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Discovering the Essential Universe

Sixth Edition

CHAPTER 2

Gravitation and the Motion of the Planets

1

In this chapter we will talk about …

- Copernicus
- Kepler
- Newton

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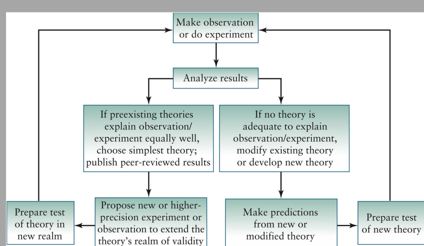

The scientific method is used to develop new scientific theories. Scientific theories are accepted when they *make testable predictions that can be verified using new observations and experiments.*

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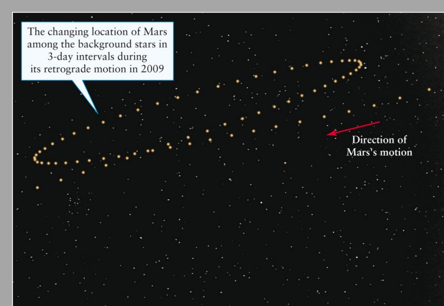

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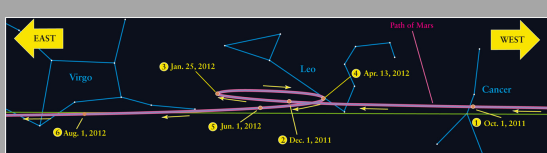
The Path of Mars



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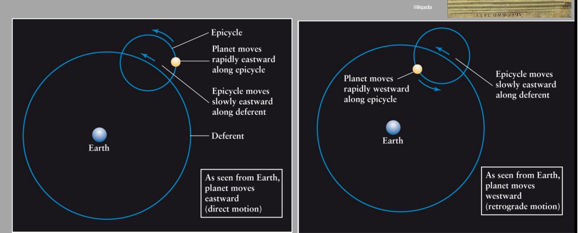
A geocentric explanation of planetary motion
- the Ptolemaic system

Ptolemy

~100 - 168 AD



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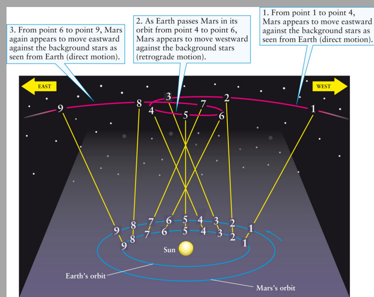

Nicolaus Copernicus developed the first complete *heliocentric* (Sun-centered) model of the solar system.

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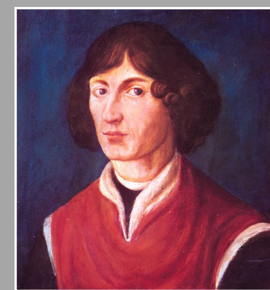
Nicolaus Copernicus devised the first comprehensive heliocentric model

- Copernicus' s heliocentric (Sun-centered) theory simplified the general explanation of planetary motions
- In a heliocentric system, the Earth is one of the planets orbiting the Sun
- The sidereal period of a planet, its true orbital period, is measured with respect to the stars



1473 -1543

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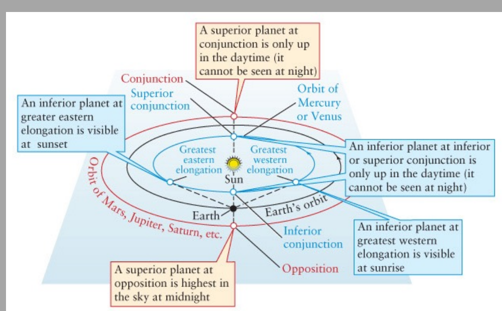

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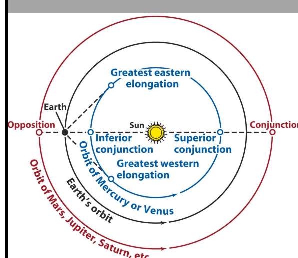
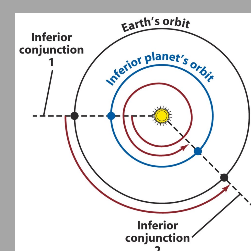
A planet' s synodic period, S, is measured with respect to the Earth and the Sun (for example, from one opposition to the next)





A planets sidereal period, P, is measured with respect to stars. In one sidereal period the planet completes a 360 deg orbit.

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Sidereal and Synodic Orbital periods

- During time S earth covers (360deg/E)S
- Inferior planet has covered (360deg/P)S
- => (360deg/P)S = (360deg/E)S +360deg
- For Inferior Planets 1/P = 1/E + 1/S
- Similarly it can be shown that:
- For Superior Planets 1/P = 1/E – 1/S

P = Sidereal Period of the planet
S = Synodic Period of planet
E = Earth' s Sidereal Period (1 year)

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Example for Mercury (inferior planet)

- 1/P = 1/E + 1/S
- 1/P = 1/365d + 1/116
- 1/P = 0.0113604 1/d
- P = 88 d

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table 4-1 Synodic and Sidereal Periods of the Planets

Planet	Synodic period	Sidereal period
Mercury	116 days	88 days
Venus	584 days	225 days
Earth	—	1.0 year
Mars	780 days	1.9 years
Jupiter	399 days	11.9 years
Saturn	378 days	29.5 years
Uranus	370 days	84.1 years
Neptune	368 days	164.9 years
Pluto	367 days	248.6 years

De revolutionibus orbium coelestium

- On the revolutions of the celestial spheres
- Published 1543 (year of his death)

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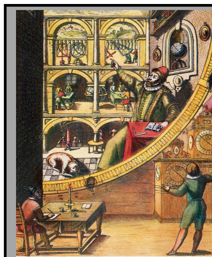
Parallax Shift

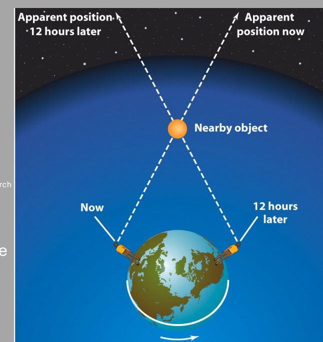


Astronomical Research Institute

Tycho Brahe
1546-1601

Wikipedia

Passion for exact empirical facts



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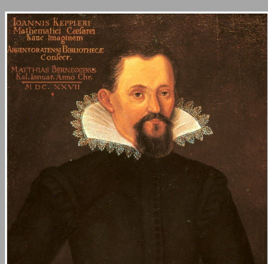
Johannes Kepler proposed elliptical paths for the planets about the Sun



- Using data collected by Tycho Brahe, Kepler deduced three laws of planetary motion:
  1. the orbits are ellipses
  2. a planet's speed varies as it moves around its elliptical orbit
  3. the orbital period of a planet is related to the size of its orbit

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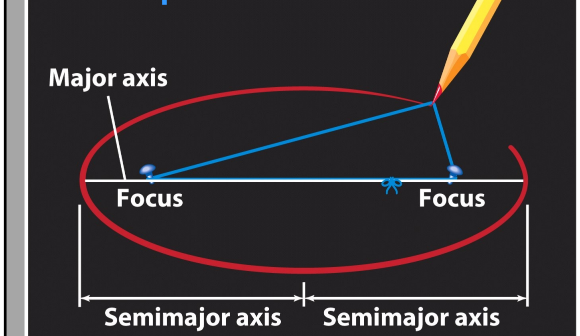
Kepler's First Law



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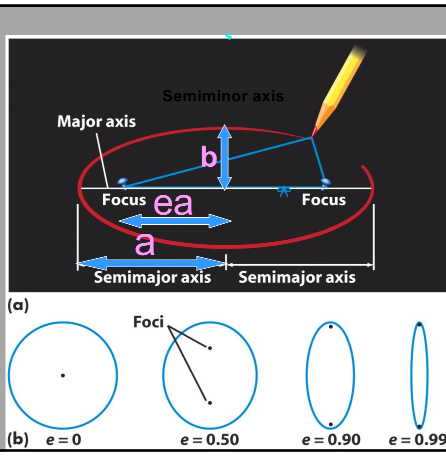

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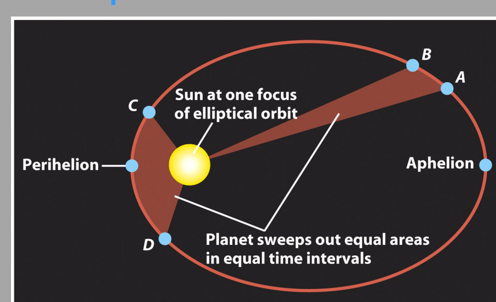
Kepler's Second Law



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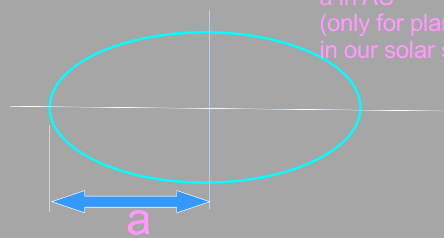
Kepler's third law

$P^2 = a^3$

P in sidereal years
a in AU
(only for planets
in our solar system)



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Kepler' s Third Law

table 4-3 A Demonstration of Kepler's Third Law ($P^2 = a^3$)

Planet	Sidereal period P (years)	Semimajor axis a (AU)	P^2	a^3
Mercury	0.24	0.39	0.06	0.06
Venus	0.61	0.72	0.37	0.37
Earth	1.00	1.00	1.00	1.00
Mars	1.88	1.52	3.53	3.51
Jupiter	11.86	5.20	140.7	140.6
Saturn	29.46	9.55	867.9	871.0
Uranus	84.10	19.19	7,072	7,067
Neptune	164.86	30.07	27,180	27,190
Pluto	248.60	39.54	61,800	61,820

$P^2 = a^3$

P = planet' s sidereal period, in years
a = planet' s semimajor axis, in AU

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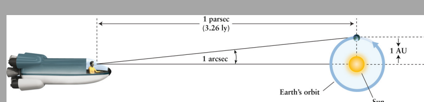

A Parsec
The parsec, a unit of length commonly used by astronomers, is equal to 3.26 ly. The parsec is defined as the distance at which 1 AU perpendicular to the observer' s line of sight makes an angle of 1 arcsec.

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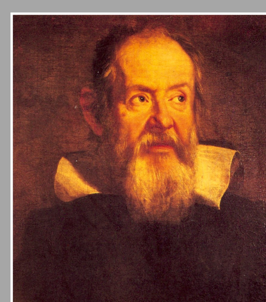
Galileo' s discoveries with a telescope strongly supported a heliocentric model



- The invention of the telescope led Galileo to new discoveries that supported a heliocentric model
- These included his observations of the phases of Venus and of the motions of four moons around Jupiter

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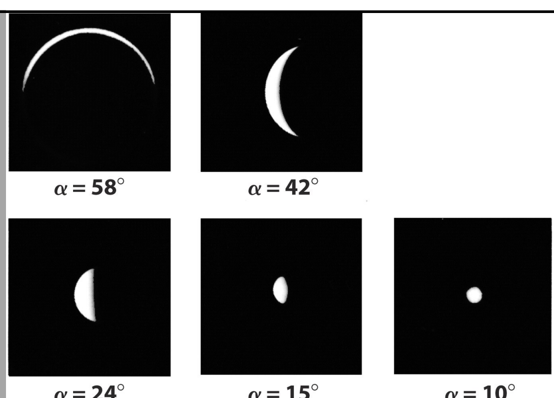

There is a correlation between the phases of Venus and the planet' s angular distance from the Sun

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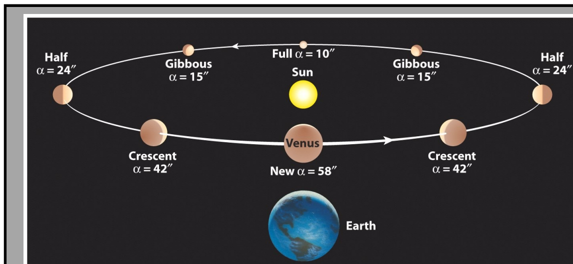

- One of Galileo' s most important discoveries with the telescope was that Venus exhibits phases like those of the Moon
- Galileo also noticed that the apparent size of Venus as seen through his telescope was related to the planet' s phase
- Venus appears small at gibbous phase and largest at crescent phase

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Isaac Newton formulated three laws that describefundamental properties of physical reality

- Isaac Newton developed three principles, called the laws of motion, that apply to the motions of objects on Earth as well as in space
- These are
  1. the ***law of inertia***: a body remains at rest, or moves in a straight line at a constant speed, unless acted upon by a net outside force
  2. ***F = m a*** (the force on an object is directly proportional to its mass and acceleration)
  3. the ***principle of action and reaction***: whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first body

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Mass vs Weight

- Mass is an intrinsic quantity and for a given object is invariant of position. It is measured in kg.
- Weight by contrast is the 'response' of mass to the local gravitational field. It is a force and measured in newtons.
- Thus while you would have the same mass on the earth and its Moon, your weight is different.
- W(eight) = m(ass) x g(ravitational acceleration)

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Newton's Law of Universal Gravitation

from Kepler's 3 laws and Newton's 3 laws

$$F = G\frac{Mm}{r^2}$$

F = gravitational force between two objects
M = mass of first object
m = mass of second object
r = distance between objects
G = universal constant of gravitation

- If the masses are measured in kilograms and the distance between them in meters, then the force is measured in newtons
- Laboratory experiments have yielded a value for *G* of
  $G = 6.673 \times 10^{-11}\ N \cdot m^2/kg^2$

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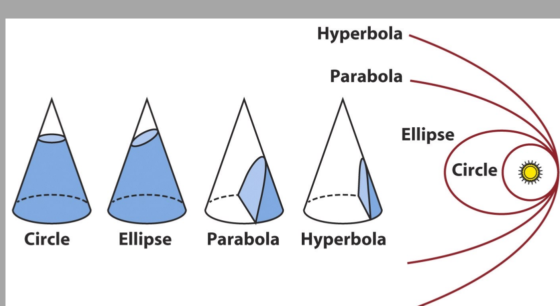
Orbits may be any of a family of curves called conic sections



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Energy

(measured in Joules, J)

- *Kinetic* energy refers to the energy a body of mass *m* has due to its speed *v*: $E_{kin} = \frac{1}{2}mv^2$.
- *For a rotating body:* $E_{kin} = \frac{1}{2}I\omega^2$

  with I, moment of inertia and L=Iω, angular momentum and ω: angular velocity.

- *Potential* energy is energy due to the position of *m* a distance *r* away from another body of mass M,

  $$E_{pot} = -G\frac{Mm}{r}$$

- Total energy, E_{tot}, is the sum of the kinetic and potential energies;

  $$E_{tot} = E_{kin} + E_{pot}$$

A body whose total energy is < 0, orbits a more massive body in a *bound*, elliptical orbit (*e* < 1).
A body whose total energy is > 0, is in an *unbound*, hyperbolic orbit (*e* > 1) and escapes to infinity.
A body whose total energy is exactly 0 just escapes to infinity in a parabolic orbit (*e* = 1) with zero velocity at the end.

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Escape velocity

- The velocity that must be acquired by a body to just escape, i.e., to have zero total energy, is called the *escape velocity*. By setting $E_k + E_p = 0$, we find:

$$v_{escape}^2 = \frac{2GM}{r}$$

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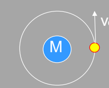
Velocity

- A body of mass *m* in a circular orbit about a (much) more massive body of mass orbits at a constant speed or the circular velocity, v_c where

$$v_c^2 = \frac{GM}{r}$$



(This is derived by equating the gravitational force with the centrifugal force, mv^2/r).

- Note that: $v_{escape}^2 = 2v_c^2$.

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Kepler's Third Law derived by Newton

$$P^2 = \frac{4\pi^2}{G(M+m)} a^3$$

P = Sidereal orbital period (seconds, s)
a = Semi-major axis planet orbit (meters, m)
M, m = mass of objects (kilograms, kg)
G = Gravitational constant : 6.673×10^{-11} Nm²/kg²

For M>>m: $P^2 = \frac{4\pi^2}{GM} a^3$

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Example: What is the mass of the Sun?

$M+m = 4\pi^2 a^3 / GP^2$

$= 4\pi^2 (1.5\ 10^{11})^3 / (6.673\ 10^{-11} (365.24 \cdot 24 \cdot 60 \cdot 60)^2)$

$= 2.0 \cdot 10^{30}$ kg

Since mass, m, of Earth is negligible, this is the mass of the sun, $M_{sun} = M + m \cong M$.

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Example: What is your weight on the Moon?

weight=force
Force=mass x acceleration
F=ma
Force is also the gravitational force
$F=GMm/r^2$
We need the acceleration, a, for the Moon to calculate your weight on the Moon
$ma= GMm/r^2$
$a= GM/r^2$

$M= 7.345 \times 10^{22}$ kg (mass of Moon)
r=1737 km (radius of Moon)
$a=6.673 \times 10^{-11} \times 7.345 \times 10^{22} / (1.737 \times 10^6)^2$
$a=1.624$ m/s² (acceleration on the Moon)

If your mass is 100 kg then your weight on the Moon is:
F= 100x1.624 =162.4 N

On Earth the acceleration is 9.81 m/s²
F=100x9.81 = 981 N
So your weight on the Moon is 162.4/981 = 0.166 times your weight on Earth

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Example: What is the escape velocity for Earth?

$v^2_{escape} = 2GM/r$
$M = 5.972 \times 10^{24}$ kg (mass of Earth)
r=6371 km (radius of Earth)

$v^2_{escape} = 2 \times 6.673 \times 10^{-11} \times 5.972 \times 10^{24} / 6.371 \times 10^3 = 11185$ m/s
$V_{escape} = 11185$ m/s = 11.185 km/s

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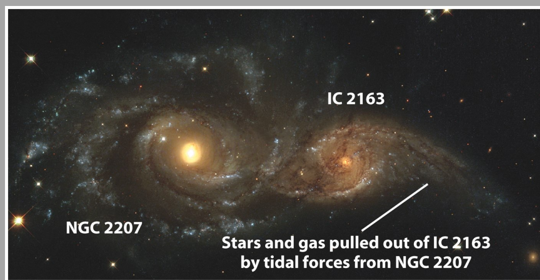
Gravitational forces between two objects produce tides



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Frontiers yet to be discovered

1) Why is the inertial mass in F=ma equal to the gravitational mass in $F=GmM/r^2$?

2) Newton's law of gravitation is not quite accurate as can be shown with precision measurements.

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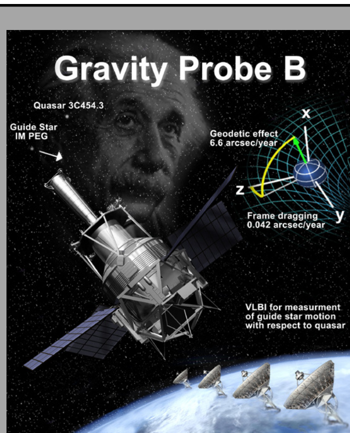

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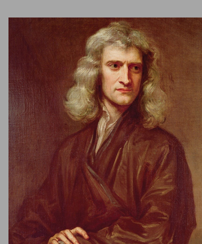
Isaac Newton (1642–1727)
Newton delighted in constructing mechanical devices, such as sundials, model windmills, a water clock, and a mechanical carriage. He received a bachelor's degree in 1665 from the University of Cambridge. While there, he began developing the mathematics that later became calculus (developed independently by the German Gottfried Leibniz). While pursuing experiments in optics, Newton constructed a reflecting telescope and also discovered that white light is actually a mixture of all colors. His major work on forces and gravitation was the tome *Philosophiae Naturalis Principia Mathematica*, which appeared in 1687. In 1704, Newton published his second great treatise, *Opticks*, in which he described his experiments and theories about light and color. Upon his death in 1727, Newton was buried in Westminster Abbey, the first scientist to be so honored.



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NEWTON'S THREE LAWS OF MOTION

- **Newton's first law**—the law of inertia: Inertia is the property of matter that keeps an object at rest or moving in a straight line at a constant speed unless acted upon by a net external force.
- **Newton's second law**—the force law: The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass.
- **Newton's third law**—the law of action and reaction: Whenever one object exerts a force on a second object, the second object exerts an equal and opposite force on the first object.

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Conservation of Angular Momentum
As this skater brings her arms and outstretched leg in, she must spin faster to conserve her angular momentum.

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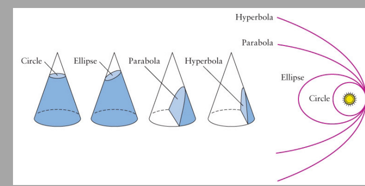

Conic Sections
A conic section is any one of a family of curves obtained by slicing a cone with a plane, as shown. The orbit of one body around another can be an ellipse, a parabola, or a hyperbola. Circular orbits are possible because a circle is just an ellipse for which both foci are at the same point.

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Halley's Comet
Halley's Comet orbits the Sun with an average period of about 76 years. During the twentieth century, the comet passed near the Sun twice—once in 1910 and again, as shown here, in 1986. The comet will pass close to the Sun again in 2061. Although dim in 1986, it nevertheless spread more than 5° across the sky, or 10 times the diameter of the Moon.

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Insight into Science

Quantify Predictions

Mathematics provides a language that enables science to make quantitative predictions that can be checked by anyone. For example, in this chapter, we have seen how Kepler's third law and Newton's universal law of gravitation correctly predict the motion of objects under the influence of the Sun's gravitational attraction.

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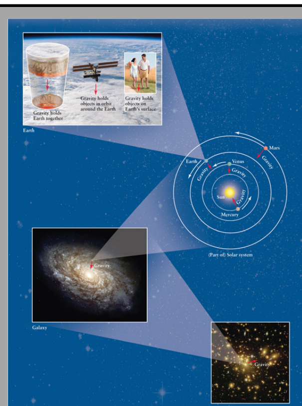
Gravity Works at All Scales
This figure shows a few of the effects of gravity here on Earth, in the solar system, in our Milky Way Galaxy, and beyond.



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