

Laboratory Exercises in Astronomy —

The Rotation of the Sun

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WHEN Galileo Galilei and a few of his contemporaries turned their new telescopes to the Sun in 1611, they were astonished to find that its supposedly pure ethereal substance was blemished by a variety of dark spots. The Jesuit astronomer Christopher Scheiner argued at first that these spots were caused by clouds interposed between the Earth and the Sun. But Galileo, by careful measurements of the spots' positions, showed convincingly that they had to be on the solar surface itself.

In 1863 Richard Christopher Carrington published his discovery, based on detailed observations over a 7½-year period from 1853 to 1861, that the average latitude of the spots depends on when they are observed in the 11-year sunspot cycle. He also found that the period of rotation depends on the latitude of the spots — it is 25 days near the equator and 28 days at latitude 45°. Thereby, Carrington deduced that the visible surface of the Sun rotates differentially and thus is not solid. This discovery helped revolutionize ideas about the nature of the Sun almost as effectively as the spectrum analysis that was going on in the same decade.

More recently, astronomers have begun to suspect that this differential rotation of the Sun's outer layers is not constant and may vary over the centuries. As we shall see in this exercise, determining the differential rotation of the Sun requires more data than we can present here. Nevertheless, we can derive a fairly accurate rotational period for the Sun, and we can use this information to gain some knowledge about the stability of sunspot groups.

The principal observational data for this exercise have been provided by the U. S. Naval Observatory, which for many years routinely photographed the solar disk around noon each day. In the late spring of 1951, when an unusually high peak of

solar activity had begun to wane, a streak of remarkably clear weather occurred in Washington, D. C., and photographs could be taken daily for many weeks. From this extraordinary sequence we have selected 12 images that show the movement of a number of rather large groups. Several individual spots exceed the Earth in size!

The photographs were also taken at a time when the Earth crossed the plane of the solar equator; thus, the spots appear to move in straight lines across the disk. This is not always the case, as may be seen in the figure below, but it provides a helpful simplification for our exercise. If you wish to derive a value for the solar rotation period from your own sunspot drawings or photographs obtained at a different season, you may have to proceed in a somewhat more complicated way.

Before beginning the exercise, it is worthwhile to examine the pictures to see the manner in which the sunspot groups are moving and changing. Can you identify any groups that disappear over the western (left) limb of the Sun and then reappear at the eastern limb a couple of weeks later? It is difficult to answer this question without making some specific measurements of the solar rotation. (Notice that the Sun appears to rotate from east to west, contrary to the defined west-to-east rotation of the Earth. Actually, both the Sun and Earth rotate counterclockwise when viewed from their north poles, but the directions for the Sun are specified the way it appears in our sky, *not* as if it were an independent globe in space.)

Although sunspots appear dark in contrast to the disk of the Sun, they are really quite brilliant. The brightness temperature of the normal solar photosphere in the center of the Sun's disk is about 6,300° Kelvin, whereas that of a sunspot is typically 4,000° K. (The brightness tempera-

ture of a photoflood bulb is 3,400° K.) Notice also that the edge of the solar disk is not as bright as the center. This phenomenon is known as limb darkening. Because of the line-of-sight geometry, the radiation reaching the Earth from the edge of the Sun's disk arises from somewhat higher and cooler layers than does the radiation from the center, thereby producing the limb darkening.

In order for the sunspots to show clearly on the photographs, the contrast and exposure must be carefully chosen, but this has the consequence that the darkened solar limb is not well defined in the images. All of the photographs are reproduced to the same scale, but the procedure described in the following section will go most smoothly if we begin with a particularly clear image, such as the one for May 22nd.

SUNSPOTS AND SOLAR ROTATION

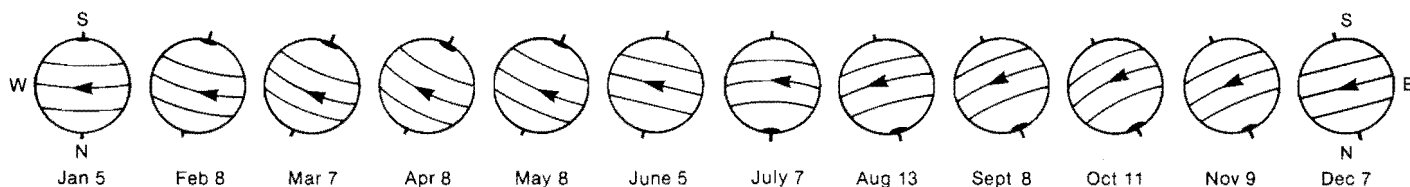
The Sun's rotation period can be determined by measuring the angular displacement of sunspots during a known time interval. The trick is to convert linear measurements of the apparent solar disk pictured on the next two pages to angular measurements of the actual solar sphere. The geometrical method we shall describe is simple yet elegant; except for a few modern touches it dates back to the time of Ptolemy.

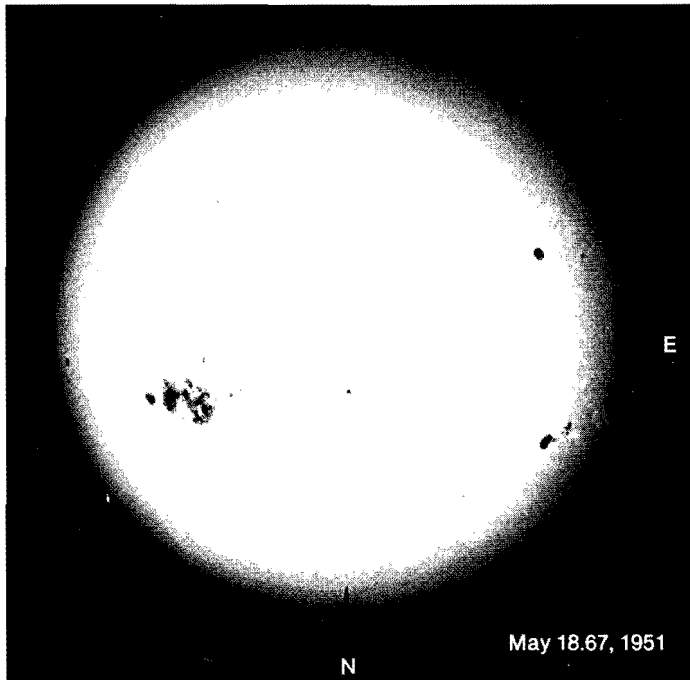
Probably the easiest way to measure the movements of the spots is to transfer their successive images onto a single circle. Our experiments show this is most easily done using a thin plastic sheet such as those sold as paper protectors or for making transparent photocopies. It is possible to work with a *very* thin piece of tracing paper, but this slows down the procedure considerably.

First, carefully measure to the nearest

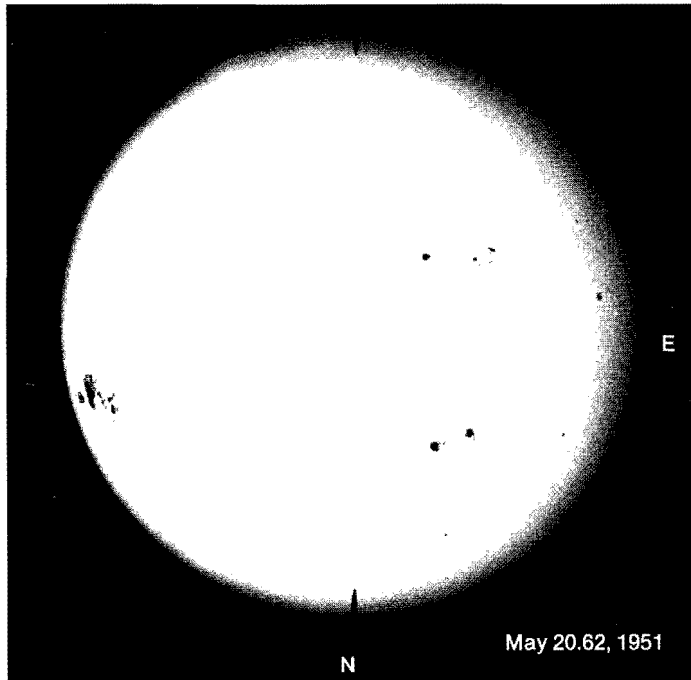
Pages 2 and 3: These photographs of the Sun were taken at the U. S. Naval Observatory in 1951. Each is labeled with the date and decimal fraction of the day. Spikes mark the Sun's rotational axis (south up, west left). This sequence roughly spans one solar rotation, providing an opportunity to watch long-lived, though ever-changing, sunspot groups disappear around one limb and reappear on the other. Several individual spots are larger than the Earth!

Below: The variation of the position angle of the Sun's axis during the year as seen in an inverting telescope from the Northern Hemisphere. The distance of the Sun's rotational pole from the limb has been exaggerated. Only in early June and December are the solar equator and the paths of sunspots seen as straight lines on the disk.

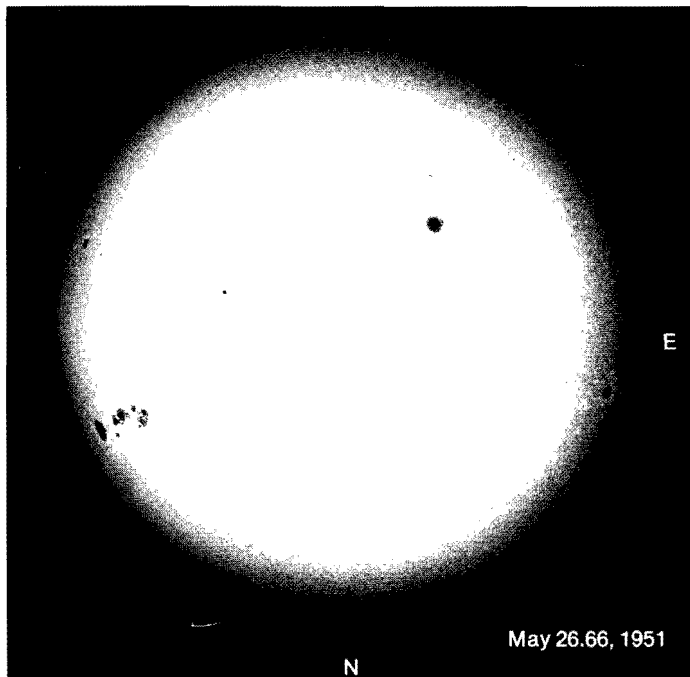




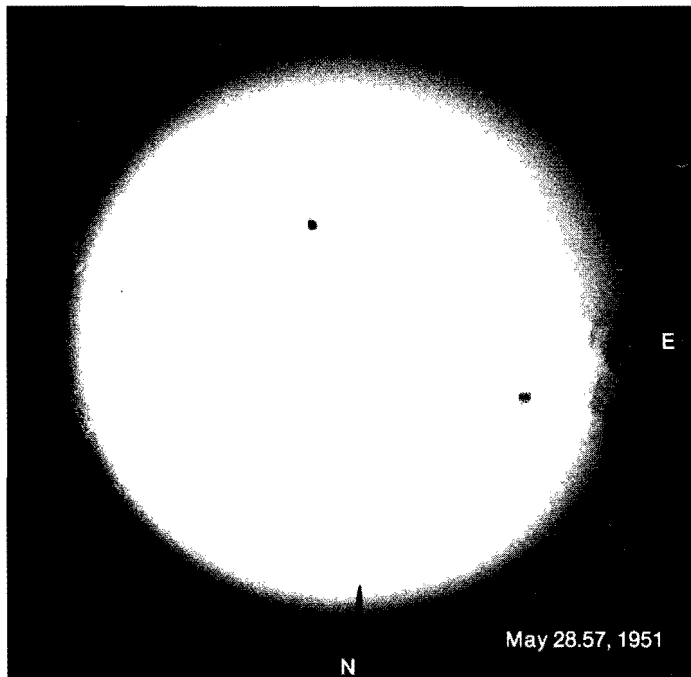
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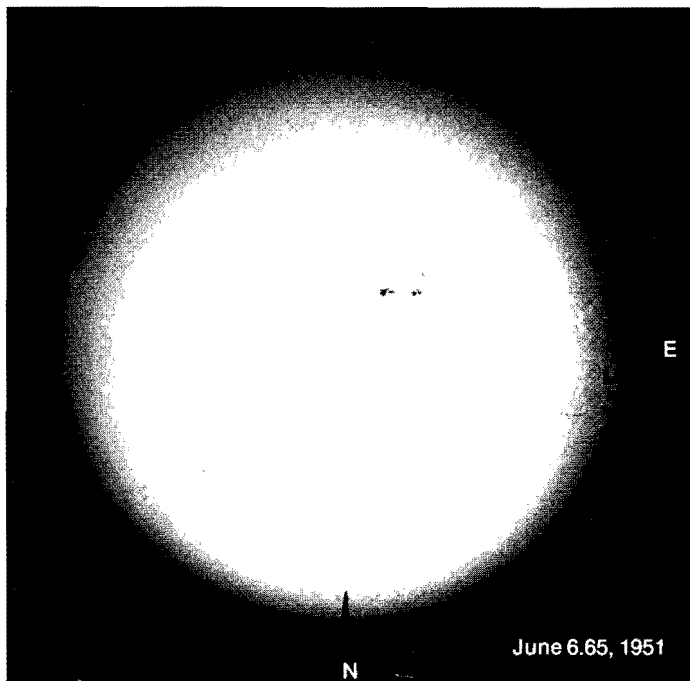
May 20.62, 1951



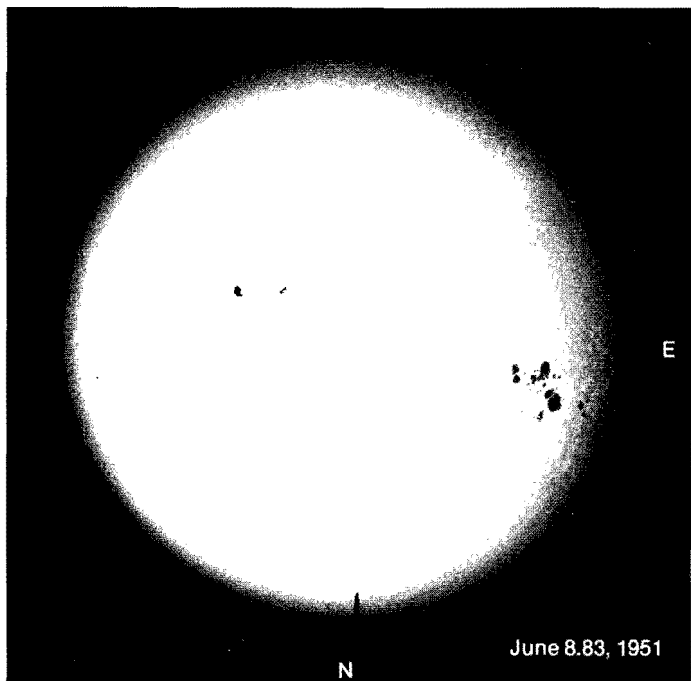
May 26.66, 1951



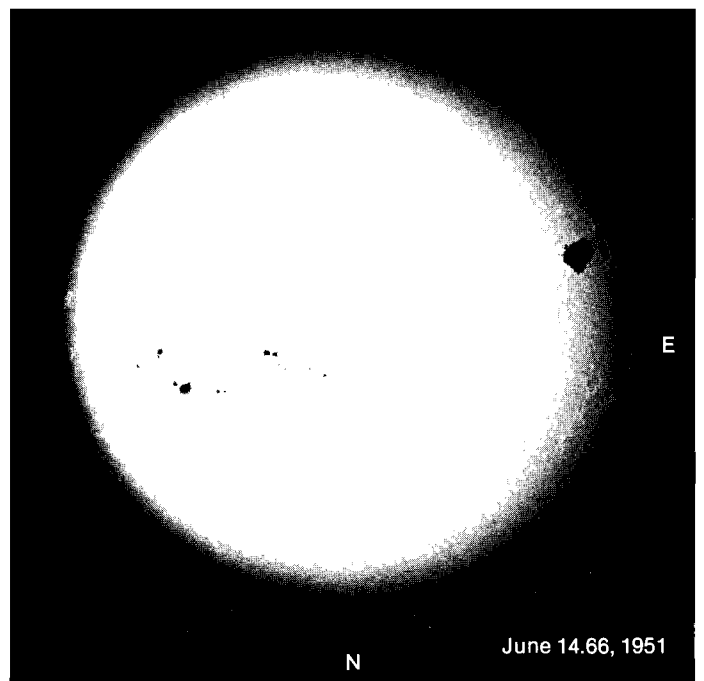
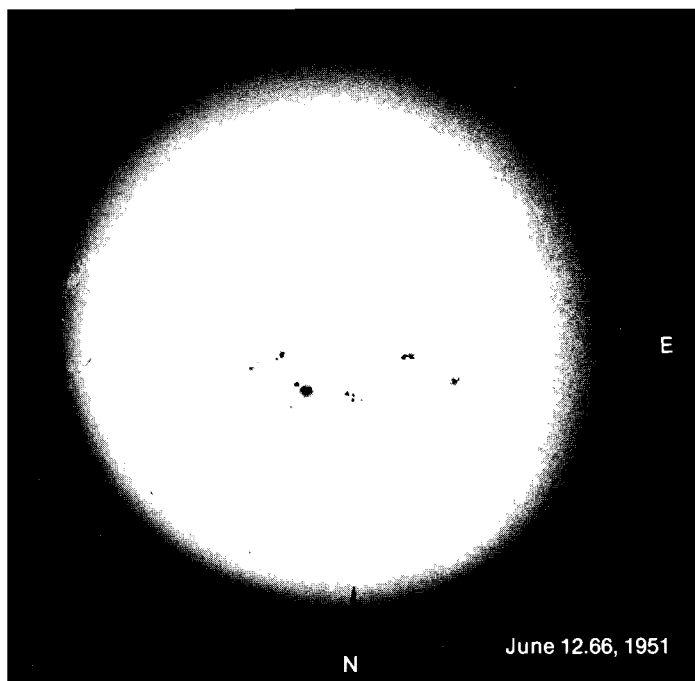
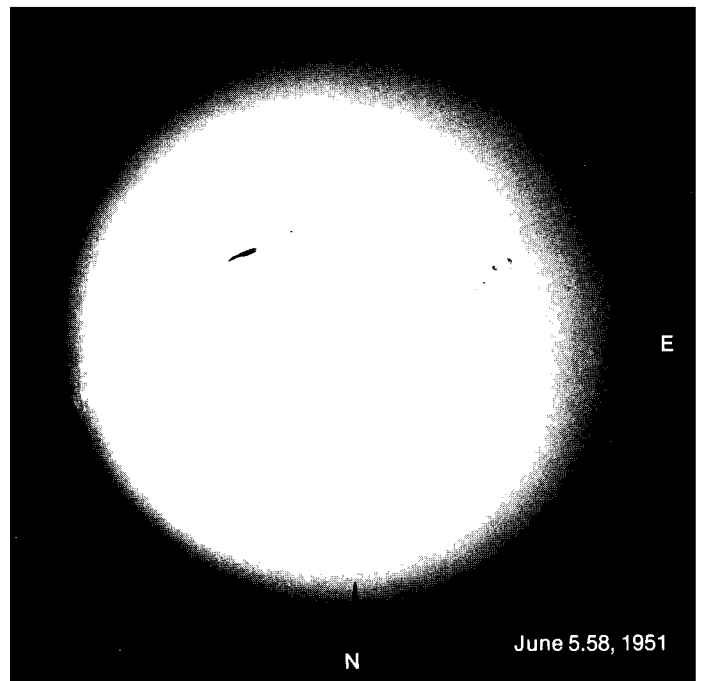
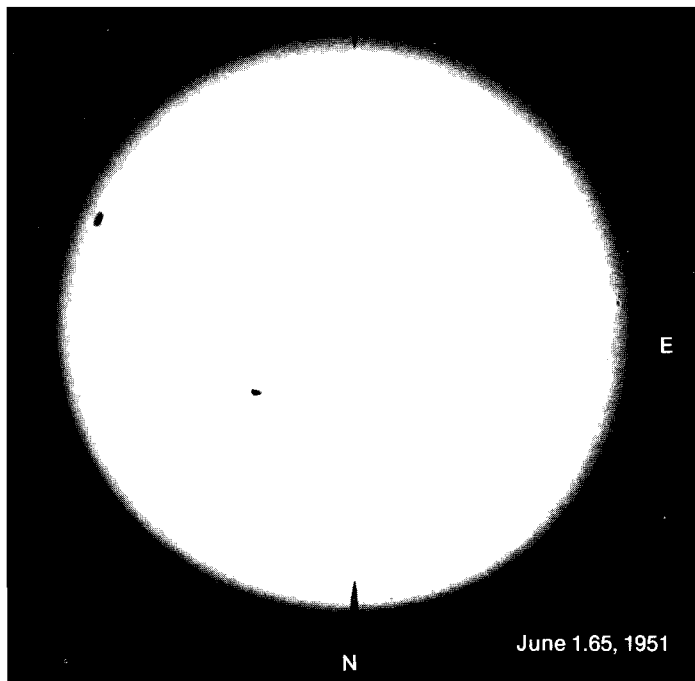
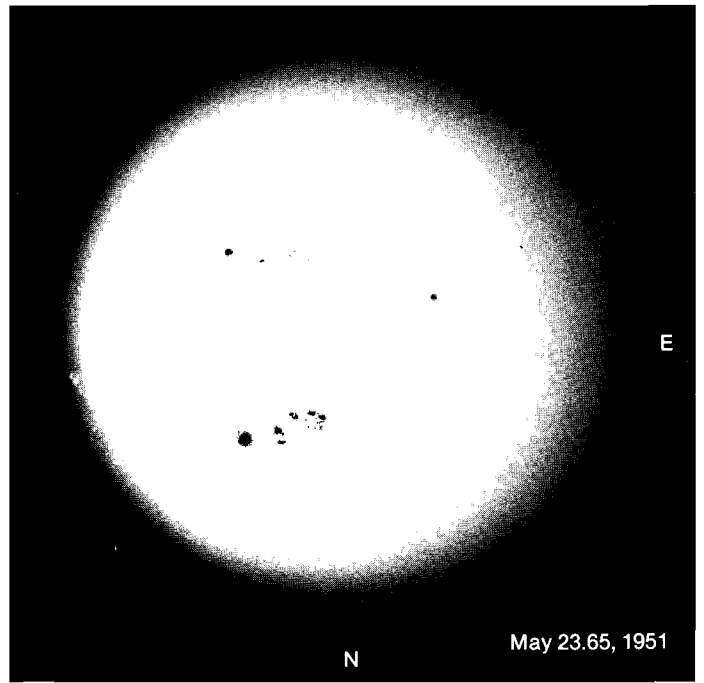
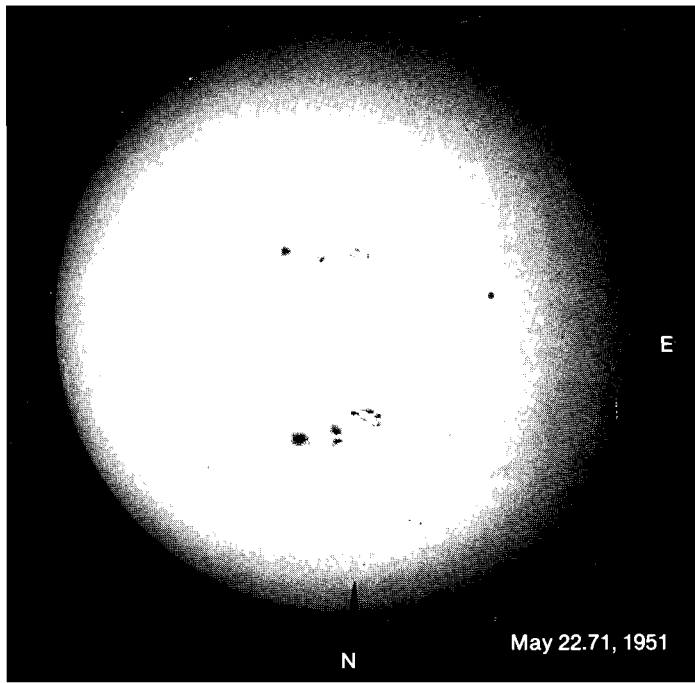
May 28.57, 1951



June 6.65, 1951



June 8.83, 1951



millimeter the diameter of the image of the Sun in the photograph (on the previous page) dated May 22, 1951. Use a compass to draw a circle of this size on your transparent plastic overlay. (If you draw with a pencil, the circle may not be very dark, but as long as you can see it you will be able to do the exercise.) Align your circle over the May 22nd image and trace the two indicator spikes that mark the orientation of the Sun's rotation axis, labeling them *N* and *S* according to the designation on the photographs. You should use a marking pen specifically intended for use on plastic, since the ink from ordinary felt-tipped pens usually smears badly.

Locate a prominent sunspot that first appears on the approaching (eastern) limb of the Sun early in the sequence. Place your transparency over the picture in which the spot is first visible and match your circle to the limb of the Sun. The spikes on the image and those on your tracing may not line up exactly. If that is the case, use the spikes to align your circle parallel to the *N-S* line, keeping the Sun's limb and your circle concentric.

Trace the position of the sunspot onto your transparency and label it with the date of the photograph. Repeat this step

for each successive photograph until the spot disappears over the receding (western) limb. Using a straightedge, draw a line from limb to limb; the line should pass as closely as possible through all of your plotted positions. This line represents the projection of the sunspot's path onto the apparent disk of the Sun. Repeat this procedure for at least two other sunspots, including one that first appears late in the photographic sequence. You should find that all your sunspot paths are parallel to each other and perpendicular to the axis of rotation. (If any of the lines are badly askew, double-check to be sure that you used the same sunspot each time you plotted successive positions across the disk.)

It should now be obvious that the spots pass by rapidly from east to west when they are near the center of the apparent disk and rather slowly near its limb. This is a consequence of foreshortening caused by the spherical globe of the Sun. Although we could calculate the amount of foreshortening by means of elementary trigonometry, we can avoid doing so by adopting the following simple graphical procedure. Imagine a globe with a circle drawn all the way around representing the positions of a sunspot carried through one

solar rotation. Now cut a slice through the globe to get a disk whose edge represents the complete circular trajectory of the sunspot. Each straight line on your plastic overlay sheet corresponds to such a circle viewed edge on.

You can draw such a circle by measuring (to a millimeter or better) the length of the straight line of a spot's apparent motion across the solar disk from limb to limb and by using this length as the diameter of the circle. (Notice that this diameter will be slightly smaller than for the original circle used on the overlay — can you see why?) In fact, you need only make a semicircle, because we can observe only the Earth-facing half of the Sun.

This conversion from the apparent linear path of a sunspot across the disk to a circular arc on the spherical Sun is best done in the following manner. On a piece of graph paper draw a horizontal line the same length as one of the sunspot paths on your transparency. Bisect this baseline and use a compass to draw a semicircle connecting its two ends. This semicircle represents the Earth-facing surface of the Sun at the latitude of the sunspot, while the line is its projection as seen on the photographs.

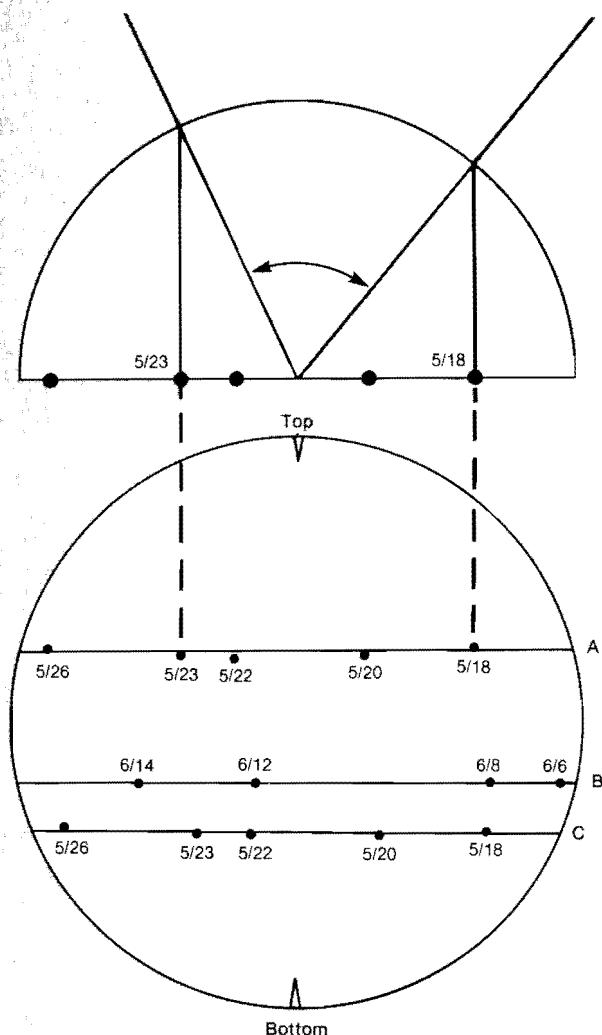
Avoiding points close to the limb, transfer two widely separated sunspot positions from the line on your tracing to the line on the graph paper, labeling them with the dates of the corresponding photographs. Draw a vertical line from each spot position to the semicircle, as shown in the figure on this page. The intersection points represent the actual positions of the sunspot on the surface of the Sun. Draw lines from these positions to the midpoint of the baseline. Using a protractor, find the angle (to the nearest tenth of a degree) *between* these radial lines. Record the result on your diagram, and then repeat the entire procedure with fresh diagrams for your other sunspots.

For each sunspot, determine the number of degrees traveled each day to the nearest tenth of a degree, and then calculate the average of your results. Using this average sunspot motion, compute the rotation period of the Sun.

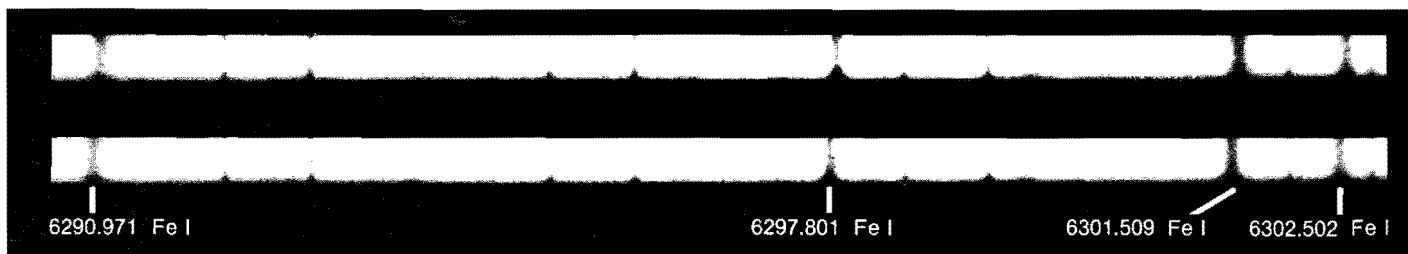
Your result gives the synodic period of the Sun. Remember that the photographs were taken from a moving planet rather than from a position "fixed" in space relative to the stars, so the period we measure is synodic rather than sidereal. The accepted synodic period in the sunspot zone is 27.3 days. If you made your measurements carefully, your result shouldn't differ from this by more than a day. To convert the synodic period to sidereal period, that is, to the solar rotation reckoned against the starry framework of space, use the relation:

$$P = S \times E / (S + E),$$

where *P* = sidereal period, *S* = synodic



Following the explanation in the text, this diagram shows the conversion from the apparent linear path of a sunspot across the solar disk to a circular arc on the spherical Sun. The circle at the bottom represents the tracing of successive sunspot positions onto a transparency. The semicircle at the top represents the Earth-facing surface of the Sun at the latitude of the spot, and should be drawn on graph paper to provide convenient vertical lines. All illustrations with this article were supplied by the authors.



These spectrograms of the Sun's limbs at 15° latitude contain dark absorption lines (whose wavelengths are given in angstroms) formed by iron in the Sun's photosphere. As the Sun rotates, one limb approaches the Earth while the other recedes, thereby causing the lines to be blue- or redshifted due to the Doppler effect. Lines that are not Doppler shifted arise from molecular oxygen in the Earth's atmosphere. Yerkes Observatory photograph.

period, and $E = 365.26$ days (one Earth sidereal year). The sidereal period corresponding to the synodic period given above is 25.4 days.

Given the angular rate of rotation, it is possible to determine whether any of the sunspot groups visible in the early photographs reappeared by the end of the sequence. If you work carefully, you should be able to identify which groups are the same, despite their considerably altered appearance.

Our photographs from 1951 show the sunspots at pretty nearly the same latitudes (15° either N or S). Clearly, if we wished to find rotation periods at different latitudes, it would be necessary to have sunspots at other distances from the solar equator. Photographs from 1949, for example, would have recorded groups farther from the equator, whereas a series from 1953 would have shown groups closer to the solar equator.

Thus, to establish the differential rotation of the Sun, several sequences like the one on pages 2 and 3 would be needed.

THE SOLAR SPECTRUM AND THE SUN'S ROTATION

Even if the Sun had no spots, it would be possible to measure its rotation rate by using the Doppler shift of its spectral lines. This physical phenomenon gives us the means to check on the rotational period by an entirely different method, or we can think of this as a rare opportunity to check up on the so-called Doppler effect itself. Furthermore, sunspots are seldom found at latitudes above 40 degrees, but we can measure the solar rotation at virtually any latitude by examining spectra taken of the approaching and receding limbs of the Sun.

When a light source moves toward or away from an observer, the lines in its spectrum are displaced by an amount proportional to the speed of approach or recession. The redshift in the spectra of distant, receding galaxies is but one astronomical example of the Doppler effect.

The illustration above from Yerkes Observatory shows the spectrum of each limb of the Sun. It is immediately apparent that the thick dark spectral lines from the two limbs are not at the same wavelengths. Because of the Sun's rotation, the

lines at the approaching limb are blue-shifted (toward shorter wavelengths) while those at the other are redshifted (toward longer wavelengths). Looking again at the spectra, can you tell which limb is approaching and which is receding?

As you examine the spectra, you should notice that not all of the spectral lines exhibit Doppler shifts. Those that do not are the so-called telluric lines, which originate in the Earth's atmosphere. Most of the telluric lines in this portion of the spectrum arise from molecular oxygen. In the illustration, the spectra are aligned so that the telluric lines match vertically, making the Doppler shift in the solar lines strikingly apparent.

Because the amount of the Doppler shift in the solar spectrum is proportional to the Sun's rotational velocity, we can determine the rotation period of the Sun by measuring this shift. This can, in principle, be done at any latitude, but the shift becomes smaller and harder to detect at very high latitudes, requiring a spectrum of much better quality than the one presented here. When the analysis is carried out properly, one finds that the Sun does indeed rotate more slowly at higher latitudes.

In this part of the exercise, we will measure the Doppler shift at 15° latitude, corresponding roughly to the position at which most of the sunspots in the accompanying photographs appear. The rotation period derived from this measurement should agree with that obtained in the first part of the exercise, corroborating Galileo's assertion that the sunspots are features on the "surface" or photosphere of the Sun.

For the four lines whose wavelengths are indicated, measure the difference between the positions of the lines at the two limbs of the Sun. This can best be done using a millimeter rule and a magnifying glass. For the spectra at each limb, estimate to the nearest tenth of a millimeter the distance between the centers of a solar line and the nearest telluric line. (Be sure to use the same telluric line in the spectra of both limbs, and, to achieve the highest accuracy, measure from the center of the thick solar line toward the center of the thin telluric line.) The Doppler shift is half the difference in the two measure-

ments. Record your measurements and calculate the average of the shifts found for the four lines.

To convert the Doppler shift from millimeters into wavelength units, you must establish the scale of the photograph in angstroms per millimeter. This is easily done using the labeled solar lines and your millimeter rule; again, try to estimate distances to the nearest 0.1 mm. Once you have the scale, find the average Doppler shift ($\Delta\lambda$) in angstroms.

Using 6300 angstroms as an average wavelength (λ) and your calculated Doppler shift, find the rotational velocity (v), which is half the velocity difference between the approaching and receding limbs. Use the mathematical statement of the Doppler effect:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where c is the speed of light (300,000 kilometers per second).

With the rotational velocity and the radius of a slice of the Sun's globe at 15° latitude ($R_{15} = 672,300$ km), it is easy to calculate the synodic rotation period S in days. (The velocity is the distance per unit time, in this case the solar circumference at 15° latitude of $2\pi R_{15}$ per rotational period S ; note that there are 86,400 seconds in a day.) How closely does your result compare with the period obtained from the sunspot photographs? With the spectrum reproduced here and the relatively simple measurements used, the difference between the periods obtained from the two methods may be as large as two or three days.

CONCLUDING REMARKS

The solar rotation rate has interesting cosmogonic and cosmological implications. For example, it is a well-known paradox that although 99.9 percent of the mass of the solar system resides in the Sun, the planets have about 99.5 percent of the system's angular momentum. For the Sun to have its proportionate share of the angular momentum, it would have to complete one rotation in about three hours. If the solar system condensed from a rotating, gravitationally collapsing nebula, then the Sun at the nebula's center should spin rapidly, just as an ice skater spins faster when the arms are drawn in. Astronomers assume

that some physical process transferred most of the early Sun's rotational angular momentum to the orbital motion of the planets.

Although our Sun rotates rather slowly, this cannot be said of all stars. Much faster rotators have been discovered through the Doppler shift. In a spinning star part of the stellar surface is approaching and another part is receding. Since the star is an unresolved point of light, its entire disk contributes to any observed absorption line in the spectrum; therefore, the line is formed from both blue- and redshifted radiation and its profile is broadened. The effect is unnoticeably small for a star rotating as slowly as the Sun, but for many *B*- and *A*-type stars the rotational broadening of the spectral lines is quite conspicuous. This phenomenon was first carefully described by Otto Struve and G. A. Shajn in 1929, and the typical rotational period implied by their measurements is less than a day for late *B*-type stars.

Curiously, solar-type stars all seem to have rotations too slow to measure by the broadening of spectral lines. Whatever mechanism has slowed the Sun seems to have been equally effective for its cousins. Recently it has become possible to measure the rotations of some of these slowly spinning stars by analyzing the periodic variations in a chromospheric line of ionized calcium. The *K*-type components of the binary star 61 Cygni rotate in 37 and 48 days, respectively. Probably some physical mechanism common to all these stars, such as strong winds of ionized particles interacting with interstellar magnetic

fields, has slowed the solar-type stars but not the hotter *B*- and *A*-type stars.

The solar rotation also has a complex connection with Einstein's general theory of relativity. He predicted a small ongoing change in the orientation of Mercury's orbit in excess of that explained by Newtonian gravitation theory, an amount that had been observed but not previously explained. However, the calculations assume that the Sun is completely spherical. If the central core (containing most of the Sun's mass) is rotating much more rapidly than the outer layers, it could be oblate, something that would disturb the Newtonian calculations and give a discrepancy with Einstein's predictions.

At present the very subtle measurements required to establish an inner oblateness of the Sun remain highly controversial. In any event, we are reminded that our determination of the solar rotation period from sunspots and the solar spectrum refers only to the outer layers. But, in fact, the Sun may have more rotation and hence more angular momentum than meets the eye!

ADDITIONAL NOTES

I. We strongly recommend the use of plastic transparencies, but very transparent tracing paper may be substituted if necessary. Office-supply stores usually carry transparencies or transparent sheet protectors (which may be cut to yield two sheets of clear plastic) ranging from 20 to 40 cents apiece. Here is a list of materials needed to work this exercise:

1. Transparency, sheet protector, or thin tracing paper
2. Sanford's "Sharpie" No. 3000 (or

similar) marking pen

3. Millimeter ruler
4. Drawing compass
5. Protractor
6. Millimeter graph paper
7. Magnifying glass

II. Here are some auxiliary exercises:

- Derive the relation between the synodic and sidereal periods. Hint: Consider the synodic period as the interval between successive alignments of the Sun's center, a given sunspot, and the Earth. Find a relation between the number of solar rotations in one synodic period and the number of Earth orbits in the same time, noticing that the next alignment will occur when the sunspot has lapped the Earth exactly once. Draw a diagram!

- The Sun's diameter is about 100 times that of the Earth. From this information, determine how many of the sunspots in the photographs have a diameter larger than the Earth's.

- In determining the Sun's rotation period in the first part of this exercise, why should sunspots close to the limb be avoided in making the measurements? The answer can be illustrated graphically.

III. Instructions for determining the solar rotation period from sunspot photographs or drawings made at a time when the Earth is not in the plane of the Sun's equator may be found in R. William Shaw and Samuel L. Boothroyd, *Manual of Astronomy* (W. C. Brown, Dubuque, Iowa, 1967).

IV. We wish to thank the U. S. Naval Observatory and Yerkes Observatory for the photographic materials, and R. W. Noyes and D. Danby for helpful advice. 