the restrictions on the call provision are met. The better is the initial information of the manager the higher is the chance that this occurs. However, the manager may not always be able to call and force conversion. The worse the initial information of the manager the greater is the chance that he will be unable to force conversion so that the new claimholders will be left holding the more valuable convertible debt. For the optimal security these two effects exactly offset each other so that the date 0 expected value of the claims for the new claimholders is independent of the manager's private information and the beliefs of the market.

Recall that for $\beta = 1$, financing with short-term debt and refinancing at date 1 also implements the same outcome as the optimal callable convertible security. However, this is no longer true when $\beta < 1$. If the manager issues short-term debt at date 0 that matures at date 1, then he has to raise cash to honor his debt obligations at that date by issuing some other security. Since there is still residual asymmetric information at date 1, the high type of the manager will still suffer from dilution at that date regardless of whether $m = m_1$ or m_2. The date 0 expected value of this date 1 dilution will be positive for the high type and may even make him unwilling to invest in the project. A similar point applies to Brennan–type floating price convertibles that pay I at date 1 regardless of the information revealed to the market.

Proposition 2 provides an example of a belief-independent security that implements the symmetric information outcome in a pooling equilibrium. There may be other efficient equilibria involving similar securities. For instance, consider the possibility that the high type θ_2 of the manager issues a callable convertible security that will be forced into conversion at date 1 when $m = m_2$ whereas the low type θ_1 issues equity. In such a candidate separating equilibrium, given that the convertible has been issued at date 0, the market infers that the manager's type is θ_2. To raise the required outlay, α and F must satisfy:

$$(1-\beta)C_2(\alpha, F) + \beta\alpha V_2 = I. \tag{16}$$

For such a separating equilibrium to exist, the type θ_1 cannot have a strict incentive to mimic type θ_2. This translates into the constraint:

$$\beta C_1(\alpha, F) + (1-\beta)\alpha V_1 \geq I. \tag{17}$$

The right hand side of (17) is the expected value of the claims that type θ_1 sells in equilibrium, whereas the left hand side is the expected value of the convertible, conditional on θ_1, if type θ_1 instead mimics θ_2. Notice that right-hand side of (17) does not depend on the kind of security θ_1 issues in equilibrium since any such security will be fairly valued at I in a separating equilibrium. In terms of Figure 1, the α and F associated with an efficient separating equilibrium must satisfy (16) and lie on L_2. To satisfy the no-mimicking constraint (17), α and F must also lie on or above the curve L_1.

In such a candidate separating equilibrium, given that such a convertible has been issued, the future signal m does not convey any information about the expected value of total cash flows to the market, which is inferred to be equal to V_2. Nevertheless, the manager's ability to force conversion will depend on the market's conjectures about his call decision. If the market conjectures that the bond will be called in order to force conversion when $m = m_2$, then the share price will equal $(1-\alpha)V_2$ at that date and state. On the other hand, if the market conjectures that the bond cannot be called or converted when $m = m_1$, the share price will equal $V_2 - C_2(\alpha, F)$. Since an option is worth more alive than dead, we see that $(1-\alpha)V_2$ is greater than $V_2 - C_2(\alpha, F)$. Consequently, if type θ_2 of the manager sets a call price $k < \alpha V_2$ and a trigger price p satisfying $p < (1-\alpha)V_2$ but $p > V_2 - C_2(\alpha, F)$, he will be able to force conversion when $m = m_2$ but not when $m = m_1$.[17] The next result shows that such a separating equilibrium exists whenever the pooling equilibrium of Proposition 2 does and vice versa.

Proposition 3 *An efficient separating equilibrium, in which type θ_2 issues a callable convertible that is forced into conversion only when $m = m_2$ and type θ_1 issues any other security, exists if and only if the efficient pooling equilibrium of Proposition 2 does. Within the class of such separating equilibria,*

[17] Although a signal does not convey new information about fundamentals, the price following m_2 is higher since the manager forces conversion, thereby increasing the value of equity. All of our results extend to a more general setting in which the signal m contains information about final cash flows over and above the manager's date 0 private information θ, i.e., the case where the conditional cash flow distribution G is of the form $G(x|\theta, m)$. In that case, the signal m_2 leads to higher prices directly due to the higher likelihood of larger future cash flows.

the no-mimicking constraint (17) binds for the one involving a convertible with the lowest conversion price ρ. Such a convertible is belief-independent.

Proof. See the Appendix. ■

Proposition 3 shows that efficient separation may occur without any need for additional frictions, such as managerial bankruptcy costs to create costly signaling. For instance, it is sufficient for type θ_2 to issue a belief-independent convertible that is identical (in terms of α and F) to the security issued in the pooling equilibrium of Proposition 2. Such a convertible satisfies (11) and (12) (equivalently, (16) and (17) with equality) and is depicted by the intersection of L_1 and L_2 in Figure 1. However, it is also possible for θ_2 to separate by issuing a convertible that is not belief-independent and has a higher F and lower α corresponding to a point on L_2 strictly above L_1. While the set of efficient equilibria involving callable convertibles is large (and differ in terms of observable implications, as we discuss below), commonly observed features of convertibles may allow one to design belief-free securities that perfectly solve the adverse selection problem.[18]

Additional frictions in the form of bankruptcy costs allow us to refine our predictions. For instance, a separating equilibrium where type θ_1 issues debt is eliminated in the presence of such costs but not one where θ_1 issues a junior security like equity. Similarly, type θ_2 will in general prefer to signal with a convertible that has the lowest face value F in the presence of bankruptcy costs. However, it then necessary to offer a higher equity share α (and so lower conversion price ρ) in order for (16) to hold. Since type θ_1 manages to force conversion to equity less often than type θ_2, any slack in the no-mimicking constraint (17) falls as F is lowered and α raised. As Figure 1 illustrates, for the convertible with the lowest F that still allows type θ_2 to separate, (17) must hold with equality and α and F must lie at the intersection of L_1 and L_2. In other words, for type θ_2 to separate from type θ_1 by issuing a callable convertible with the lowest expected bankruptcy costs, it is necessary and sufficient to issue the belief-independent convertible with the lowest conversion price.[19] Our next result provides comparative static properties of such securities with respect to the parameters of the model.

Proposition 4 *For the belief- independent callable convertible with the lowest conversion price, $\frac{\partial \alpha}{\partial \beta} \geq 0$, $\frac{\partial F}{\partial \beta} < 0$ and $\frac{\partial \rho}{\partial \beta} < 0$. Furthermore, for β sufficiently large, $\frac{\partial \alpha}{\partial I} > 0$, $\frac{\partial F}{\partial I} > 0$ and $\frac{\partial \rho}{\partial I} \geq 0$.*

In equilibrium, β represents the chance that the high type θ_2 of the manager will be able to force conversion leaving the investors holding equity that is less valuable. For fixed F, as β rises type θ_2

[18] In addition to efficient equilibria, there may also exist inefficient equilibria involving underinvestment similar to the one characterized by Myers and Majluf (1984). Extensions of standard forward induction refinements, such as the Intuitive Criterion of Cho and Kreps (1987), will in general fail to refine the equilibrium set absent other frictions.

[19] In general, L_1 may intersect L_2 more than once but at the intersection corresponding to the smallest F, L_1 must cut L_2 from above. Such an intersection corresponds to the belief-independent convertible with the lowest conversion price. See the Appendix for details.

has to raise α in order to compensate investors for the higher chance of a conversion forcing call. For the low type θ_1 however, the probability of a conversion forcing call is given by $1-\beta$. As β rises type θ_1 may not be able to force conversion with a higher probability and investors may end up holding a convertible that is more valuable than equity. For fixed α, type θ_2 can then afford to lower the face value F and still be able to raise the required outlay I at date 0. In terms of Figure 1, a rise in β has the effect of twisting L_2 upwards and L_1 leftwards. Since L_1 must intersect L_2 from above at the point corresponding to the belief-independent convertible with the lowest conversion price, a rise in β raises α, lowers F, and so lowers ρ.

Consider next the effect of a larger issue size I (relative to firm value) on the equity share α. The manager is likely to force conversion into equity when good news arrives in the market at date 1, which is highly correlated with the manager having good information θ_2 at date 0 when β is high. Since the issue must be valued at I for each possible type of the manager, it follows that α must rise in order to compensate investors for the higher outlay I when the likely type of the firm is θ_2. This, however, is not enough to compensate investors when the likely type of the firm is θ_1. Indeed, since the manager is unlikely to be able to force conversion at date 1 when he has bad information θ_1 at date 0, the option value of converting to debt must also rise. This implies that F must also rise as I rises. Indeed, to the extent that the debt part is risky, the rise in F must be proportionately higher than the rise in α, implying that the conversion price ρ must also be weakly increasing in I.

5 Mandatory Convertibles

The previous section shows that belief-independent callable convertible securities can efficiently solve the adverse selection problem without any need for signaling and even in the absence of frictions such as the costs of financial distress and bankruptcy. When the costs of financial distress are significant, however, it is not clear that such securities manage to avoid these costs. Stein (1992) argues in a model with three managerial types and sufficiently large variation in the likelihood of bankruptcy across managerial types, that one may be able to construct a separating equilibrium in which the best manager issues debt that is safe. Such debt is costly to mimic for managers with worse information because of the large probability of bankruptcy. So the middle type of manager issues a callable convertible while the worst type issues equity. When the manager issues a convertible, he manages to avoid financial distress since he can force conversion with probability 1 in Stein's model. The lowest type does not mimic the middle type since he cannot ensure conversion to equity. One can construct similar equilibria in a richer version of our model with bankruptcy costs. However, in the presence of noise in financial markets it is not possible to guarantee conversion for any type of the manager due to the presence of call restrictions. This implies that financial distress costs cannot necessarily be avoided by issuing callable standard convertibles. An alternative approach that deals with both adverse selection and financial distress costs is to consider floating-price convertibles with conversion ratios that depend on date 1 endogenous variables like the market value of equity or the stock price, including mandatory convertibles that are automatically converted to equity. In this section, we characterize the optimal belief-independent mandatory convertible.

We also extend the basic model in Section 4 by letting the manager's private information take more than two values, i.e., we let θ take values in the set $\{\theta_1, ..., \theta_N\}$, $N \geq 2$ with $\Pr[\theta = \theta_i] = \lambda_i$ and assume that suitable generalizations of (1)—(3) hold for all θ. Even absent considerations of financial distress as in the model of Section 4, when there are more than two types, one convertible bond with a fixed conversion ratio will not be able to implement the symmetric information outcome for all types. One solution is to let the manager issue multiple convertibles with differing face values, conversion ratios and call restrictions, perhaps sequentially in a manner reminiscent of the MCI and HGS cases discussed earlier. While this approach is also feasible, a simpler approach is to consider a single issue of only one mandatory convertible.

With more than two managerial types, we also need to extend the information structure for the date 1 public signal m. We suppose that at date 1, there is an analyst who is either informed (i.e., knows θ_i) with probability γ, or uninformed with probability $1 - \gamma$, with $\gamma \in (0, 1]$. The analyst's type is private information and he makes a public announcement $m \in \{m_1, ..., m_N\}$ given his type on date 1, after observing the date 0 decisions of the manager. The message m_i is to be interpreted as a statement by the analyst that the state of the world is θ_i. We assume that when the analyst is informed, he is also truthful, i.e., sends message m_i when the state is θ_i. When the analyst is uninformed, we let σ_i denote the probability with which he sends message m_i at date 1 with $\sigma = (\sigma_1, ..., \sigma_N)$.[20] While all results of this section go through for arbitrary specifications of σ provided γ is large enough, in what follows we provide a simple endogenization of σ. We do so by assuming that the uninformed analyst chooses his disclosure strategy σ in order to maintain a reputation for expertise, i.e., he seeks to maximize the market's posterior posterior probability that he is informed and truthful.

As before, we will look for perfect Bayesian equilibria of this game. Let $\mu_i^0(S)$ denote the uninformed analyst's (as well as the market's) date 0 beliefs that the type of the manager is θ_i given that a security $S \in \mathbf{S}$ has been issued by the manager. Let $\mu_i(m, S)$ denote the date 1 beliefs of the market that $\theta = \theta_i$ given a message m sent by the analyst and given S has been issued at date 0. Let $\nu(m, S)$ denote the date 1 beliefs of the market that the analyst is informed, given that the date 0 security is S and that he has sent a message m. Finally, let:

$$\overline{V}(m, S) = E[X|m, S] = \sum_{i=1}^{N} \mu_i(m, S) V_i, \tag{18}$$

be the date 1 market value of the expected cash flows of the firm given that the analyst's message is m and that the security issued is S.

[20] In terms of the notation of the previous section where $N = 2$, we have $\beta = \gamma + (1 - \gamma)\sigma_i$ and $\sigma_1 = \sigma_2 = \frac{1}{2}$.

We will look for a pooling equilibrium where each type of the manager issues the same floating price mandatory convertible bond at date 0. Such a security, denoted by $S^* = (\alpha^*, V^*)$, consists of a *vector* of equity shares $\alpha^* = (\alpha_1^*, ..., \alpha_N^*)$ together with a vector $V^* = (V_1^*, ..., V_N^*)$ of cut-off levels for the date 1 market value of the firm. The interpretation is that the security is converted to α_i^* shares when the date 1 market value of the firm is V_i^*.[21]

In order to state our result, it will be convenient to define:

$$\widehat{V} = \left[\sum_{i=1}^{N} \lambda_i \frac{1}{V_i}\right]^{-1}. \tag{19}$$

$\widehat{V}$ is the inverse of the average equity shares sold in the symmetric information world per dollar of investment. Let:

$$\gamma^* = \max\left[1 - \frac{\widehat{V}}{V_N}, \frac{I}{V_1}\frac{\widehat{V} - V_1}{(\widehat{V} - I)}\right]. \tag{20}$$

Proposition 5 *For all $\gamma > \gamma^*$ there exists a pooling equilibrium that implements the first-best efficient outcome where the manager invests by issuing mandatory convertible $S^* = (\alpha^*, V^*)$ satisfying:*

$$V_i^* = \gamma V_i + (1 - \gamma)\overline{V}, \tag{21}$$

and

$$\alpha_i^* = \frac{I}{\gamma}\left[\frac{1}{V_i} - (1 - \gamma)\frac{1}{\widehat{V}}\right] \in (0, 1), \tag{22}$$

for all $i = 1, ..., N$.

Proof. See the Appendix. ■

In the pooling equilibrium neither the market nor the uninformed analyst will infer anything about θ from the date 0 choice of securities. In the Appendix, we first solve for the equilibrium behavior of the uninformed analyst. We show that in order to maximize the market's posterior probability of his expertise, he announces m_i with probability $\sigma_i = \lambda_i$, the probability he attaches to the informed analyst sending message m_i. As a result, the market will attach probability γ to the analyst being informed after any message m_i and so the market value of the firm $\overline{V}(m_i, S^*)$ will be equal to V_i^* for each m_i. The new claimholders will obtain a share α_i^* when the date 1 market value of the firm equals V_i^*.

[21] We can equally let the conversion ratios depend on the date 1 stock price of the firm, instead of the total market value, without affecting anything. Mandatory conversion allows us to ignore the debt part of the claim. All results will carry over if we instead use a non–mandatory floating–price convertible. Since the conversion ratio in such a security floats with the stock price, we can choose a sufficiently low face value for the debt part of such a security, in order to guarantee voluntary conversion. This also allows us to eliminate the need for restrictive call provisions. Call provisions forcing conversion may still be attached, however, in order to make sure that conversion happens.

Given this equilibrium behavior, the conversion ratios α^* will be chosen in such a way that the expected value of the claims sold will equal I regardless of the private information of the manager. Since the manager of type θ_i attaches probability $\gamma + (1-\gamma)\lambda_i$ to the message m_i and a probability $(1-\gamma)\lambda_j$ to a message $m_j \neq m_i$, we must have that α^* solves:

$$[\gamma + (1-\gamma)\lambda_i]\alpha_i^* V_i + (1-\gamma)\sum_{j\neq i}\lambda_j \alpha_j^* V_i = I,$$

or, equivalently,

$$\gamma\alpha_i^* + (1-\gamma)\sum_{j=1}^{N}\lambda_j\alpha_j^* = \frac{I}{V_i}, \tag{23}$$

for all $i = 1, ..., N$. Equation (23) has a simple interpretation—the expected equity share sold by type θ_i must equal the share $\frac{I}{V_i}$ that would be sold by this type in the first–best world. The solution to the system (23) is given by (22). When γ is greater than its threshold value γ^*, the solution is admissible, i.e., $\alpha_i^* \in (0,1)$ for all i. To support the pooling equilibrium, we assume that if any other security is issued at date 0, everyone attaches probability 1 to type θ_1.

Note that:

$$\alpha_i^* - \alpha_j^* = \frac{I}{\gamma}\left[\frac{1}{V_i} - \frac{1}{V_j}\right]. \tag{24}$$

Thus, α_i^* is decreasing in i—the more optimistic is the market, the lower is the share sold. Furthermore, it is easily checked that the market value $\alpha_i^* V_i^*$ of the claims sold when $m = m_i$ is also decreasing in i. Intuitively, the higher the type of the manager, the greater is the chance that a favorable m will be disclosed in date 1. To keep the expected value of the claims sold constant across manager types, the market value of the claims sold must be decreasing in the date 1 market value of the company. This property of the floating price convertible is identical to the corresponding property of the callable convertible security characterized in Section 4.2.

Note moreover that for $i > j$, the difference $\alpha_i^* - \alpha_j^*$ (as well as $\alpha_i^* V_i^* - \alpha_j^* V_j^*$) is decreasing in γ. The less the probability that the analyst is informed, the more sensitive must be the date 1 market value of shares sold to the analyst's message, in order to keep the date 0 expected value independent of θ. Finally, since the firm initially has one share outstanding, after the conversion the share price p_i^* will be given by $(1-\alpha_i^*)V_i^*$, which is increasing in i. The more optimistic is the market at date 1, the higher will be V_i^*, the total value of the firm. Furthermore, the lower will be α_i^* the number of shares sold and so the total number of shares outstanding. For both these reasons, the stock price will be higher in a more optimistic market.

6 Empirical Implications

In this section we discuss the empirical implications of our results, relating them both to the existing theoretical and empirical literature on convertibles, as well as providing some novel testable predictions.

Forcing calls, call restrictions, and 'implied' call delay. Our results emphasize the role of conversion forcing calls and the need for restrictions on such call provisions in the presence of noise in the market. As the experiences of HGS and MCI illustrate, there is evidence that managers seek to use call provisions to force conversions when they are able to, and that call restrictions frequently constrain such attempts. Indeed, Lewis, Rogalski and Seward (1998a) document that about 95% of convertible bonds issued in the U.S. are callable. Furthermore, between 80% to 90% of these have restrictions that prevent calls unless the stock price is higher than a trigger price. For instance, 44 out of the 49 securities in the sample analyzed in the Morgan Stanley Dean Witter U.S. Convertible Research Report (Iyer et al., 2000), covering the period April 1999 to March 2000, have such call restrictions, usually referred to as 'soft' or 'provisional' call restrictions in the industry. Support for such widespread prevalence of call restrictions can also be found in the work of Asquith (1995) and Lewis, Rogalski and Seward (1998b).

When an equity-aligned manager satisfies such call restrictions, our results imply that the manager should call and thereby extinguish a valuable put option, at least in the absence of secondary effects arising out of tax shields or short-term movements in the stock price during the call notice period. This may appear to be inconsistent with results first obtained by Ingersoll (1977b), who shows that convertibles are called only in the presence of a large call premium (i.e., the conversion value exceeding the call price by an average of 44%). This suggests that the callability option was in the money for quite a while before the actual call decision, implying in turn that managers delay the call decision.

Such an 'implied' call delay is not, however, inconsistent with our results once one takes into account 'soft' call restrictions that prevent calls unless there is a significant premium over the conversion price. Indeed, as Asquith and Mullins (1991) and Asquith (1995) show, after accounting for tax shield effects, a large portion of the observed call delay can be explained once 'hard' call restrictions are taken into account. Such hard call restrictions take the form of call protection periods (typically one to three years). To the best of our knowledge, there has been no documentation of the role of soft call restrictions in explaining call premiums. Our results also suggest that it may be possible to empirically distinguish between early conversion forcing calls and conversion decisions that occur later in the bond's life once hard and soft call restrictions have expired.

Price effects around call dates. A significant portion of the empirical literature has focused on the behavior of stock prices around call dates. For instance, Mikkelson (1981) points out that the announcement of convertible debt calls is followed by a decline in the stock price. The subsequent empirical literature (Mazzeo and Moore (1992), Byrd and Moore (1996), and Ederington and Goh (2001)) finds that such a decline is typically short–lived and is more likely to be related to liquidity effects arising out of an increase in the number of available shares rather than due to asymmetric information effects. Since we abstract away from liquidity considerations, our results do not predict any significant decline or increase in the stock price after the call announcement. We also show that, in line with the empirical evidence (see, e.g., Campbell, Ederington and Vankudre (1991)), calling firms should experience high earnings or some other good news prior to call announcements that allows the manager to satisfy the soft call restrictions.

Announcement effects of convertible issues. The empirical evidence on the announcement effect of convertible security issues suggests that there is a small (around 2%) negative effect on the stock price (e.g., Dann and Mikkelson (1984)) on average. This is larger in magnitude than the effect associated with debt issues but smaller than that associated with equity issues. Such announcement effects have often been interpreted in terms of the information content of security issues about firm prospects. In the pooling equilibrium that we focus on, there should not be any significant announcement effect of the issue since the security issued does not signal any information in equilibrium. In contrast, in the efficient separating equilibrium of Proposition 3, the announcement of a convertible issue should cause a price impact that is favorable relative to the equity issued by the low type. Making our results fully consistent with the information content interpretation of announcement effects would require a richer version of our model where, in the spirit of Stein (1992), the manager issues debt, convertibles or equity depending on his information about future firm prospects.[22]

It is fair to point out, however, that there is considerable cross-sectional variation in the announcement effect of convertible issues. For instance, private placements are associated with a positive announcement effect of around 2% (Fields and Mais, 1991) and, even for public issues, the magnitude of the announcement effect is less pronounced for issues that have a lower credit rating (Mikkelson and Partch, 1986). For non-U.S. markets, Kang and Stulz (1996) document a positive announcement effect for convertible issues in Japan whereas Abhyankar and Dunning (1999) find a positive effect for U.K. issues when the proceeds are used for capital expenditures. All this suggests that the reasons underlying the announcement effects of convertible issues are varied. Apart from differences in the relative importance of adverse selection, the available evidence may also reflect the market's concerns about potential agency issues, such as empire-building and the associated free cash flow problem, the role of certification and monitoring on managerial incentives, as well as other institutional factors. Finally, it is also well documented that hedge funds buy significant portions of convertible securities and simultaneously hedge their positions by short selling the underlying stock at the time of the convertible issue (see, e.g., L'habitant (2002) and Agarwal et al. (2005), and Choi et al. (2006)). This may create a negative pressure on stock prices at announcement due to liquidity reasons.

[22] We refrain from formally pursuing such an extension in order to avoid reproducing the original contributions of Stein (1992) and Nyborg (1995) in our setting.

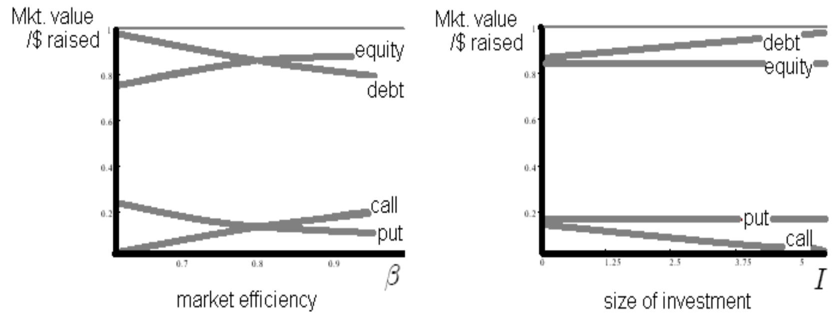

Figure 2: Relative market values

Properties of belief-independent convertibles: conversion prices and option values. As summarized by Proposition 4, the belief independent convertibles that we derive have very specific comparative static properties. Figure 2 illustrates these comparative statics. In the figure we assume that conditional on θ_i the distribution of the cash flows (from the new project and assets in place combined) is uniform in the interval $[0, 2V_i]$ and plot date 0 market values per dollar raised. The curve labelled 'equity' shows the fraction of cash raised via the (total) equity component of the convertible with the balance coming from the embedded 'put' option to convert to debt (adjusted for the probability of a forcing call). Similarly, the curve labelled 'debt' shows the fraction of cash raised via the risky debt component of the convertible (again, adjusted for the possibility of a forcing call) with the balance coming from the embedded 'call' option to convert to equity.

In the left panel of Figure 2, we plot these four relative market values as a function of β. Consistent with Proposition 4, as β rises the equity share α rises while the face value F and conversion price ρ falls implying a higher call option value to convert to equity (relative to debt) and a lower put option value to convert to debt (relative to equity).[23]

Depending on the precise interpretation of the date 1 signal m, the parameter β has a number of different interpretations. If one interprets m as information about the firm's future prospects that the manager can predict at date 0 (e.g., the outcome of a patent filing), β measures the accuracy of the manager's date 0 information. Alternatively, the signal m may be viewed as an analyst announcement in which case β may be interpreted as a measure of the informational efficiency of the market. Firms with greater analyst following (and lower dispersion in analyst forecasts) should therefore be expected to have higher βs. Similarly, firms that have a larger share of institutional blockholders should also have more informationally efficient market prices or higher βs. Consequently, convertibles issued by such firms should have higher call option value, lower conversion price, and smaller debt value. On the

[23] For the left panel of Figure 2, we use parameter values $\lambda_1 = \lambda_2 = 1/2$, $V_1 = 8$, $V_2 = 10$, and $I = 2$, in which case a belief-independent convertible exists as long as β is higher than 0.504. For the right panel of the figure, we fix $\beta = 0.75$ and vary I from 0 to 5, keeping all other parameters fixed.

other hand, firms with greater intrinsic uncertainty surrounding their operations should issue more out of the money options. To the extent that smaller bid-ask spreads and higher liquidity are related to lower asymmetric information in the market, as first argued by Kim and Verrecchia (1994), smaller spreads and higher liquidity should be associated with convertibles that have lower conversion prices.

In the right panel of Figure 2, we depict the comparative static properties of the optimal belief-independent convertible with respect to the required outlay I. As I rises, the NPV of the project falls all else equal regardless of the manager's private information. As the figure shows, the relative value of the call option to convert to equity must fall and the belief independent convertible becomes more like debt. For β large enough the equity share α is linear in I so that the relative shares of the equity and put option components do not vary as much. Therefore, we should expect a higher face value of debt and higher conversion prices for convertible issues that are larger relative to firm size and this effect should be higher for issues that make use of riskier bonds.

A fall in I all else equal is qualitatively similar to a rise (independently of the manager's private information) in the NPV of the project all else equal. In such cases, the informational advantage of the manager at date 0 concerns mainly the value of assets in place. This is likely to capture a situation of a young growth firm with unique intangible assets that are more likely to be misvalued by the market due to, for instance, uncertainty about their alternative uses. We therefore predict that firms with profitable growth opportunities but intangible assets should issue convertibles that have a larger share of the embedded call option to convert to equity relative to the debt component. The relative shares of the equity and embedded put option components should not respond as much to changes in the value of growth opportunities relative to those of assets in place.

What would be the effect of an observable increase in the idiosyncratic volatility of cash flows? If such a change is mean-preserving (and does not depend on the manager's private information), then it should not affect total firm value, at least absent capital structure effects.[24] When β is large and the high type manages to force conversion with a high probability, an increase in idiosyncratic volatility should have a small effect on the equity share α. A rise in volatility will, however, raise the value of the embedded put option in the convertible. Since the low type of the manager can force conversion only with a low probability, this implies that the face value F must fall in order to keep the total value of the optimal convertible independent of the private information of the manager. Since α is unchanged while F falls, the conversion price ρ must then fall. The net effect of a rise in idiosyncratic volatility is to raise the value of the embedded call option to convert to equity relative to the debt component. The relative shares of the equity and embedded put option components will not vary as

[24] If the change in cash flow volatility changes the systematic risk of the firm, this will no longer be true. In such cases, the effect on the optimal convertible will depend on the relative sizes of the change in the idiosyncratic and systematic components of volatility. Similarly, if the firm has existing senior debt, even a change in idiosyncratic risk only may change the value of the (levered) equity.

much since neither the value of the equity component nor the sum of the values of the equity and put option components change.

Mandatory convertibles Mandatory convertibles first appeared in the late 1980s. One common feature of all observed mandatory convertibles is that for some interval of the underlying share price, the conversion ratio is a decreasing function of the share price. This feature qualitatively matches our optimal mandatory convertible. However, the slope (i.e., change in conversion ratio as a function of the underlying stock price) of the optimal security that we characterize is larger in absolute terms than observed mandatory convertibles. This could be due to the fact that floating price and mandatory convertibles give rise to agency problems on the part of the investors in the issue (mostly convertible hedge funds and similar institutions).[25] This potential for manipulation creates a trade-off that possibly limits their use in mitigating the adverse selection problem without affecting bankruptcy costs. Such a tradeoff is conceptually similar to Lutz's (1989) argument against warrant contracts for consumer durables discussed in Section 2.

In the present context, the possibility of stock price manipulation also provides insight on a trade-off that firms may face in choosing between a standard callable convertibles and mandatory or floating price convertibles. While mandatory convertibles have lower expected bankruptcy costs, standard callable convertibles may come with lower expected agency costs arising out of stock price manipulation. In the first place, this is likely to be true because mandatory convertibles use the price average over about 20 trading days, so that it is comparatively easy for investors to manipulate the stock price and obtain a favorable conversion ratio. For standard convertibles, this benefit of stock price manipulation is absent since the conversion ratio is fixed. Furthermore, if the potential manipulator intends to prevent a conversion forcing call, he needs to keep the stock price low over a period of years, as opposed to a few trading days. Finally, by avoiding a forced conversion, the investor in a standard convertible ends up holding a debt-like contract. The associated agency costs between equity and debt (e.g., underinvestment and risk shifting) may be costly for the investor and lower his incentives to engage in manipulation. Therefore, among firms that issue convertibles in order to mitigate an adverse selection problem, we would expect those that are in better financial health to prefer standard callable convertibles to mandatory or floating price convertibles. This is because such firms face lower expected costs of financial distress. Firms with higher expected costs of financial distress should prefer mandatory convertibles, especially when the high liquidity of the underlying stock lowers the chances of successful manipulation by investors. Indeed, Arzac (1997) documents that firms issuing mandatory convertibles are often highly leveraged or temporarily troubled and face significant likelihood of

[25] For instance, in a 1988 legal case, Home Shopping Network Inc. argued that Drexel Burnham Lambert Inc. and certain Drexel clients manipulated the price of its stock in order to obtain a high conversion ratio, a case that was settled out of court.

financial distress.

7 Conclusion

We show that when the asymmetry of information is imperfectly resolved over time, commonly used securities such as callable convertible preferred stock or debt can perfectly solve the adverse selection problem. By conditioning call and conversion decisions on the future public resolution of the manager's current private information, such securities make the value of the claim insensitive to the private information of the manager. We call such securities belief-independent convertibles. The manager prefers to force conversion whenever he is able to, but may not be able to force conversion due to the presence of call restrictions. Complete mitigation of adverse selection can also be achieved by a floating price and mandatory convertibles.

In our model, the manager never obtains additional information over the course of time. In a setting where the manager does obtain information over time, the adverse selection problem of date 0 can still be perfectly solved by issuing belief-independent convertibles such as those we analyze. This is because the date 0 private information of the manager is the best estimate of his expected future information. Since the value of a belief-independent convertible does not depend on the date 0 private information of the manager, such securities will not cause any dilution in expectation at date 0, eliminating the possibility of underinvestment. Nevertheless, a model with new managerial private information at each date may yield other interesting dynamic predictions especially when interacted with a richer set of considerations, such as managerial incentives and multiple rounds of financing. Another promising extension is to consider the case where the manager is able to engage in signal-jamming, for instance by affecting the access to information for outside analysts. Novel empirical implications may also emerge in a such model in terms of the trade-off between the agency costs of signal-jamming and the adverse selection problem.

8 Appendix

Proof of Proposition 2

1. Existence of a Solution to (11) **and** (12)

Define the function $\xi(\alpha, F; \beta)$ as:

$$\xi^1(\alpha, F; \beta) = \beta C_1(\alpha, F) + (1-\beta)\alpha V_1 - I \tag{25}$$

and

$$\xi^2(\alpha, F; \beta) = (1-\beta) C_2(\alpha, F) + \beta\alpha V_2 - I. \tag{26}$$

Recall that $\xi(\alpha^*, F^*; 1) = 0$ where α^* and F^* solve (5) and (4) respectively. We wish to use the implicit function theorem to demonstrate the existence of an admissible solution to (25) and (26) when β is high enough. To do this we need to show that the Jacobian of ξ with respect to α and F when evaluated at $(\alpha^*, F^*; 1)$ is non–singular.

Denoting partial derivatives with subscripts, we obtain:

$$\begin{aligned}
\xi^1_\alpha(\alpha^*, F^*; 1) &= \frac{\partial C_1(\alpha^*, F^*)}{\partial \alpha} \\
\xi^1_F(\alpha^*, F^*; 1) &= \frac{\partial C_1(\alpha^*, F^*)}{\partial F} \\
\xi^2_\alpha(\alpha^*, F^*; 1) &= V_2 \\
\xi^2_F(\alpha^*, F^*; 1) &= 0
\end{aligned}$$

It follows that the relevant Jacobian is non–singular iff $\frac{\partial C_1(\alpha^*, F^*)}{\partial F} \neq 0$. But since:

$$C_i(\alpha, F) = \int_0^F x dG(x|\theta_i) + F \int_F^{F/\alpha} dG(x|\theta_i) + \alpha \int_{F/\alpha}^\infty x dG(x|\theta_i),$$

we obtain:

$$\frac{\partial C_1(\alpha^*, F^*)}{\partial F} = \left[G(\frac{F^*}{\alpha^*}|\theta_1) - G(F^*|\theta_1) \right] > 0,$$

since $\alpha^* \in (0, 1)$ and $F^* > 0$.

2. Existence of a pooling equilibrium

To show that pooling with such a security is indeed an equilibrium, we proceed backwards in time.

Date 1, $m = m_1$

In this case, if the market conjectures that the bond will not be converted, then the share price will be given by the left–hand side of (15). As a result, the manager will not be able to call the bond and so, from (13) it follows that it will not be converted. We also allow the manager to buy back the security in the market by issuing another security. For any security S that is issued to buy back the debt, we assume that the market puts probability 1 on the type for whom $C_i(\alpha, F) - S_i$ is the maximum, where $S_i \equiv E[S(X)|\theta_i]$. Given such beliefs, it is straightforward to check that both types of the manager will either not want to issue such a security to buy back the existing claims, or will not be able to do so.

Date 1, $m = m_2$

In this case, if the market conjectures that the bond will be converted, then the share price will be given by the right–hand side of (15). From (13) and (14), the manager will call to force conversion regardless of his private information and investors will convert when the security is called. If instead the manager tries to buy back the security and issue other claims, S, then, as above, the market attaches beliefs putting probability 1 on the type θ_i for whom $\alpha V_i - S_i$ is the maximum. No type of the manager will find such a deviation profitable.

Date 0

Given the call and conversion decisions of date 1 above, from (11) and (12) it follows that the market value of the security at date 0 will equal 1 dollar, the required outlay for the project. As a result, the manager, regardless of his private information, will be able to raise the required funds. The date 0 expected payoff of the existing shareholders will thus be equal to $V_i - 1 > A_i$ for each θ_i. Consequently, the manager will find it profitable to invest. Finally, we suppose that at date 0, if any type of the manager deviates by issuing some other security then the market puts probability 1 on type $\theta = \theta_1$. As a result, no type of the manager will find such a deviation profitable. ■

Proof of Proposition 3

Suppose an efficient pooling equilibrium exists, i.e., there exists a solution $\alpha \in (0, 1)$ and $F > 0$ to (11) and (12). Such a solution satisfies (16) and (17). It follows that an efficient separating equilibrium exists where type θ_2 issues a convertible with such α and F and with call price $k < \alpha V_2$ and call restriction p with $V_2 - C_2(\alpha, F) < p < (1 - \alpha)V_2$ and type θ_1 issues any other security with expected value given θ_1 equal to I.

For the other direction, notice first that each of the two equations (11) and (12) yield a solution $\alpha = L_1(F; I, \beta)$ and $\alpha = L_2(F; I, \beta)$ respectively, where L_i is non-increasing in F and increasing in I. The latter fact allows us to conclude that the no-mimicking constraint for type θ_1 in the separating equilibrium (17) can be written as $\alpha \geq L_1(F; I, \beta)$, whereas the valuation equation (16) (which is identical to (12)) can be written as $\alpha = L_2(F; I, \beta)$. Notice next that for all $i = 1, 2$, $L_i(0; I, \beta) = \frac{I}{V_i} \in (0, 1)$ since $C_i(\alpha, 0) = \alpha V_i$. That is, in the (F, α) plane, L_1 has a higher intercept on the vertical (α) axis than L_2. If the pooling equilibrium does not exist, then L_1 and L_2 do not intersect for any $F > 0$, i.e., L_2 lies everywhere below L_1. That is, for any α, F with $\alpha = L_2(F)$ (satisfying (16)), we must have $\alpha < L_1(F)$ (violating (16)), implying the separating equilibrium also cannot exist.

For the second part of the result, note that L_1 and L_2 may in general intersect more than once. Since L_1 has a higher intercept on the vertical (α) axis than L_2 however, L_1 must cut L_2 from above at the left most intersection, i.e., the one with the lowest F and highest α and so lowest ρ. Since in a

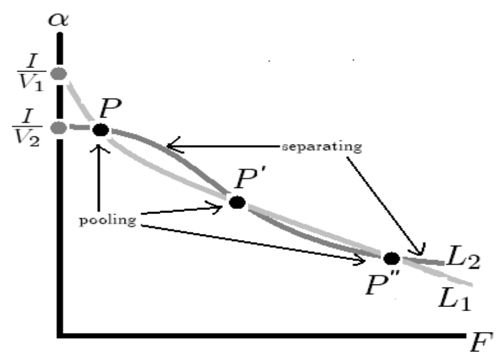

Figure 3: Multiplicity of Equilibria

separating equilibrium α and F must lie on L_2 and on or above above L_1, the result follows from the fact that L_1 is steeper than L_2 at the left-most intersection, as Figure 3 depicts. ■

Proof of Proposition 4

We use Figure 3 for the first part. As β rises, the intercept $\frac{I}{V_i}$ of L_i on the vertical axis does not change. Since $\alpha V_i < C_i(\alpha, F)$ for all $i = 1, 2$, as β rises for each F, one needs a (weakly) lower α to satisfy (11) and a (weakly) higher α to satisfy (12), i.e., L_2 twists up and L_1 twists down. Since at the left most intersection of L_1 and L_2, L_1 is steeper than L_2, it follows that α must rise and F must fall, implying $\rho = \frac{F}{\alpha}$ must fall as β rises. The weak inequality on $\frac{\partial \alpha}{\partial \beta}$ follows from the observation that L_1 may intersect L_2 on a horizontal segment of L_2.

For the second part, note first that $\frac{\partial C_i}{\partial \alpha} = \int_{\frac{F}{\alpha}}^{\infty} x dG(x|\theta_i)$ and $\frac{\partial C_i}{\partial F} = G(\frac{F}{\alpha}|\theta_i) - G(F|\theta_i)$. Totally differentiating (11) and (12) with respect to I and using Cramer's Rule, it is easy to verify that:

$$\frac{\partial \alpha}{\partial I} = \frac{\beta \frac{\partial C_1}{\partial F} - (1-\beta)\frac{\partial C_2}{\partial F}}{\beta \frac{\partial C_1}{\partial F}(\beta V_2 + (1-\beta)\frac{\partial C_2}{\partial \alpha}) - (1-\beta)\frac{\partial C_2}{\partial F}((1-\beta)V_1 + \beta \frac{\partial C_1}{\partial \alpha})}$$

$$\frac{\partial F}{\partial I} = \cdot \frac{\beta V_2 + (1-\beta)\frac{\partial C_2}{\partial \alpha} - (1-\beta)V_1 - \beta \frac{\partial C_1}{\partial \alpha}}{\beta \frac{\partial C_1}{\partial F}(\beta V_2 + (1-\beta)\frac{\partial C_2}{\partial \alpha}) - (1-\beta)\frac{\partial C_2}{\partial F}((1-\beta)V_1 + \beta \frac{\partial C_1}{\partial \alpha})}$$

As β becomes large $\frac{\partial \alpha}{\partial I}$ converges to $\frac{1}{V_2} > 0$ and $\frac{\partial F}{\partial I}$ converges to $\frac{V_2 - \frac{\partial C_1}{\partial \alpha}}{\frac{\partial C_1}{\partial F} V_2} > 0$ since $V_2 > V_1 \geq \frac{\partial C_1}{\partial \alpha}$. It follows that for β large $\frac{\partial \alpha}{\partial I}, \frac{\partial F}{\partial I} > 0$.

Further, $\frac{\partial \rho}{\partial I} \geq 0$ if and only if $\frac{1}{\alpha}\frac{\partial \alpha}{\partial I} \leq \frac{1}{F}\frac{\partial F}{\partial I}$. Using the expressions for $\frac{\partial \alpha}{\partial I}$ and $\frac{\partial F}{\partial I}$ obtained above some algebra yields the equivalent condition $\beta \int_0^F x dG(x|\theta_1) \geq (1-\beta) \int_0^F x dG(x|\theta_2)$. By first-order stochastic dominance, if $G(F|\theta_1) = 0$, then $G(F|\theta_2) = 0$, implying $\int_0^F x dG(x|\theta_1) = \int_0^F x dG(x|\theta_2) = 0$ and so $\frac{\partial \rho}{\partial I} = 0$. Otherwise, $\int_0^F x dG(x|\theta_1) > 0$ and so $\frac{\partial \rho}{\partial I} > 0$ if β is large enough. ■

Proof of Proposition 5

We begin our construction of the pooling equilibrium by considering the strategy of the uninformed analyst at date 1, on the equilibrium path. Since all types of the manager pool by issuing the same convertible S^*, neither the analyst nor the market learns anything about θ from the date 0 financing decision. As a result, $\mu_i^0(S^*) = \lambda_i$ for all i. Since the informed analyst discloses the truth, this implies that the posterior probability that the market attaches to the analyst being informed after a message m_i is:

$$\nu(m_i, S^*) = \frac{\lambda_i \gamma}{\lambda_i \gamma + \sigma_i(S^*)(1-\gamma)}. \tag{27}$$

Since the uninformed analyst wants to maximize the posterior probability that he is informed, it follows that:

$$\sigma_i(S^*) = \lambda_i \text{ and } \nu(m_i, S^*) = \gamma \text{ for all } i = 1, ..., N, \tag{28}$$

in equilibrium. To see this, note first that $\nu(m_i, S^*)$ cannot vary across messages m_i— if there exist messages m_i and m_j such that $\nu(m_i, S^*) > \nu(m_j, S^*)$, the uninformed analyst will strictly prefer to send message m_i (i.e., $\sigma_i(S^*) = 1$) implying that $\nu(m_i, S^*) = \frac{\lambda_i \gamma}{\lambda_i \gamma + (1-\gamma)} < 1 = \nu(m_j, S^*)$, a contradiction. So, we must have $\nu(m_i, S^*) = \kappa$ for some constant $\kappa \in [0, 1]$ for all $i = 1, ..., N$. From (27), we then obtain:

$$\sigma_i(S^*)(1-\gamma)\kappa = \lambda_i \gamma(1-\kappa),$$

for all i. Since $\sum_i \sigma_i(S^*) = 1$, it follows that $\kappa = \gamma$ and $\sigma_i(S^*) = \lambda_i$ for all i.

Having established the equilibrium behavior of the uninformed analyst, we now turn to the date 1 market value of the firm $\overline{V}(m_i, S^*)$ after a message m_i. Note that:

$$\mu_i(m, S^*) = \begin{cases} \gamma + (1-\gamma)\lambda_i & \text{if } m = m_i \\ (1-\gamma)\lambda_i & \text{otherwise} \end{cases} \tag{29}$$

Thus,

$$\overline{V}(m_i, S^*) = \gamma V_i + (1-\gamma)\overline{V}. \tag{30}$$

Since $\overline{V}(m_i, S^*) = V_i^*$ for all i, the security S^* entitles the new shareholders to convert to α_i^* shares when the market value of the security is $\overline{V}(m_i, S^*)$.

Next, we turn to the choice of the equity shares α^*. Since $\sigma_i(S^*) = \lambda_i$ for all i, type θ_i of the manager knows that the analyst's message will be m_i with probability $\gamma + (1-\gamma)\lambda_i$ and will be equal to m_j with probability $(1-\gamma)\lambda_j$ for $j \neq i$. We want to choose α^* such that the expected value of the claims sold in equilibrium is equal to the outlay of 1, for each type of the manager. That is, α_i^* must solve:

$$[\gamma + (1-\gamma)\lambda_i]\alpha_i^* V_i + (1-\gamma)\sum_{j \neq i} \lambda_j \alpha_j^* V_i = I,$$

for all $i = 1, ..., N$. Re–arranging we obtain,

$$\gamma \alpha_i^* + (1-\gamma)\sum_{j=1}^{N} \lambda_j \alpha_j^* = \frac{I}{V_i}, \tag{31}$$

for all $i = 1, ..., N$. Multiplying by λ_i and summing over i, we obtain$\sum_{j=1}^{N} \lambda_j \alpha_j^* = \frac{I}{\widehat{V}}$. Using this in (31), we obtain (22). It is easy to check that if $\gamma > \max[1 - \frac{\widehat{V}}{V_N}, \frac{I}{V_1} \frac{\widehat{V} - V_1}{(\widehat{V} - I)}]$ then $\alpha_i^* \in (0, 1)$ for all i.

Given the equilibrium behavior derived above, the date 0 expected value of the claims sold by type θ_i of the manager is seen to be equal to I, by construction. Thus, the date 0 market value of the security will also equal I and the expected payoff to the old claimholders will equal $V_i - I > A_i$ for all $i = 1, ..., N$. This implies that no type of the manager will prefer to under–invest.

Note that the manager is allowed to buy back the security S^* in the market by issuing some other security after a message m_i. For any security S that is issued to buy back the convertible, we assume that the market puts probability 1 on the type θ_j for whom $\alpha_i^* V_j - S_j$ is the maximum. Given such beliefs, it is straightforward to check that all types of the manager will either not want to issue such a security to buy back the existing claims, or will not be able to do so.

It remains to check that no type of the manager will want to deviate at date 0 by issuing a different security S'. We suppose that if any such security S' is issued by any type of the manager, then the market attaches probability 1 to type θ_1, i.e., $\mu_1^0(S') = 1$. It follows that $\mu_1(m, S') = 1$ for all m so that $\overline{V}(m, S') = V_1$ for all m. Given such beliefs, it is straightforward to verify that no type of the manager will find such a deviation profitable, and we omit the details. ■

References

[1] Abhyankar, A. and A. Dunning (1999), "Wealth effects of convertible bond and convertible preference share issues: An empirical analysis of the UK market," *Journal of Banking and Finance,* 23, 1043-1065.

[2] Agarwal, V., Fung, W., Loon, Y.C. and N. Naik (2005), "Risk and Return in Convertible Arbitrage Strategies," *mimeo*, London Business School.

[3] Akerlof, G. (1970), "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 89, 488-500.

[4] Arzac, E. (1997), "PERC, DECS, and Other Mandatory Convertibles," *Journal of Applied Corporate Finance*, 10, 54-63.

[5] Asquith, P. (1995), "Convertible Bonds Are not Called Late," *Journal of Finance*, 50, 1275-1290.

[6] Asquith, P. and D. Mullins (1991), "Convertible Debt: Corporate Call Policy and Voluntary Conversion," *Journal of Finance*, 46, 1273-1289.

[7] Brennan, M. (1986), "Costless Financing Policies under Asymmetric Information," *mimeo*, UCLA.

[8] Brennan, M. and A. Kraus (1987), "Efficient Financing under Asymmetric Information," *Journal of Finance*, 42, 1225-1243.

[9] Brennan, M. and E. Schwartz (1987), "The Case for Convertibles," *Journal of Applied Corporate Finance*, 1, 55-64.

[10] Brennan, M. and E. Schwartz (1977), "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," *Journal of Finance*, 32, 1699-1715.

[11] Brennan, M. and E. Schwartz (1980), "Analyzing Convertible Bonds," *Journal of Financial and Quantitative Analysis*, 15, 907-929.

[12] Byrd, A. and W. Moore (1996), "On the Information Content of Calls of Convertible Securities," *Journal of Business*, 68, 89-101.

[13] Campbell, C.; Ederington, L. and P. Vankudre (1991), "Tax Shields, Sample Selection Bias, and the Information Content of Conversion Forcing Bond Calls," *Journal of Finance*, 46, 1291-1324.

[14] Cho, I. K. and D. Kreps (1987), "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 102, 181-231.

[15] Choi, D., Getmansky M. and H. Tookes (2006), "Who shorts? Convertible Bond Arbitrage, Liquidity and Stock Returns," *mimeo*, Yale School of Management.

[16] Constantinides, G. and B. Grundy (1989), "Optimal Investment with Stock Repurchase and Financing Signals," *Review of Financial Studies*, 2, 445-465.

[17] Cornelli, F. and O. Yosha (2003), "Staged Financing and the Role of Convertible Debt," *Review of Economic Studies*, 70, 1-32.

[18] Dann, L. and W. Mikkelson (1984), "Convertible Debt Issuance, Capital Structure Change and Financing Related Information: Some New Evidence," *Journal of Financial Economics*, 13, 157-186.

[19] Dybvig, P. and J. Zender (1991), "Capital Structure and Dividend Irrelevance with Asymmetric Information," *Review of Financial Studies*, 4, 201-19.

[20] Ederington, L. and J. Goh (2001), "Is a Convertible Bond Call Really Bad News?" *Journal of Business*, 74, 459-476.

[21] Fields, L. and E. Mais (1991), "The Valuation Effect of Private Placements of Convertible Debt," *Journal of Finance*, 46, 1925-32.

[22] Francis, J.C; Toy W.W. and Whittaker, J.G. (ed.), (2000), "The Handbook of Equity Derivatives," Wiley, New York.

[23] Fulghieri, P. and D. Lukin (2001), "Information Production, Dilution Costs, and Optimal Security Design," *Journal of Financial Economics*, 61, 3-42.

[24] Graham, J.R. and C. Harvey (2001), "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics*, 60, 187-243.

[25] Green, R. (1984), "Investment Incentives, Debt and Warrants," *Journal of Financial Economics*, 13, 115-136.

[26] Greenwald, B. (1984), "MCI Communications Corp.," *Harvard Business School case 9-284-057*, Boston, MA.

[27] Grossman, S. (1981), "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law and Economics*, 461-83.

[28] Harris, M. and A. Raviv (1985), "A Sequential Signaling Model of Convertible Debt Call Policy," *Journal of Finance*, 41, 1263–1281.

[29] Harris, M. and A. Raviv (1991), "The Theory of Capital Structure," *Journal of Finance*, 46, 297–355.

[30] Ingersoll, J. (1977a), "A Contingent Claims Valuation of Convertible Securities," *Journal of Financial Economics*, 4, 289-322.

[31] Ingersoll, J. (1977b), "An Examination of Corporate Call Policy on Convertible Securities," *Journal of Finance*, 32, 463-478.

[32] Innes, R.D. (1990), "Limited Liability and Incentive Contracting with Ex-ante Action Choices," *Journal of Economic Theory*, 52, 45-67.

[33] Iyer, A. S., Cichocki, A., Decker, D., Pandit M., Rudnick, D. J., and M. Schwartz (2000), "Making the Most of Make-Whole Provisions," Morgan Stanley Dean Witter U.S. Convertible Research Report.

[34] Jensen, M.C. and W.H. Meckling (1976), "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3, 305-360.

[35] Kang, J., K. and R.M. Stulz (1996), "How different is Japanese corporate finance? An investigation of the information content of new equity issues," *Review of Financial Studies,* 9, 109-139.

[36] Kim, O., and R. Verrecchia (1994). "Market liquidity and volume around earnings announcements," *Journal of Accounting and Economics,* 17, 41-67.

[37] Lewis, C. M., Rogalski, R. J. and J. K. Seward (1998a), "Agency Problems, Information Asymmetries, and Convertible Debt Security Design," *Journal of Financial Intermediation*, 7, 32-59.

[38] Lewis, C. M., Rogalski, R. J. and J. K. Seward (1998b), "Understanding the Design of Convertible Debt," Journal of Applied Corporate Finance, 11, 45-53.

[39] L'habitant, F. S. (2002), "Hedge Funds", Wiley, West Sussex, England.

[40] Maskin, E. and J. Tirole (1992),"The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica*, 60, 1-42.

[41] Lutz, N. (1989), "Warranties as Signals Under Consumer Moral Hazard," *Rand Journal of Economics*, 20, 239-255.

[42] Mazzeo, M. and W. Moore (1992), "Liquidity Costs and Stock Price Response to Convertible Security Calls," *Journal of Business*, 65, 353-369.

[43] Mikkelson, W. (1981), "Convertible Calls and Security Returns," *Journal of Financial Economics*, 9, 237-264.

[44] Mikkelson, W. and M. Partch (1986), "Valuation Effects of Security Offerings and the Issuance Process," *Journal of Financial Economics*,15, 31-60.

[45] Myers, S. and N. Majluf (1984), "Corporate Financing and Investment Decisions When Firms Have Information That Investors Do not Have," *Journal of Financial Economics*, 13, 187-221.

[46] Nachman, D. C. and T. H. Noe (1994), "Optimal Design of Securities under Asymmetric Information,"*Review of Financial Studies*, 7, 1-44.

[47] Nyborg, K. (1995), "Convertible Debt as Delayed Equity: Forced versus Voluntary Conversion and the Information Role of Call Policy," *Journal of Financial Intermediation*, 4, 358-395.

[48] Persons, J. C., 1994, "Renegotiation and the Impossibility of Optimal Investment," Review of Financial Studies, 7, 419-449.

[49] Repullo, R. and J. Suarez (1998), "Venture Capital Finance: A Security Design Approach," *CEPR Discussion Paper*, No. 1738.

[50] Stein, J. (1992), "Convertible Bonds as Backdoor Equity Financing," *Journal of Financial Economics*, 32, 3-21.