Neil F. Comins • William J. Kaufmann III

Discovering the Essential Universe

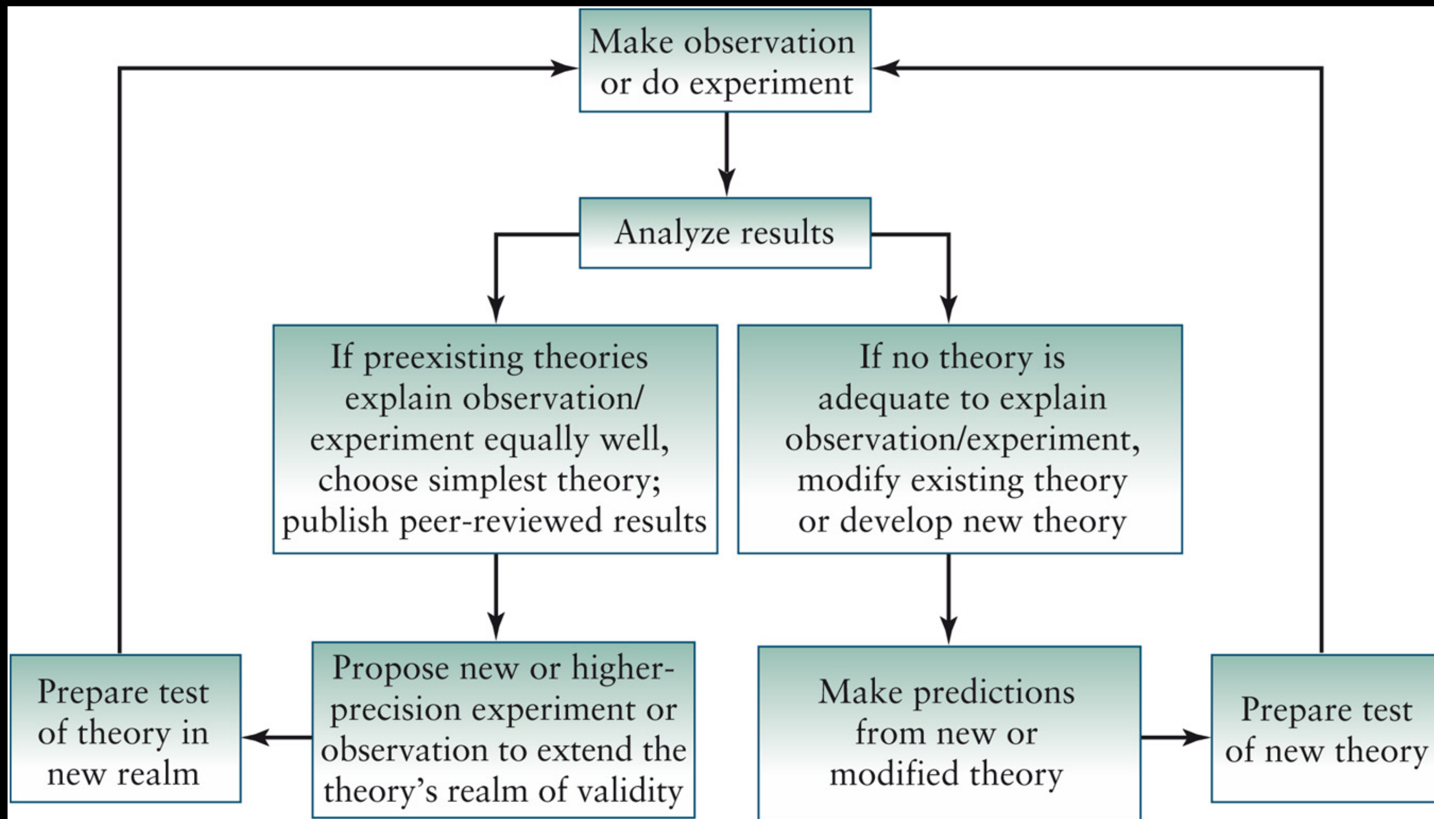
Sixth Edition

CHAPTER 2

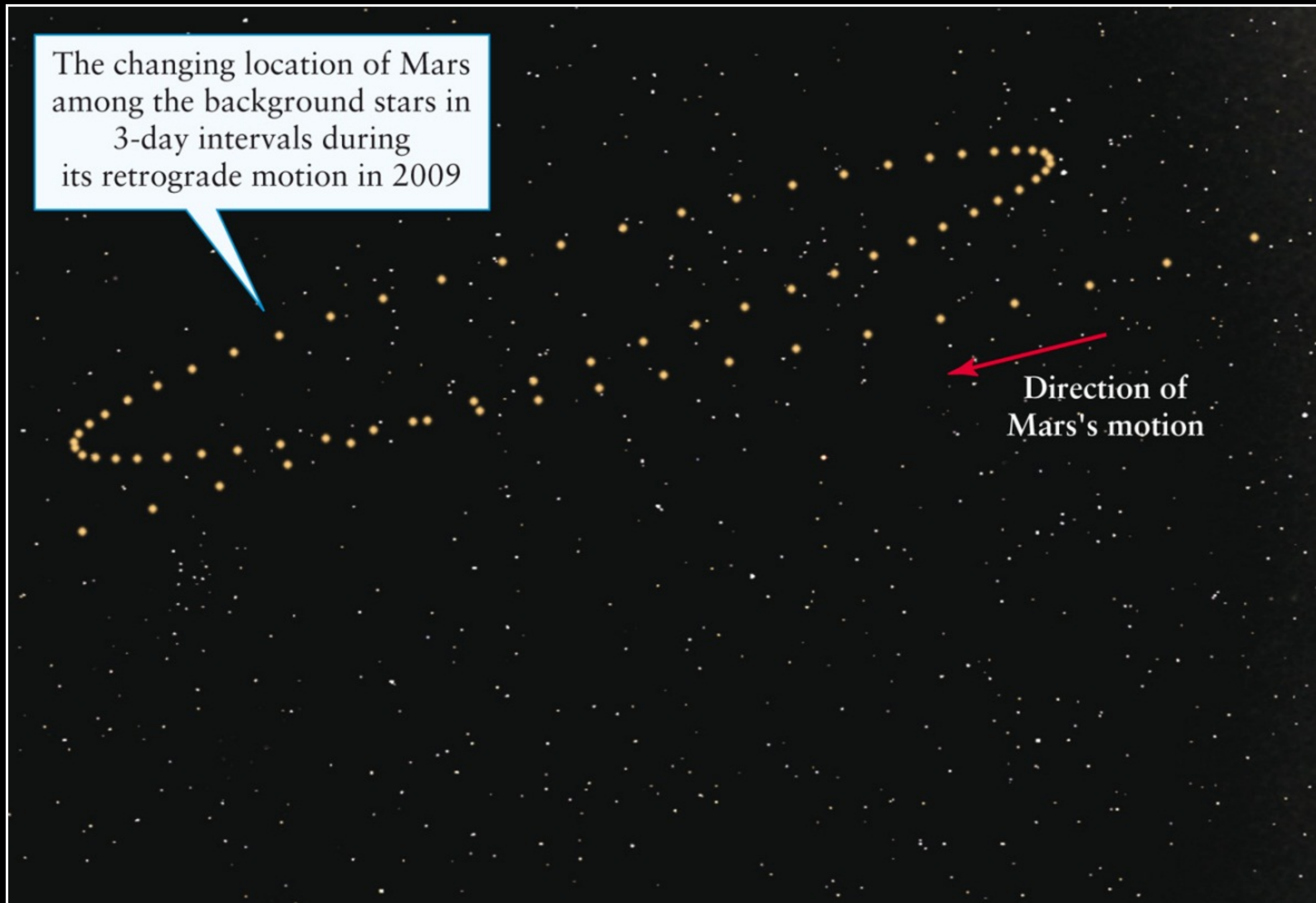
Gravitation and the Motion of the Planets

In this chapter we will talk about …

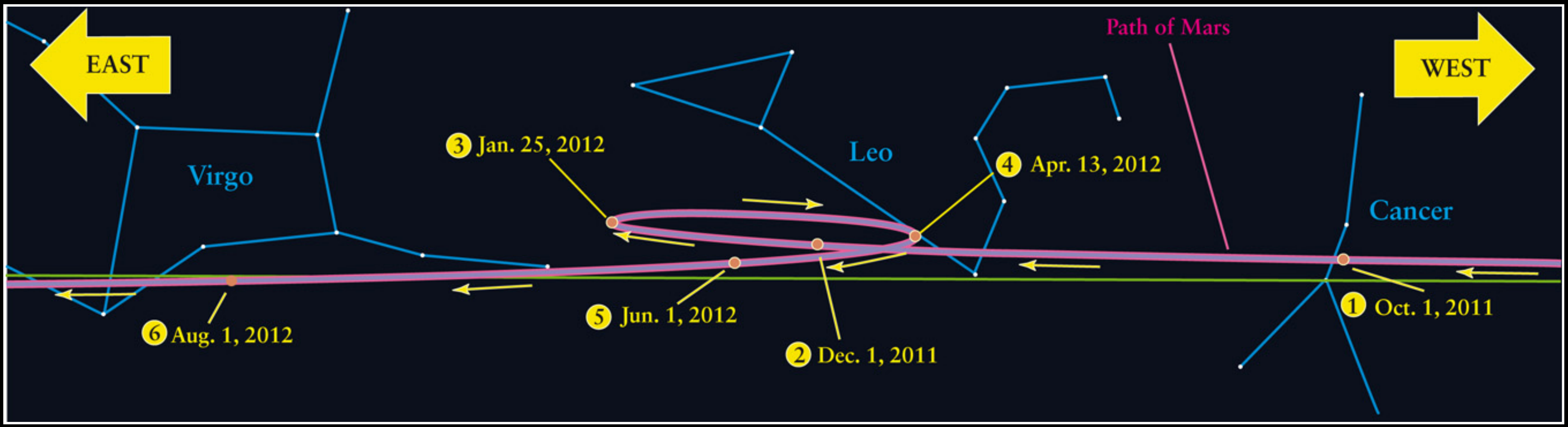
- Copernicus
- Kepler
- Newton



The scientific method is used to develop new scientific theories. Scientific theories are accepted when they *make testable predictions that can be verified using new observations and experiments*.


The changing location of Mars among the background stars in 3-day intervals during its retrograde motion in 2009
Direction of Mars's motion

The Path of Mars

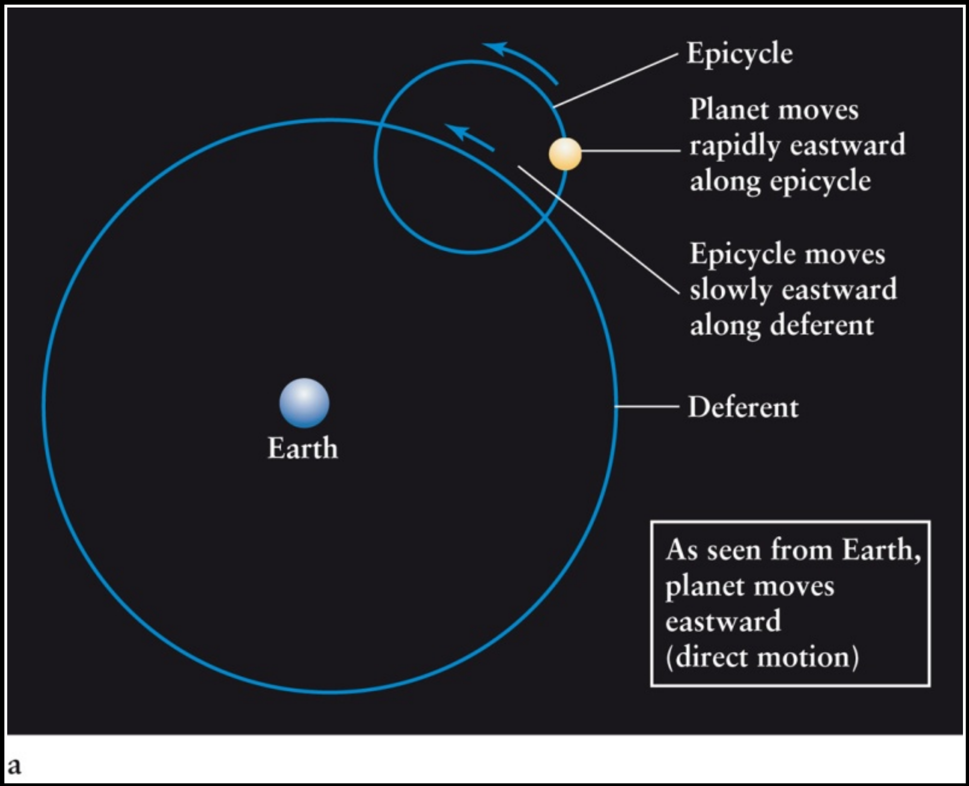


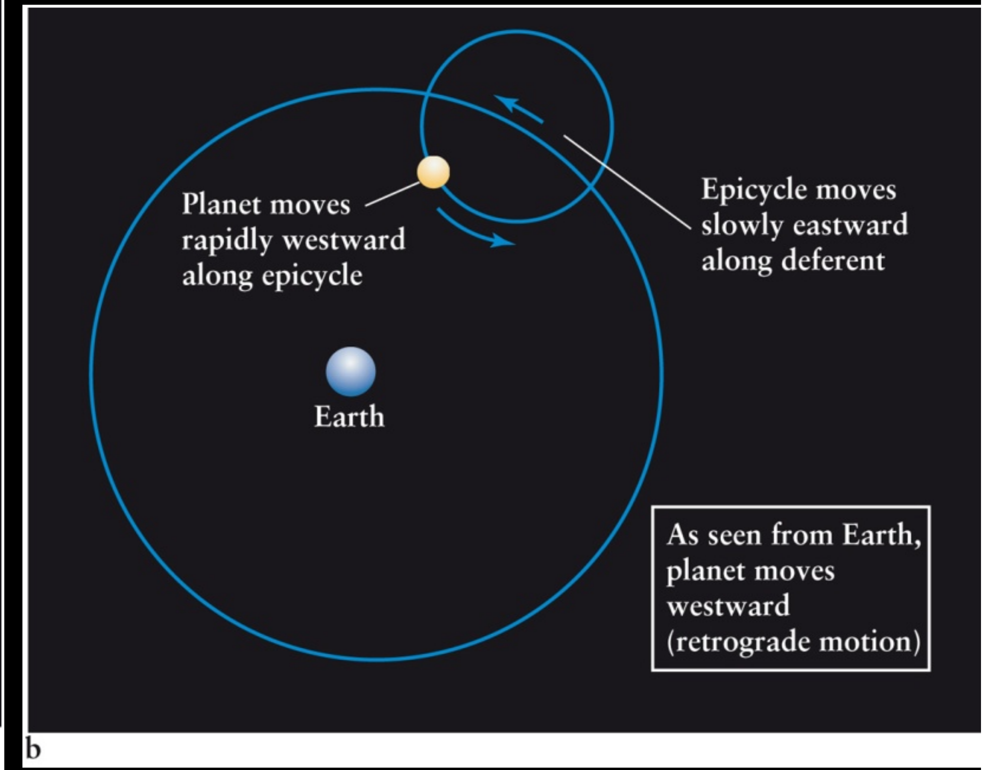
A geocentric explanation of planetary motion - the Ptolemaic system

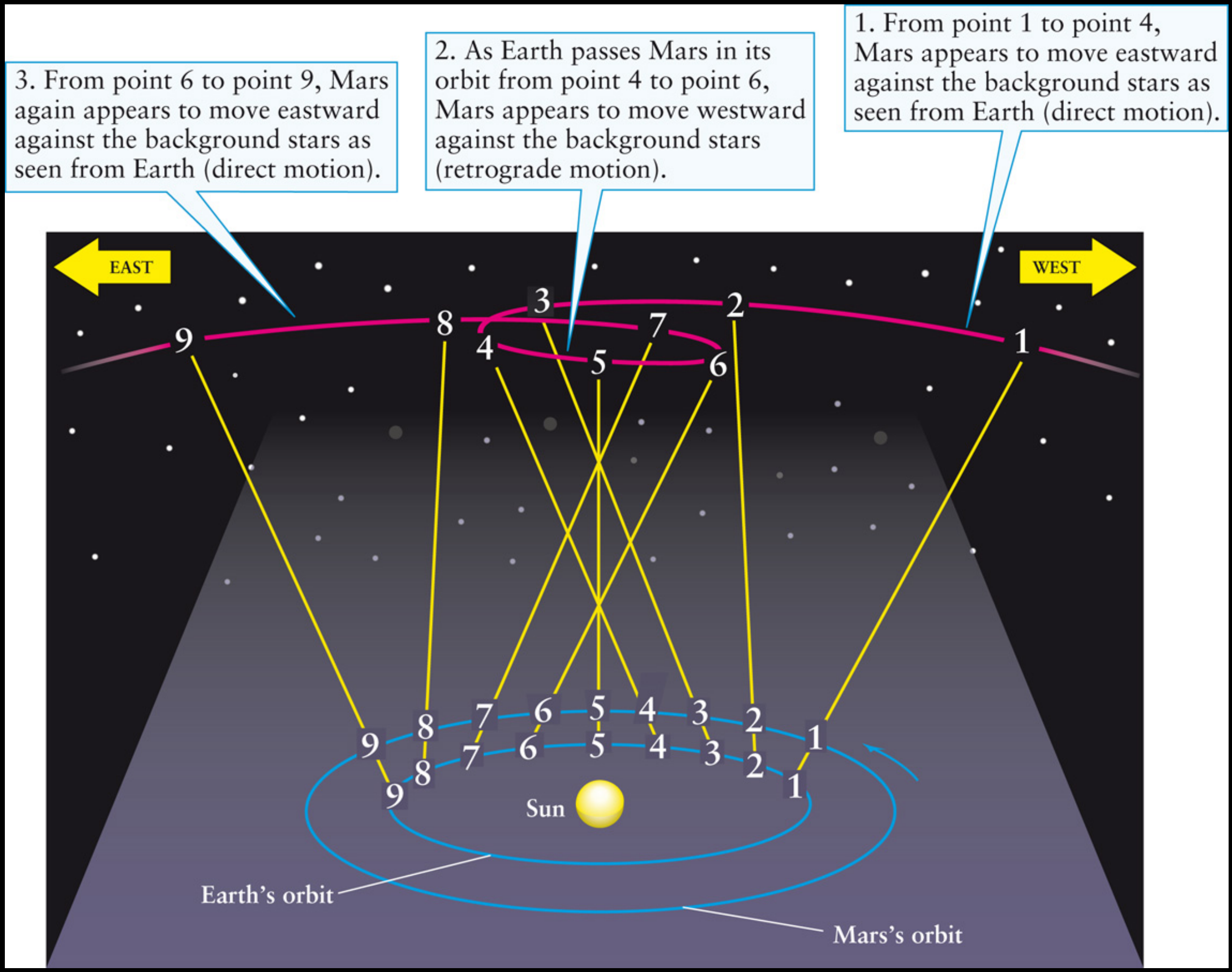
Ptolemy

~100 ~168 AD

Wikipedia



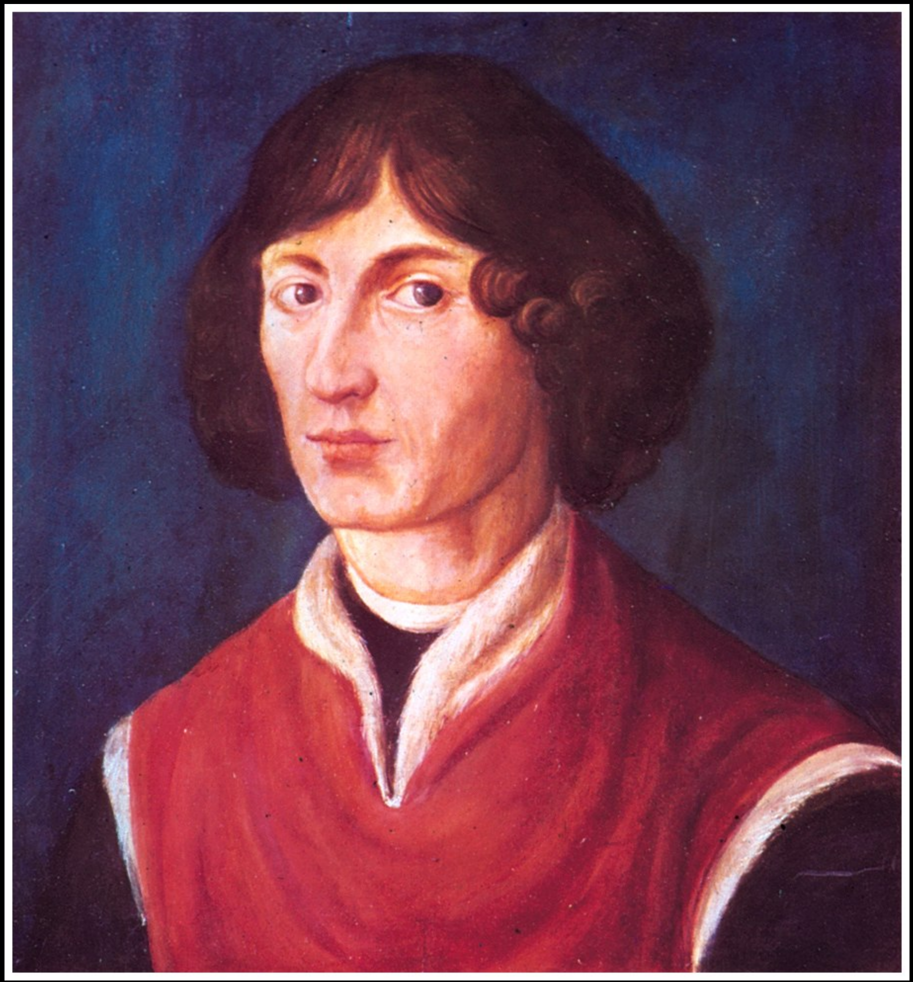




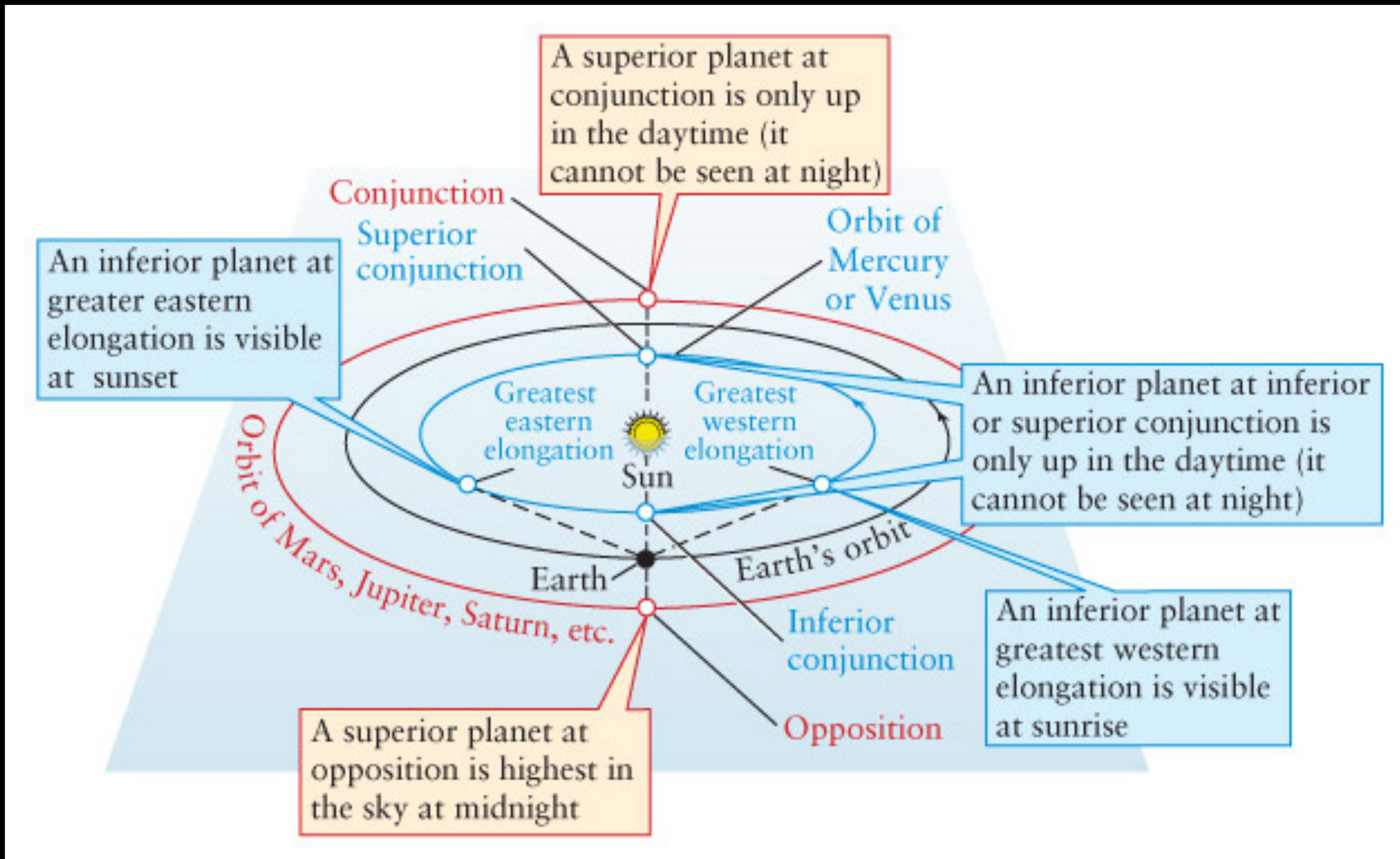
Nicolaus Copernicus developed the first complete *heliocentric* (Sun-centered) model of the solar system.

Nicolaus Copernicus devised the first comprehensive heliocentric model

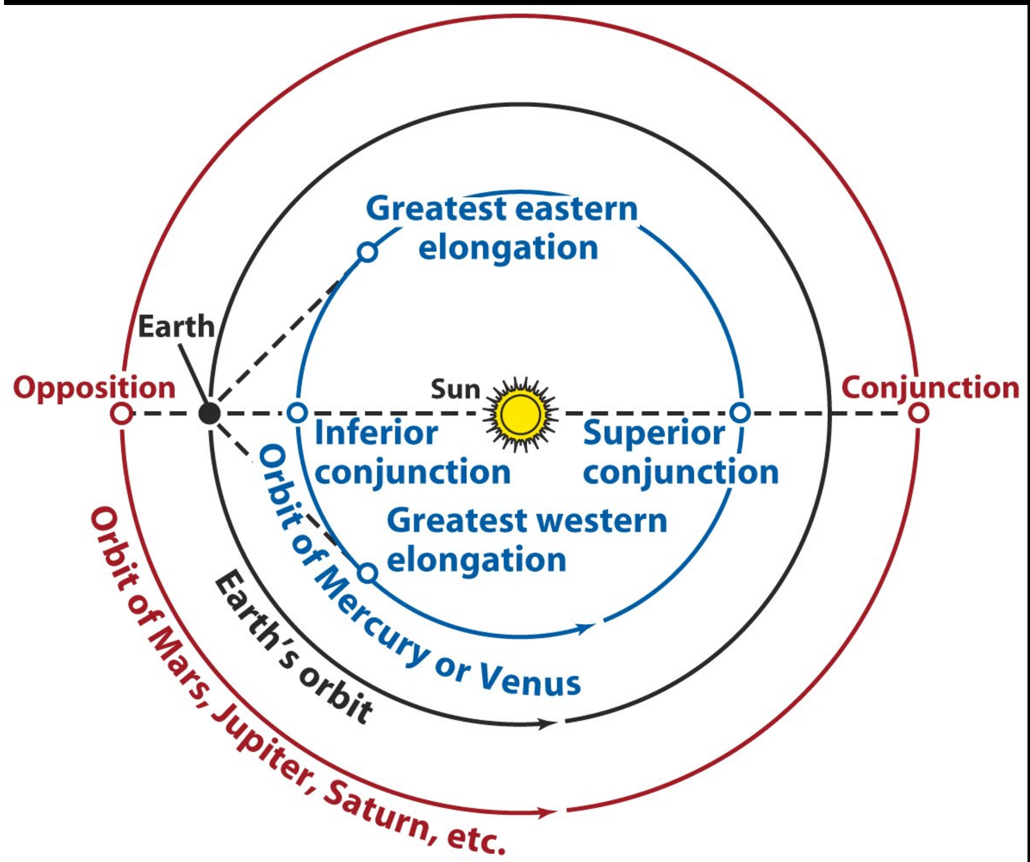
- Copernicus's heliocentric (Sun-centered) theory simplified the general explanation of planetary motions
- In a heliocentric system, the Earth is one of the planets orbiting the Sun
- The sidereal period of a planet, its true orbital period, is measured with respect to the stars

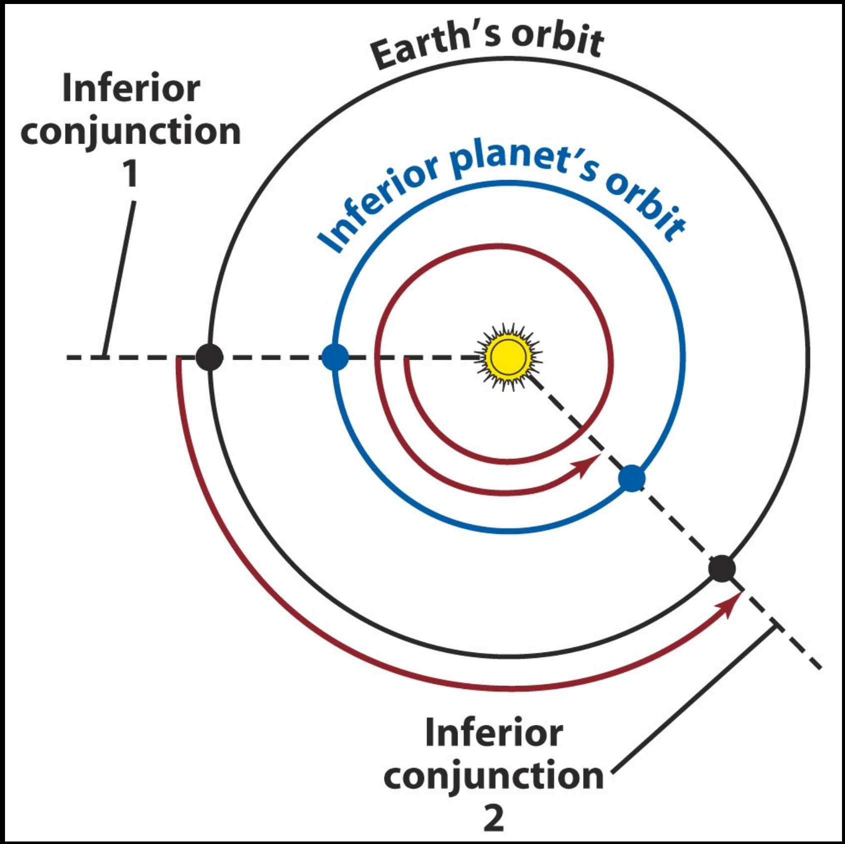


1473 -1543


A superior planet at conjunction is only up in the daytime (it cannot be seen at night)
Conjunction
Superior conjunction
Orbit of Mercury or Venus
An inferior planet at greater eastern elongation is visible at sunset
Greatest eastern elongation
Greatest western elongation
Sun
An inferior planet at inferior or superior conjunction is only up in the daytime (it cannot be seen at night)
Orbit of Mars, Jupiter, Saturn, etc.
Earth
Earth's orbit
Inferior conjunction
An inferior planet at greatest western elongation is visible at sunrise
Opposition
A superior planet at opposition is highest in the sky at midnight

A planet's synodic period, S, is measured with respect to the Earth and the Sun (for example, from one opposition to the next)





A planets sidereal period, P, is measured with respect to stars. In one sidereal period the planet completes a 360 deg orbit.

Sidereal and Synodic Orbital periods

- During time S earth covers (360deg/E)S
- Inferior planet has covered (360deg/P)S
- => (360deg/P)S = (360deg/E)S +360deg
- For Inferior Planets $1/P = 1/E + 1/S$
- Similarly it can be shown that:
- For Superior Planets $1/P = 1/E – 1/S$

P = Sidereal Period of the planet

S = Synodic Period of planet

E = Earth's Sidereal Period (1 year)

Example for Mercury (inferior planet)

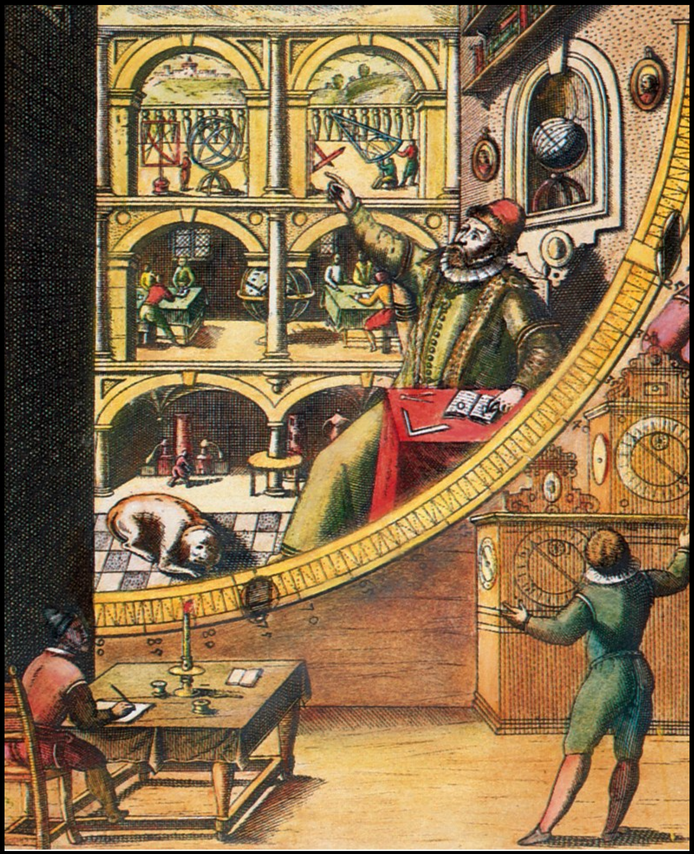
- 1/P = 1/E + 1/S
- 1/P = 1/365d + 1/116
- 1/P = 0.0113604 1/d
- P = 88 d

table 4-1 **Synodic and Sidereal Periods of the Planets**

Planet	Synodic period	Sidereal period
Mercury	116 days	88 days
Venus	584 days	225 days
Earth	—	1.0 year
Mars	780 days	1.9 years
Jupiter	399 days	11.9 years
Saturn	378 days	29.5 years
Uranus	370 days	84.1 years
Neptune	368 days	164.9 years
Pluto	367 days	248.6 years

De revolutionibus orbium coelestium

Parallax Shift



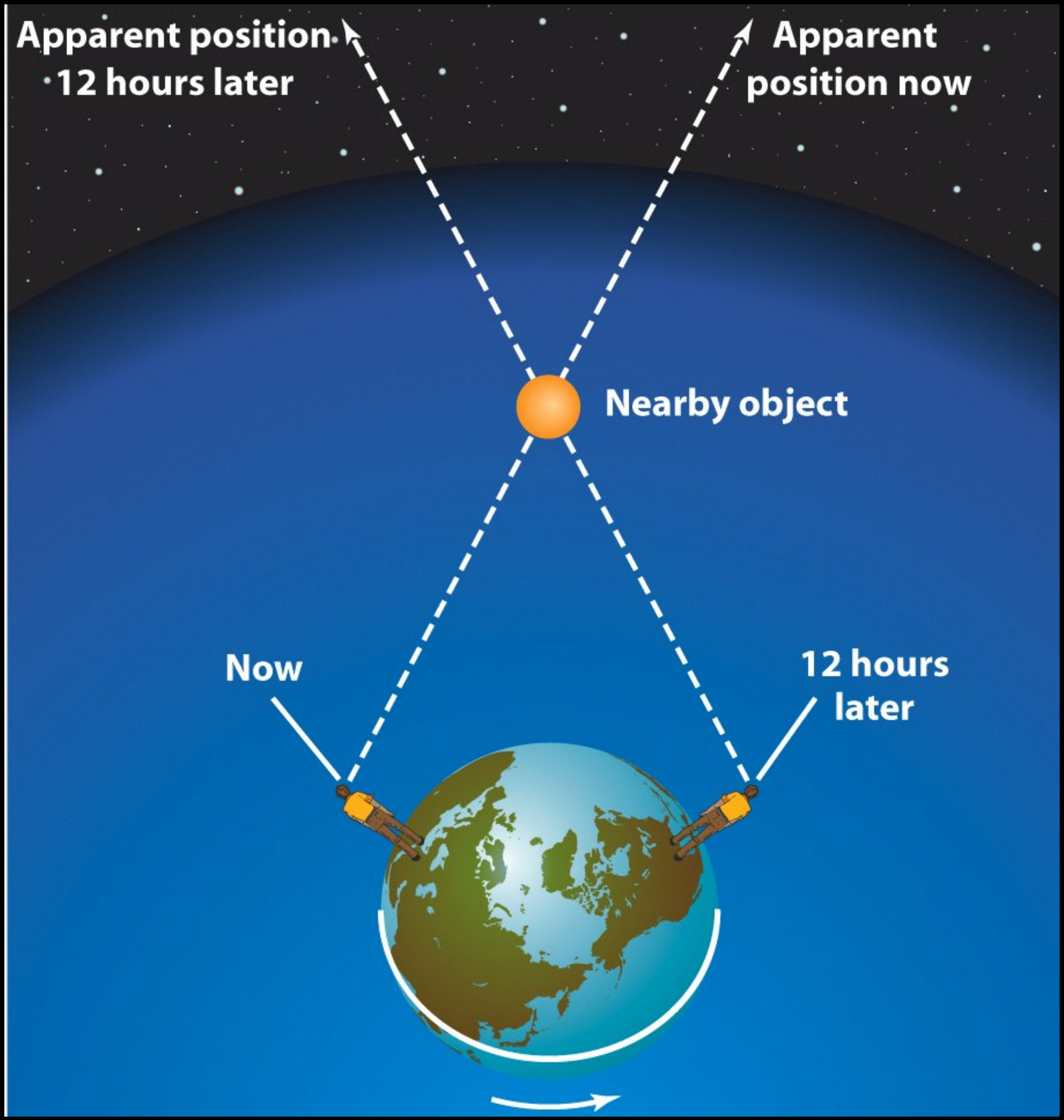
Astronomical Research institute

Tycho Brahe

1546-1601

Passion for exact empirical facts

Wikipedia

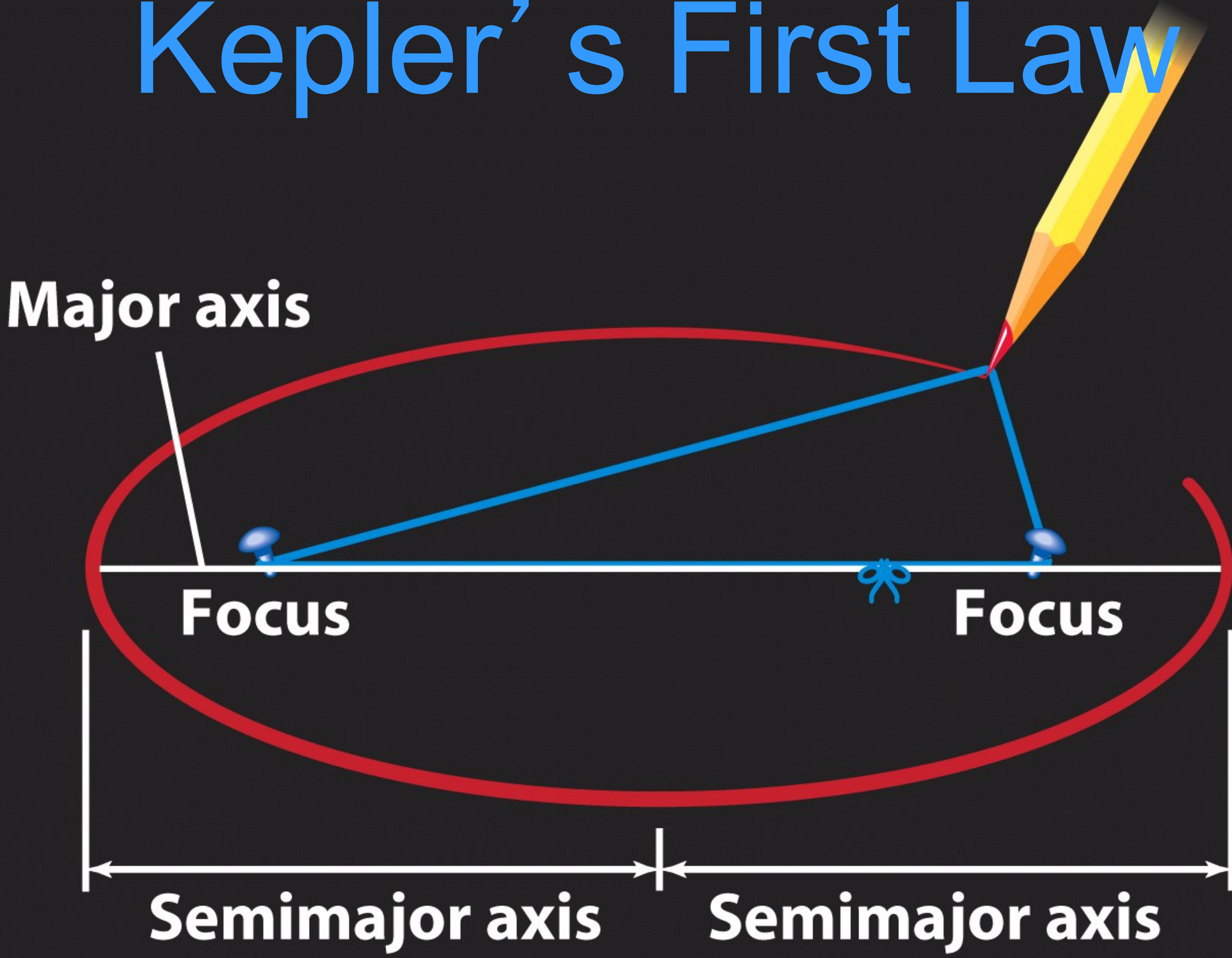


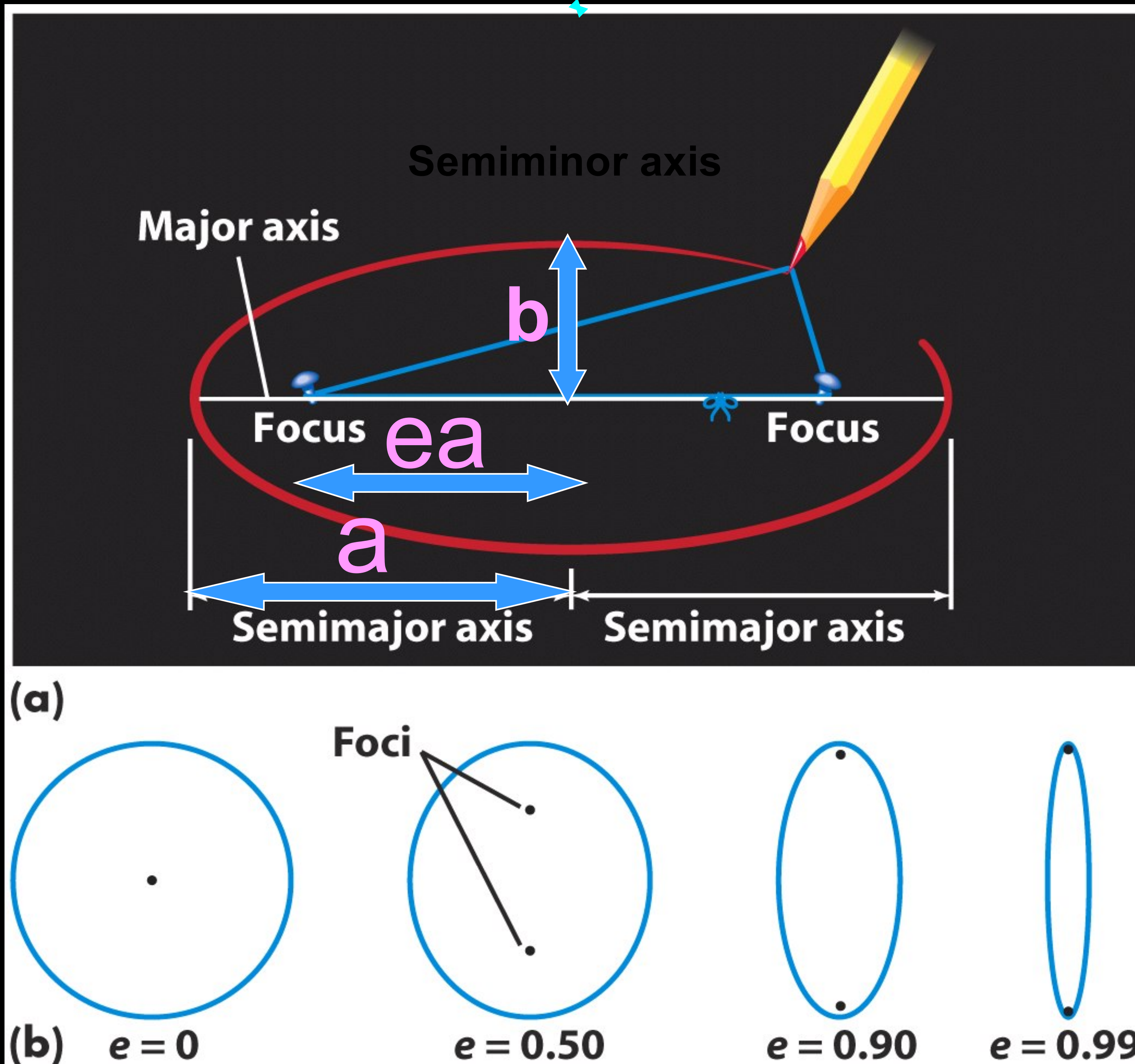
Johannes Kepler proposed elliptical paths for the planets about the Sun

- Using data collected by Tycho Brahe, Kepler deduced three laws of planetary motion:

1. the orbits are ellipses
2. a planet's speed varies as it moves around its elliptical orbit
3. the orbital period of a planet is related to the size of its orbit

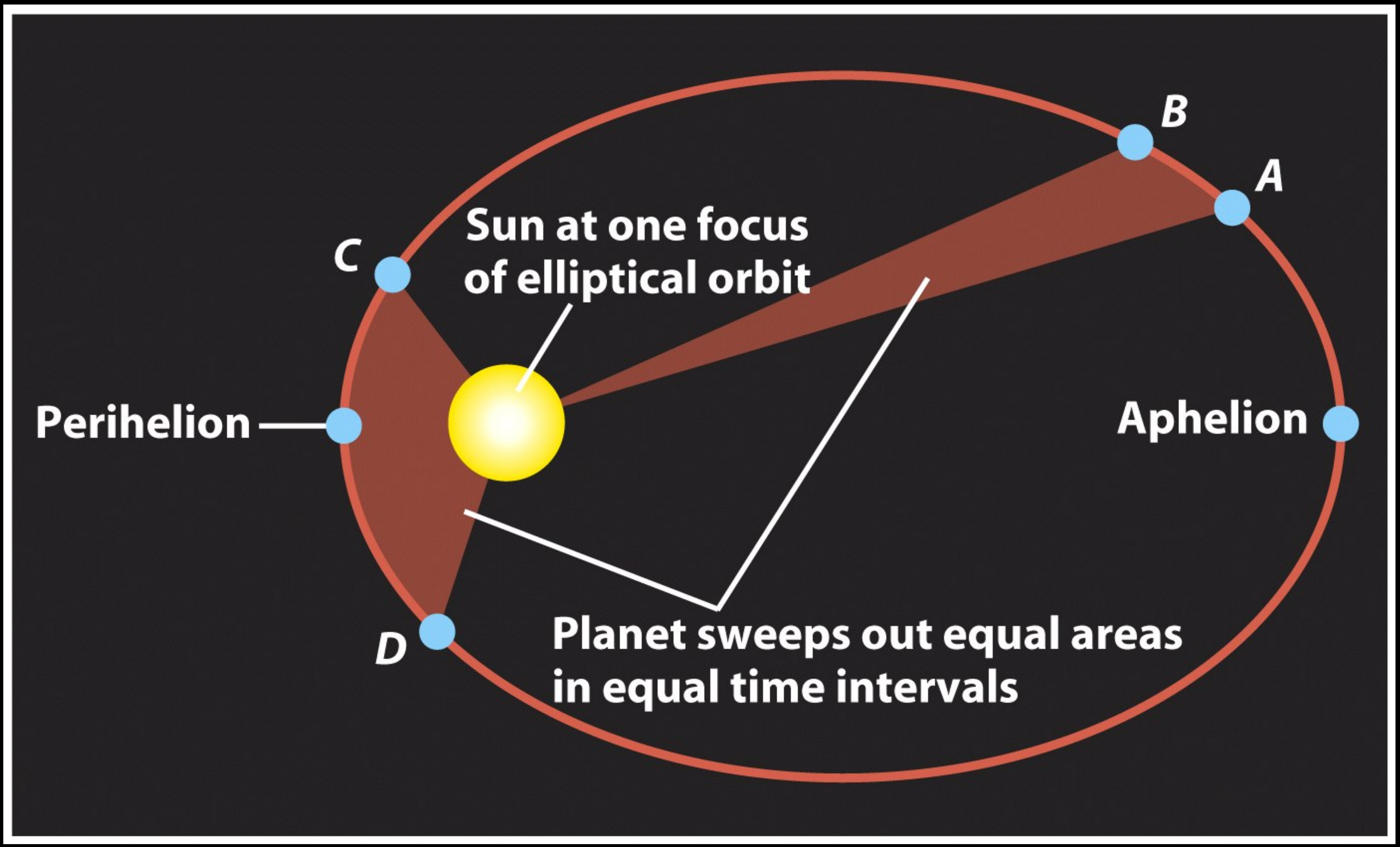
Kepler’s First Law





Seminor axis
Major axis
b
Focus
ea
Focus
a
Semimajor axis
Semimajor axis
(a)
Foci
(b)
$e = 0$
$e = 0.50$
$e = 0.90$
$e = 0.99$

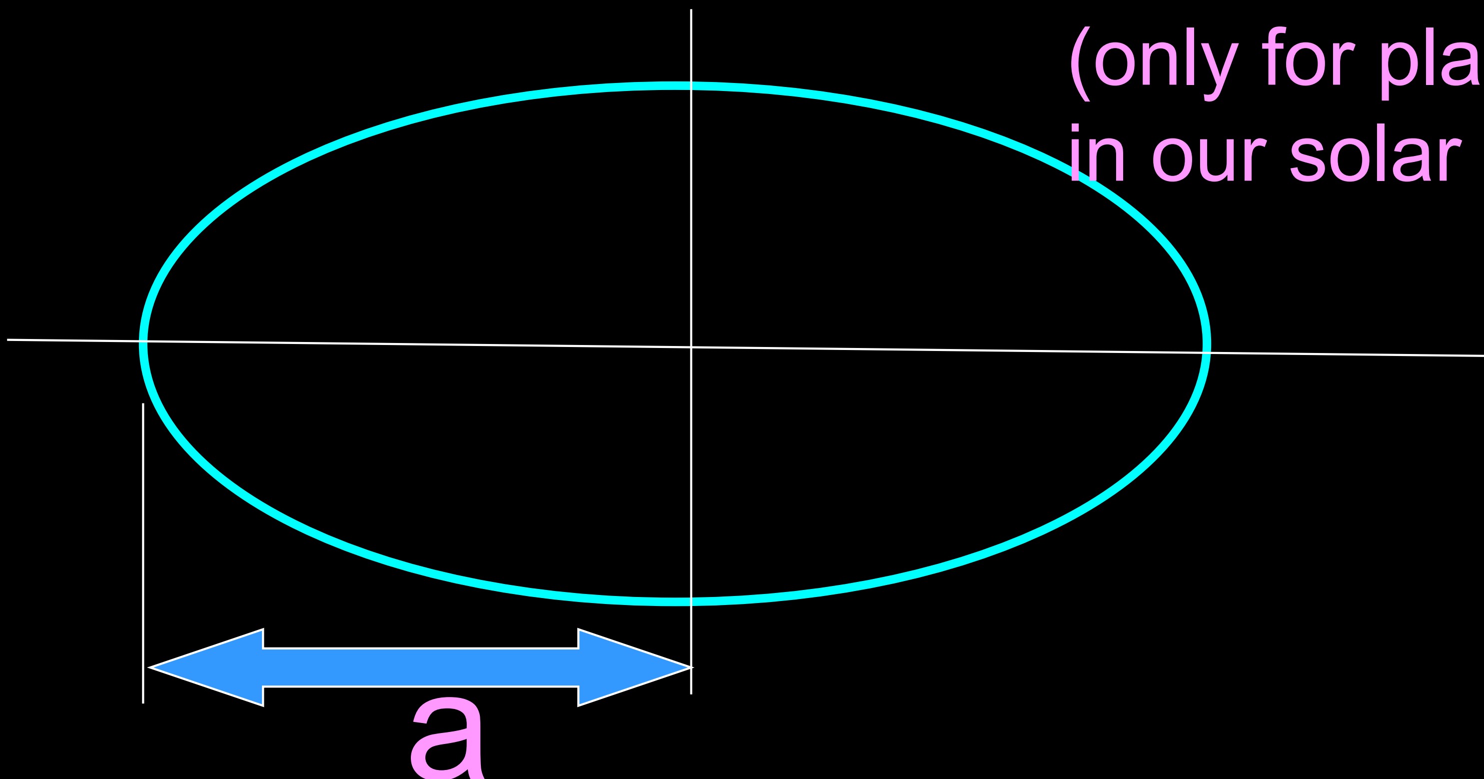
Kepler’s Second Law



Kepler's third law

$$P^2 = a^3$$

P in sidereal years
a in AU
(only for planets
in our solar system)



Kepler's Third Law

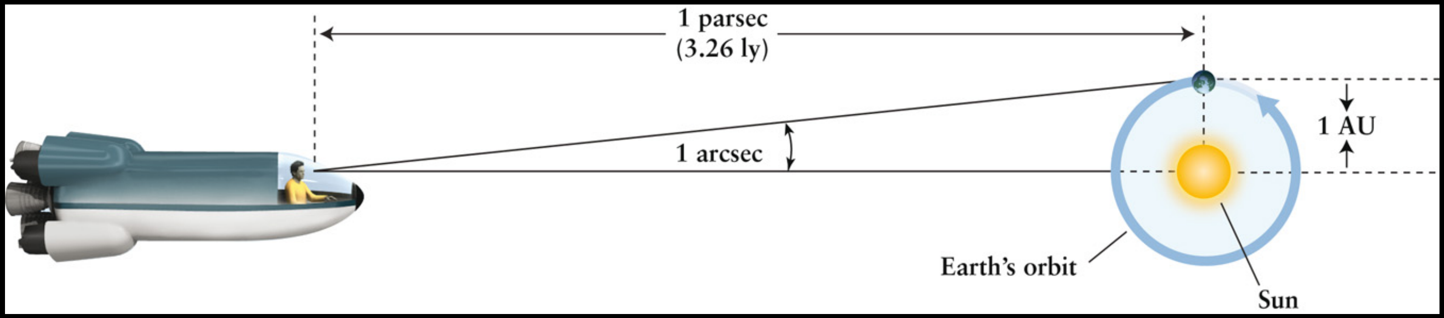
table 4-3 **A Demonstration of Kepler's Third Law ($P^2 = a^3$)**

Planet	Sidereal period P (years)	Semimajor axis a (AU)	P^2	a^3
Mercury	0.24	0.39	0.06	0.06
Venus	0.61	0.72	0.37	0.37
Earth	1.00	1.00	1.00	1.00
Mars	1.88	1.52	3.53	3.51
Jupiter	11.86	5.20	140.7	140.6
Saturn	29.46	9.55	867.9	871.0
Uranus	84.10	19.19	7,072	7,067
Neptune	164.86	30.07	27,180	27,190
Pluto	248.60	39.54	61,800	61,820

$$P^2 = a^3$$

P = planet's sidereal period, in years

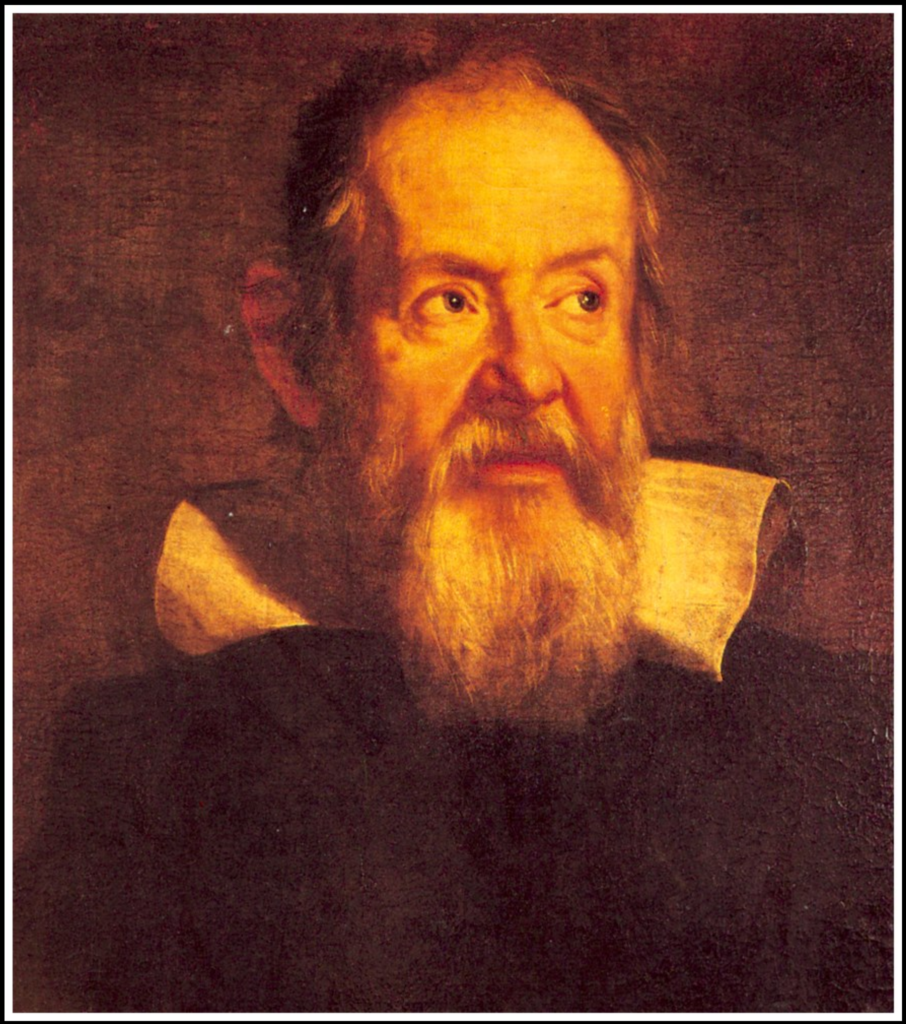
a = planet's semimajor axis, in AU



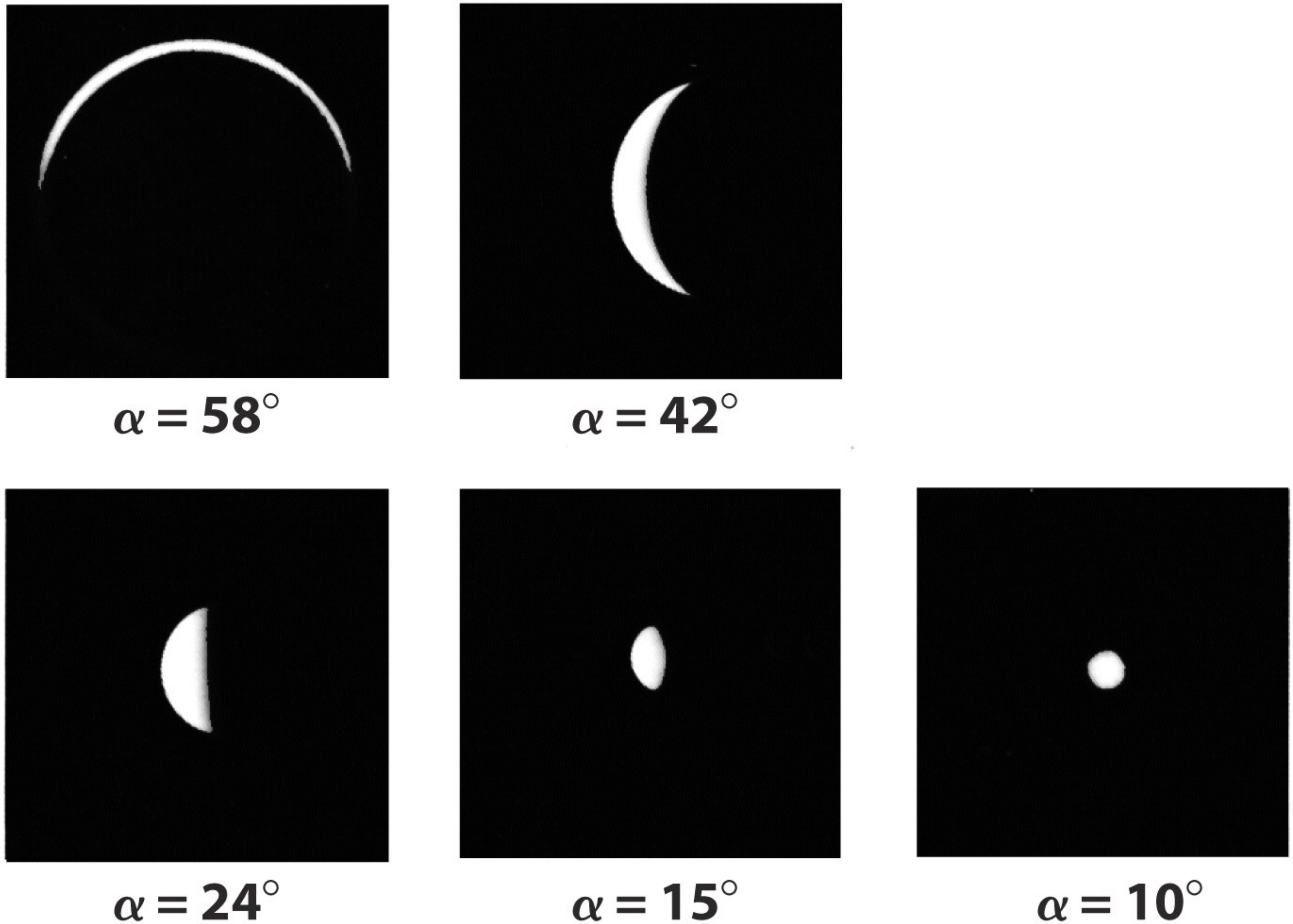
A Parsec

The parsec, a unit of length commonly used by astronomers, is equal to 3.26 ly. The parsec is defined as the distance at which 1 AU perpendicular to the observer's line of sight makes an angle of 1 arcsec.

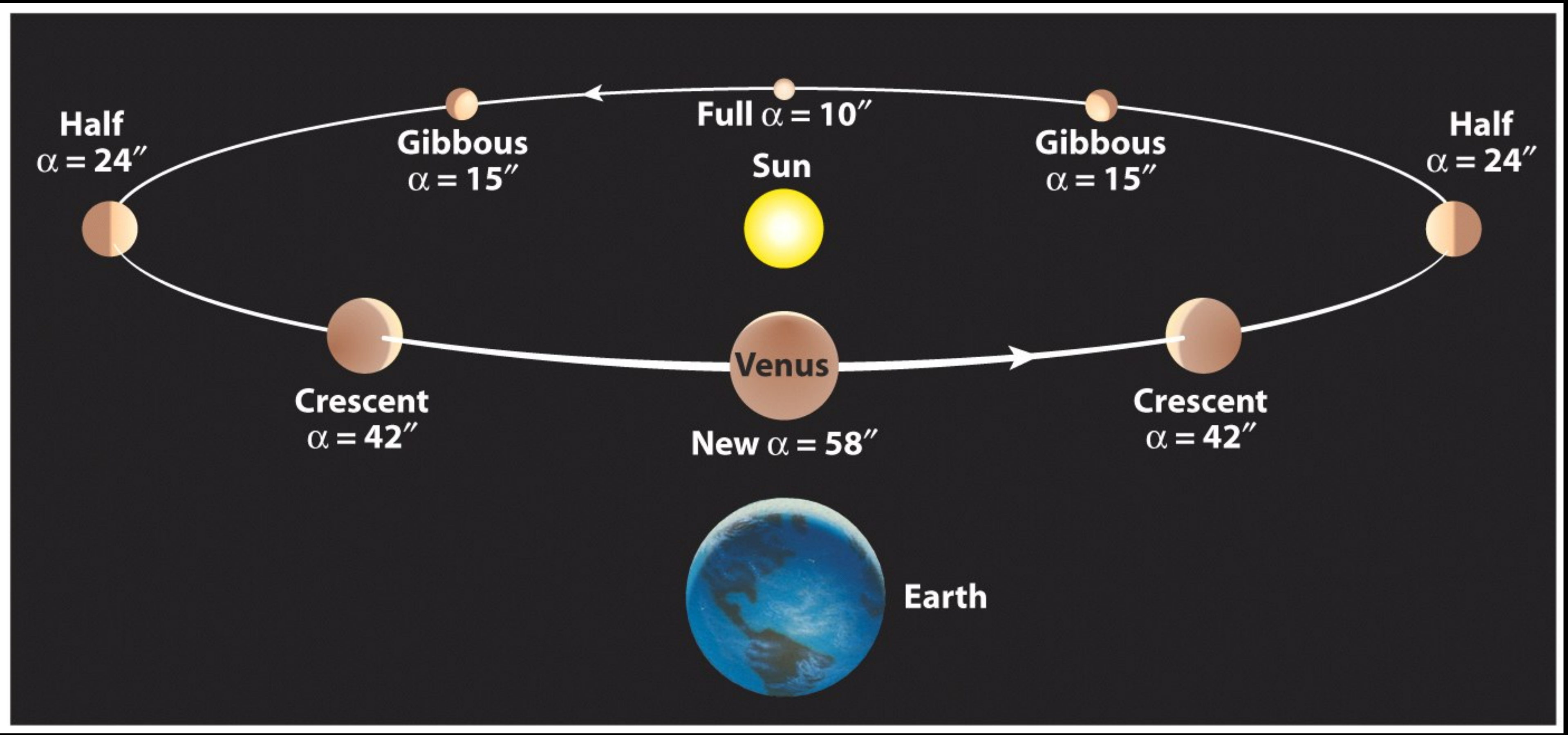
Galileo's discoveries with a telescope strongly supported a heliocentric model



- The invention of the telescope led Galileo to new discoveries that supported a heliocentric model
- These included his observations of the phases of Venus and of the motions of four moons around Jupiter



There is a correlation between the phases of Venus and the planet's angular distance from the Sun



- One of Galileo's most important discoveries with the telescope was that Venus exhibits phases like those of the Moon
- Galileo also noticed that the apparent size of Venus as seen through his telescope was related to the planet's phase
- Venus appears small at gibbous phase and largest at crescent phase

Isaac Newton formulated three laws that describe fundamental properties of physical reality

- Isaac Newton developed three principles, called the laws of motion, that apply to the motions of objects on Earth as well as in space
- These are
  1. the ***law of inertia***: a body remains at rest, or moves in a straight line at a constant speed, unless acted upon by a net outside force
  2. ***F = m a*** (the force on an object is directly proportional to its mass and acceleration)
  3. the ***principle of action and reaction***: whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first body

Mass vs Weight

- Mass is an intrinsic quantity and for a given object is invariant of position. It is measured in kg.
- Weight by contrast is the 'response' of mass to the local gravitational field. It is a force and measured in newtons.
- Thus while you would have the same mass on the earth and its Moon, your weight is different.
- W(eight) = m(ass) x g(ravitational acceleration)

Newton's Law of Universal Gravitation

from Kepler's 3 laws and Newton's 3 laws

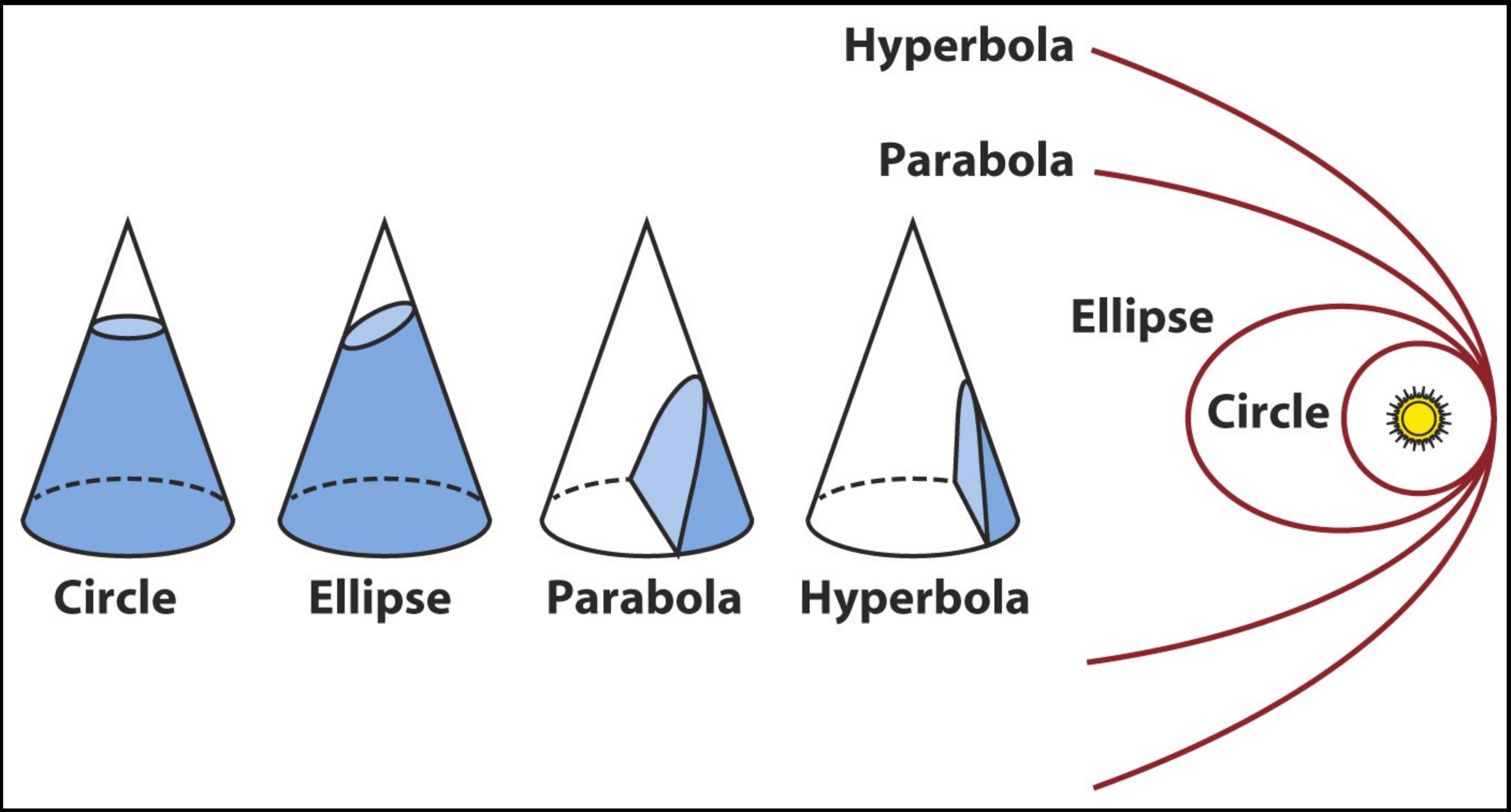
$$F = G\frac{Mm}{r^2}$$

F = gravitational force between two objects
M = mass of first object
m = mass of second object
r = distance between objects
G = universal constant of gravitation

- If the masses are measured in kilograms and the distance between them in meters, then the force is measured in newtons
- Laboratory experiments have yielded a value for G of

$$G = 6.673 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Orbits may be any of a family of curves called conic sections



Energy

(measured in Joules, J)

- *Kinetic* energy refers to the energy a body of mass *m* has due to its speed *v*: $E_{kin}=\frac{1}{2}mv^2$.
- *For a rotating body:* $E_{kin}=\frac{1}{2}I\omega^2$

  with I, moment of inertia and L=Iω, angular momentum and ω: angular velocity.

- *Potential* energy is energy due to the position of *m* a distance *r* away from another body of mass M,

$$E_{pot} = -G\frac{Mm}{r}$$

- Total energy, E_{tot}, is the sum of the kinetic and potential energies;

$$E_{tot} = E_{kin} + E_{pot}$$

A body whose total energy is < 0, orbits a more massive body in a *bound*, elliptical orbit ($e < 1$).

A body whose total energy is > 0, is in an *unbound*, hyperbolic orbit ($e > 1$) and escapes to infinity.

A body whose total energy is exactly 0 just escapes to infinity in a parabolic orbit ($e = 1$) with zero velocity at the end.

Escape velocity

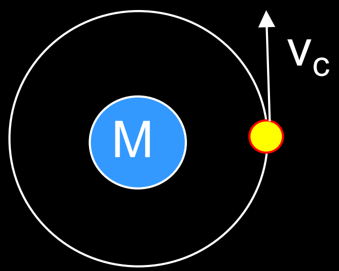
- The velocity that must be acquired by a body to just escape, i.e., to have zero total energy, is called the *escape velocity*. By setting $E_k + E_p = 0$, we find:

$$v_{escape}^2 = \frac{2GM}{r}$$

Velocity

- A body of mass *m* in a circular orbit about a (much) more massive body of mass orbits at a constant speed or the circular velocity, v_c where

$$v_c^2 = \frac{GM}{r}$$



(This is derived by equating the gravitational force with the centrifugal force, mv^2 / r).

- Note that: $v_{escape}^2 = 2v_c^2$.

Kepler's Third Law derived by Newton

$$P^2 = \frac{4\pi^2}{G(M+m)} a^3$$

P = Sidereal orbital period (seconds, s)
a = Semi-major axis planet orbit (meters, m)
M, m = mass of objects (kilograms, kg)
G = Gravitational constant : 6.673 x 10^{-11} Nm^2/kg^2

For M>>m:

$$P^2 = \frac{4\pi^2}{GM} a^3$$

Example: What is the mass of the Sun?

$M+m = 4\pi^2 a^3 / GP^2$

$= 4\pi^2 (1.5\ 10^{11})^3 / (6.673\ 10^{-11} (365.24 \cdot 24 \cdot 60 \cdot 60)^2)$

$= 2.0 \cdot 10^{30}$ kg

Since mass, m, of Earth is negligible, this is the mass of the sun, $M_{sun} = M + m \cong M$.

Example: What is your weight on the Moon?

weight=force

Force=mass x acceleration

$F=ma$

Force is also the gravitational force

$F=GMm/r^2$

We need the acceleration, a, for the Moon to calculate your weight on the Moon

$ma= GMm/r^2$

$a= GM/r^2$

$M= 7.345\times10^{22}$ kg (mass of Moon)

$r=1737$ km (radius of Moon)

$a=6.673\times10^{-11} \times 7.345\times10^{22}/ (1.737\times10^{6})^2$

$a=1.624\ m/s^2$ (acceleration on the Moon)

If your mass is 100 kg then your weight on the Moon is:

$F= 100\times1.624 =162.4$ N

On Earth the acceleration is $9.81\ m/s^2$

$F=100\times9.81 = 981$ N

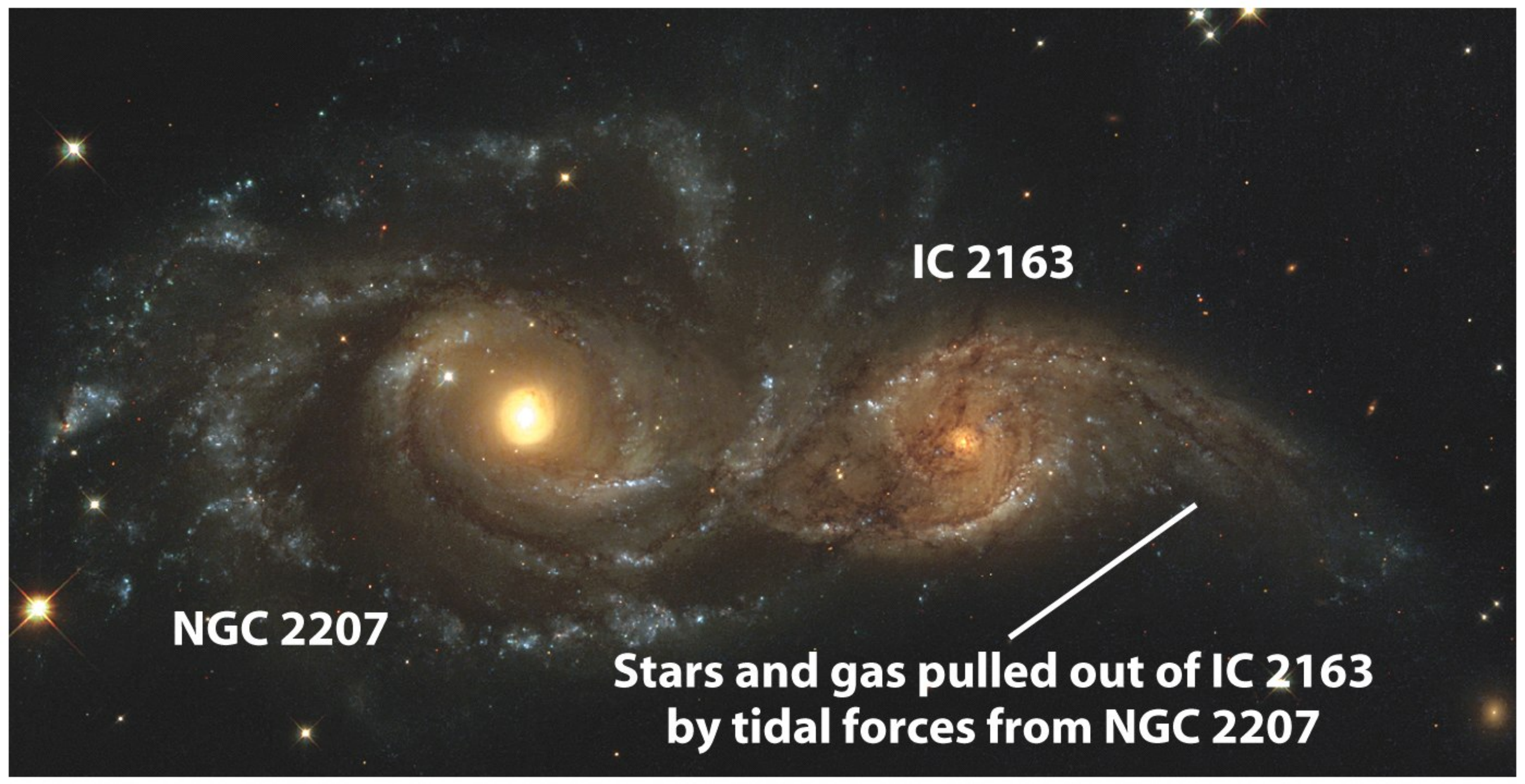
So your weight on the Moon is 162.4/981 = 0.166 times your weight on Earth

Example: What is the escape velocity for Earth?

$v^2_{escape} = 2GM/r$
$M = 5.972x10^{24}$ kg (mass of Earth)
r=6371 km (radius of Earth)

$v^2_{escape} = 2 \times 6.673x10^{-11} \times 5.972x10^{24} / 6.371 \times 10^3 = 11185$ m/s
$V_{escape} = 11185$ m/s = 11.185 km/s

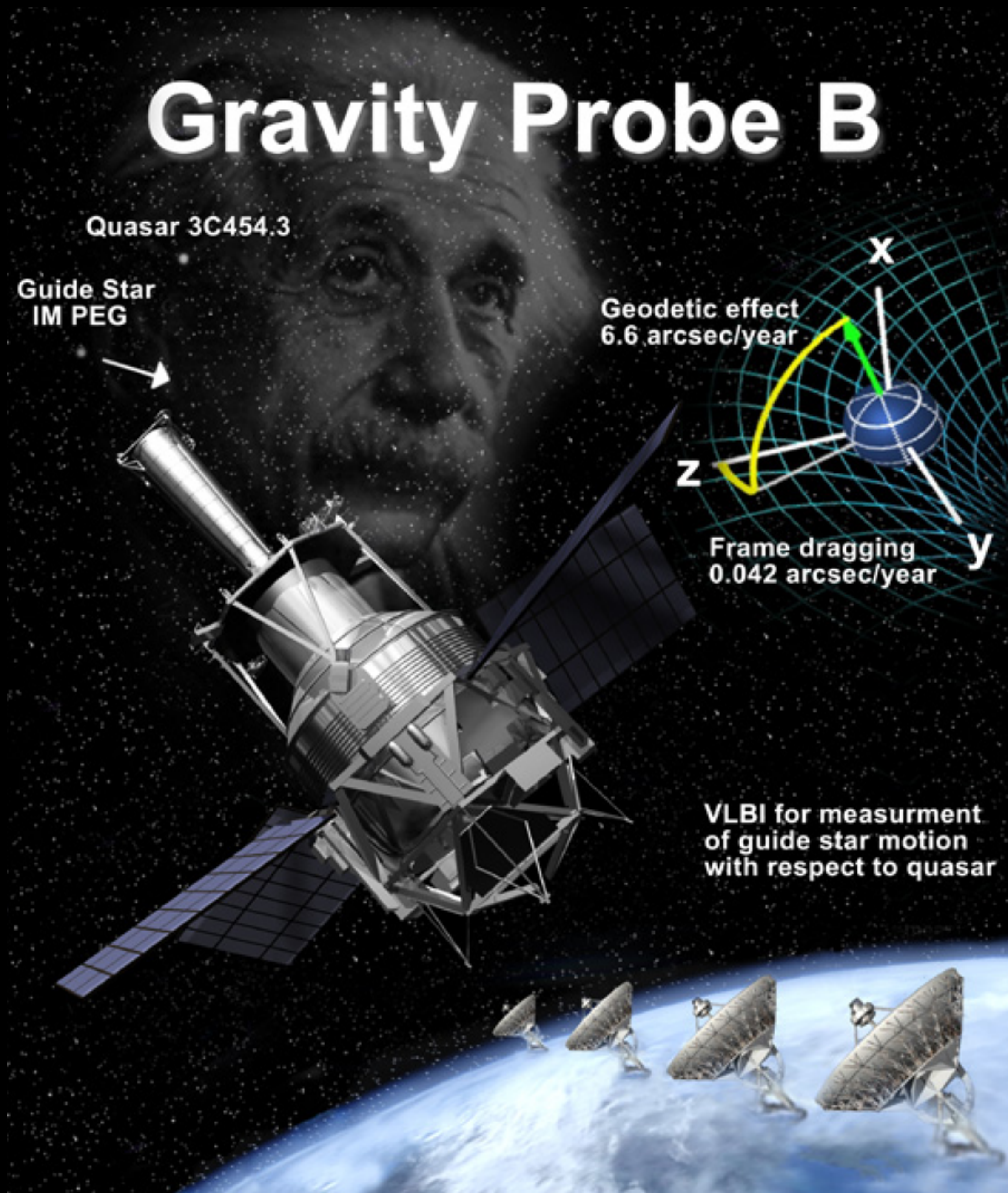
Gravitational forces between two objects produce tides



Frontiers yet to be discovered

1) Why is the inertial mass in $F=ma$ equal to the gravitational mass in $F=GmM/r^2$?

2) Newton's law of gravitation is not quite accurate as can be shown with precision measurements.


Gravity Probe B
Quasar 3C454.3
Guide Star
IM PEG
Geodetic effect
6.6 arcsec/year
x
z
y
Frame dragging
0.042 arcsec/year
VLBI for measurment
of guide star motion
with respect to quasar