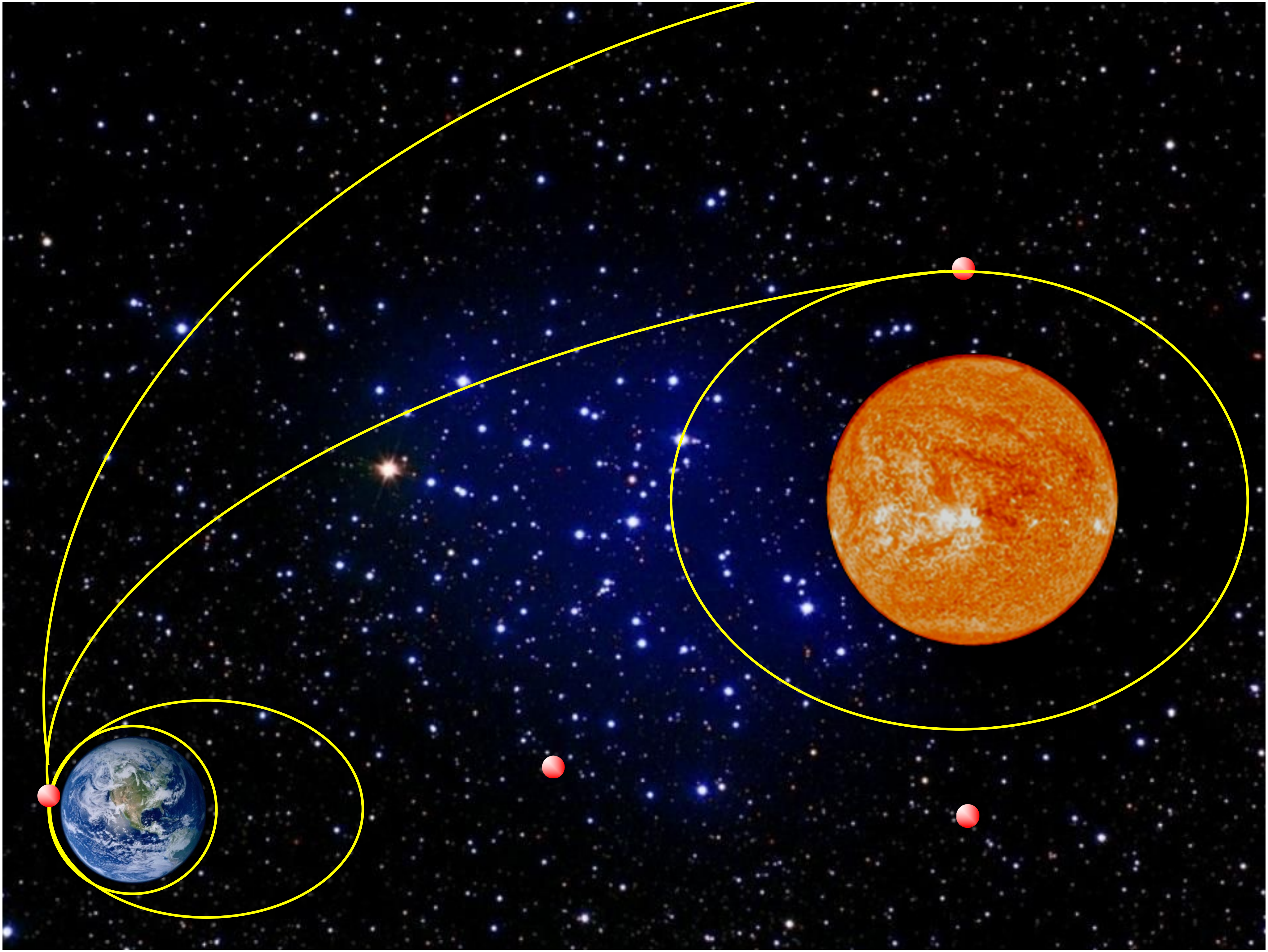
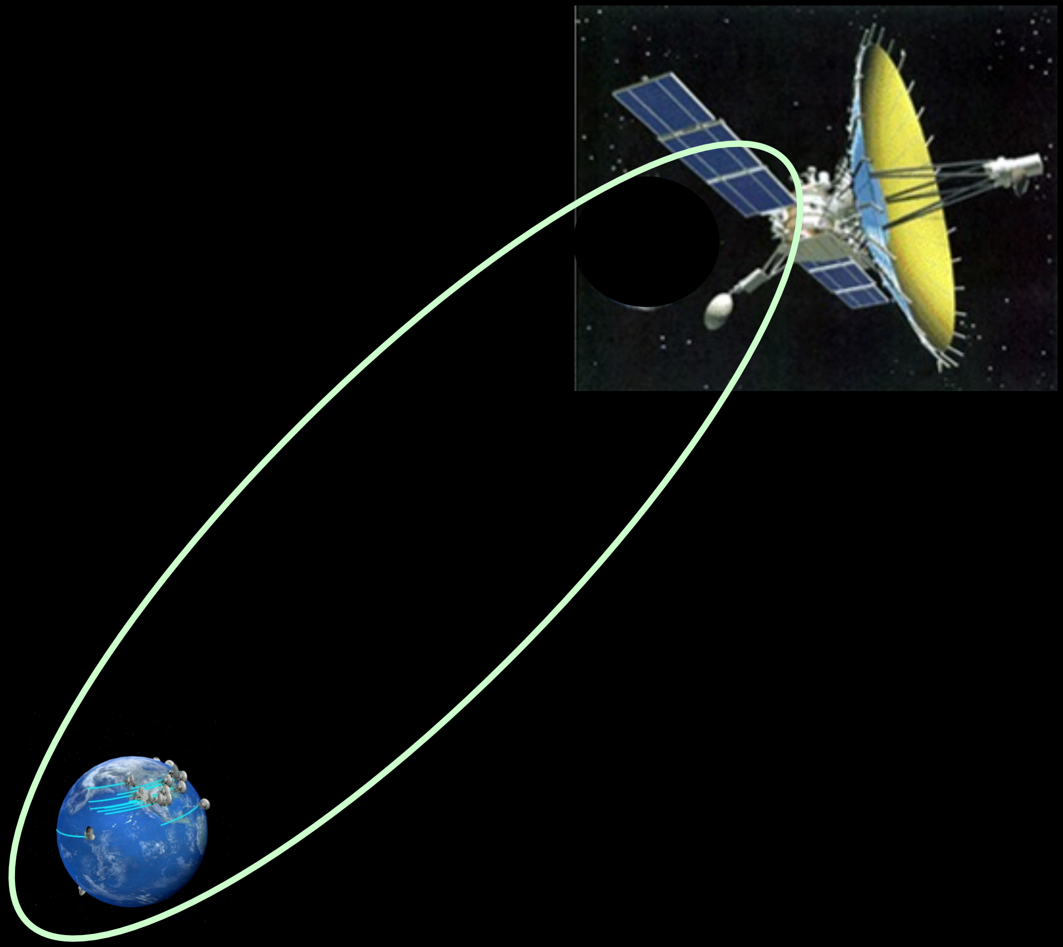
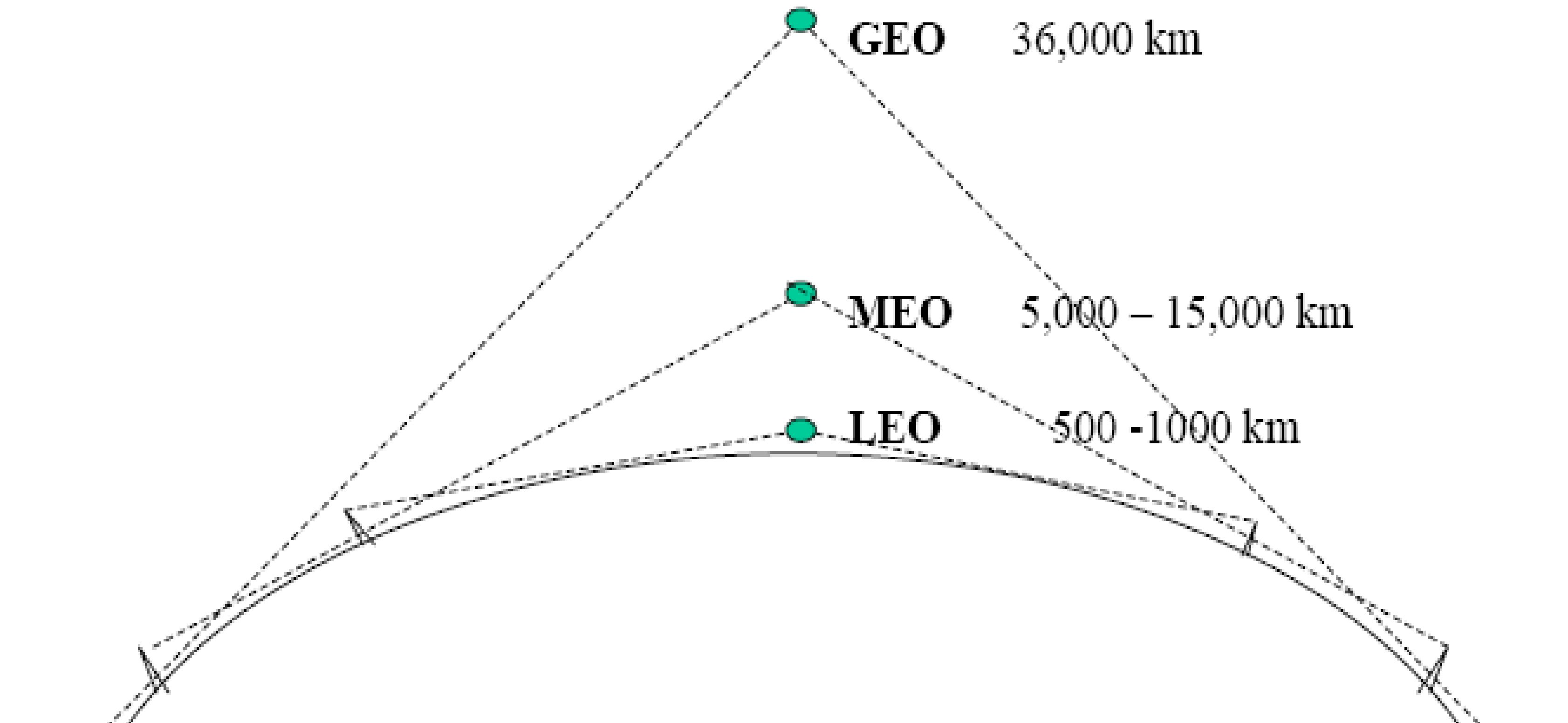


2.Orbital aspects of Satellite Communications
2.1 Orbits

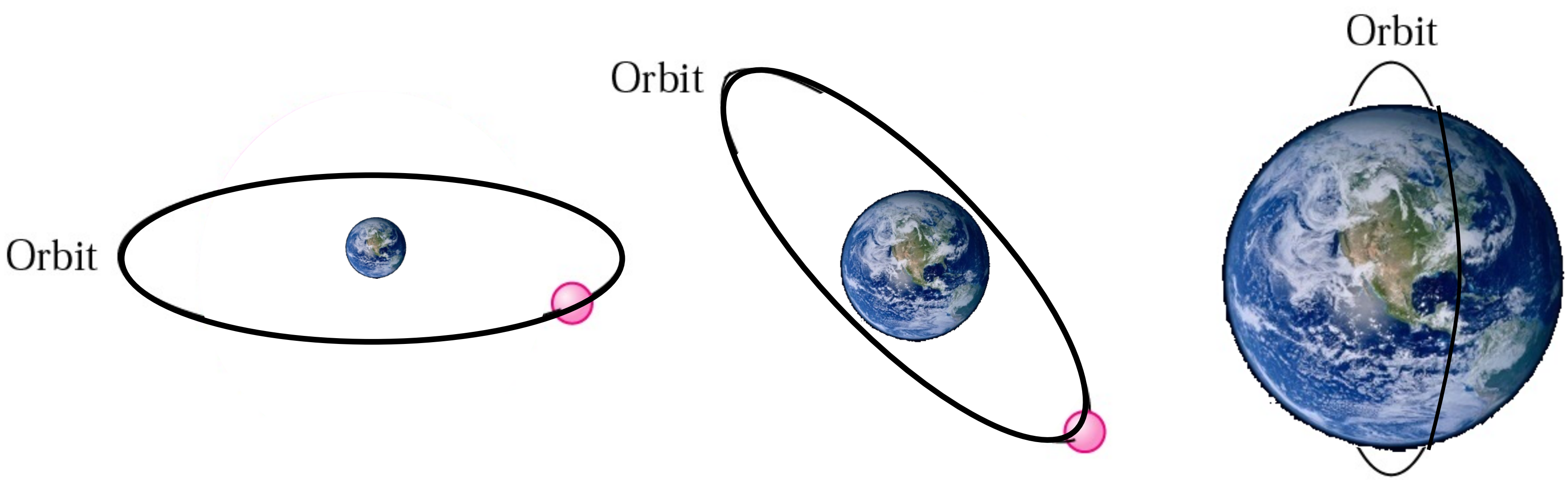


Satellite Orbits overview



Orbit Type	Height	Period
LEO - Low Earth Orbit	500-1000 Km	1.6 to 1.8 hrs
MEO - Medium Earth Orbit	8,000 to 12,000 km	approx. 6 hrs (at 10,000 km).
GEO - Geostationary	approx. 36,870 km	approx. 24 hrs

Three basic satellite orbits



a. Equatorial-orbit satellite

b. Inclined-orbit satellite

c. Polar-orbit satellite

Example: Geostationary orbit

GPS

Weather satellites and Sarsat

Advantages and disadvantages

- Satellites in geostationary orbit:
-
- *Advantages:*
- Satellite remains stationary, no tracking equipment for earth station necessary
- Satellite is visible 24h per day
- Large coverage area: large number of earth stations can communicate
- Almost no Doppler shift – keeps electronics simple
-
- *Disadvantages:*
- Latitudes north of 81.5 deg are not covered
- Great distance – received signal is weak
- Launch cost higher than for low altitude orbits
- Only one geostationary orbit is possible
-
-
- Satellites in inclined orbits:
-
- Molniya series i=65 deg, P=12h - provides communication services to the northern regions of Russia
- Military satellites
- Global Navigation Satellite Systems, e.g. GPS satellites, i=63 deg, P=12h, 3 orbital planes oriented at 120 deg angles w.r.t. each other, 3 x 8 satellites, at least 6 satellites are visible from any point on earth at any given time.
-
-
- Satellites in polar orbits:
-
- Tiros – N series (historical: 1960 – 1966)
- NOAA, P=102 min, altitude about 800 km. Also provides SARSAT service
-
- SARSAT service
- Based on utilization of Doppler shift and known satellite orbit
-
-

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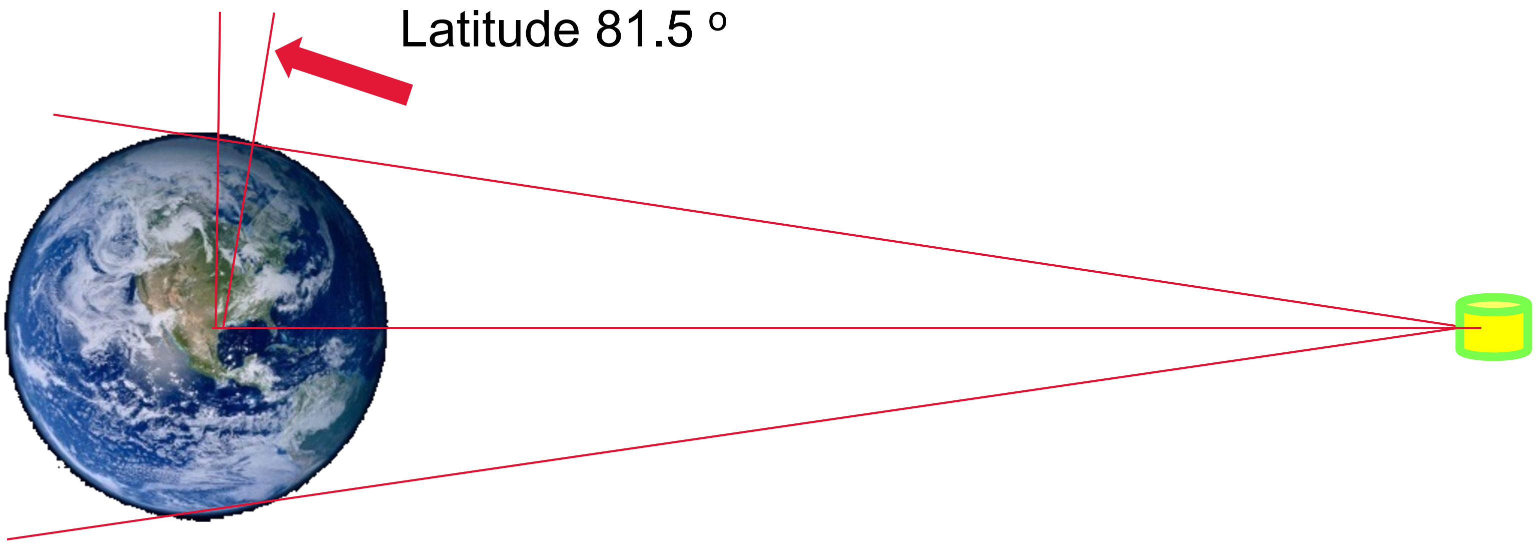
Geostationary satellite

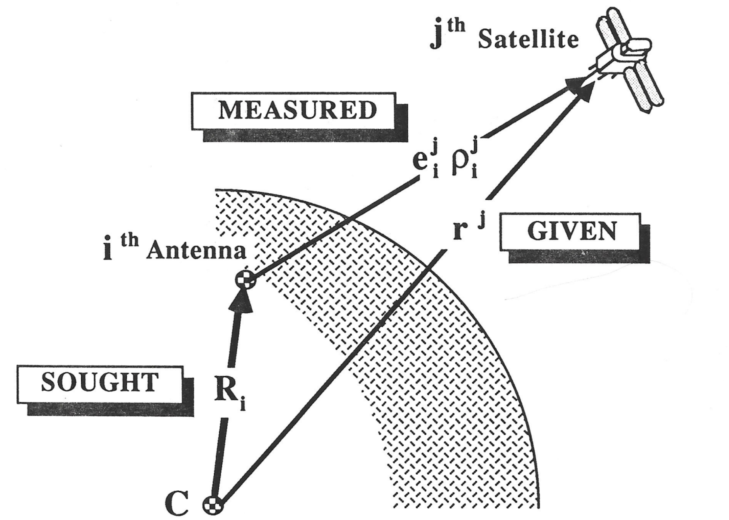
- .



Global Navigation Satellite Systems

1.07

BASIC CONCEPT OF SATELLITE POINT POSITIONING



$$R_i = r^j - e_i^j \rho_i^j$$

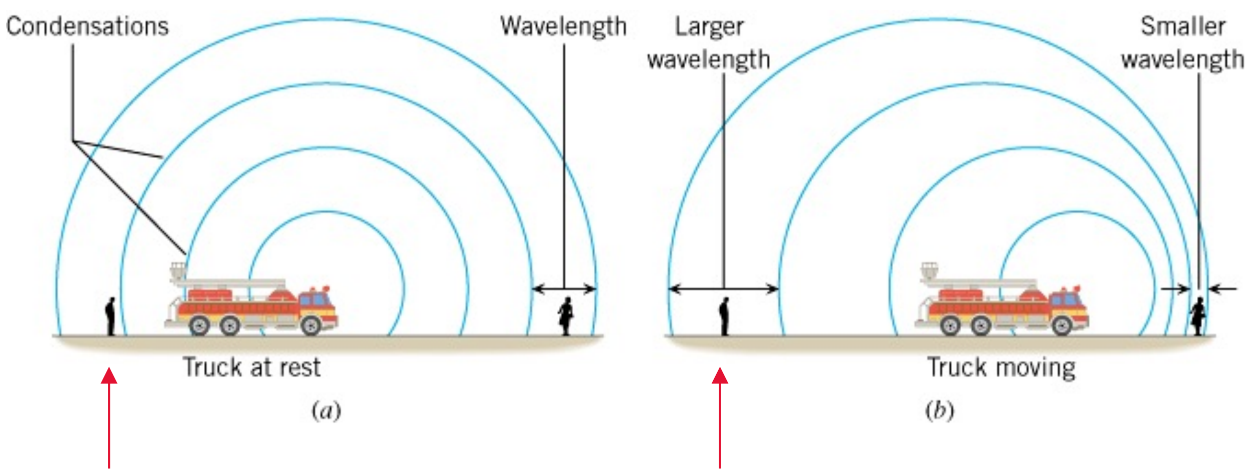
R_i is the position vector of the i^{th} antenna

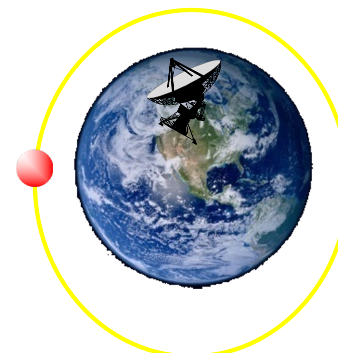
r^j is the position vector of the j^{th} satellite

$e_i^j \rho_i^j$ is the range vector between the two.

© Canadian GPS Associates, May 1987

The Doppler Effect





f_0

$$f = f_0\left(1 - \frac{v}{c}\right).$$

f, f_0 : frequency
v: velocity
c: speed of light

Increasing frequency: blueshift
Decreasing frequency: redshift

$$\Delta f = |f - f_0| = \frac{v}{c} f_0 \Rightarrow \frac{\Delta f}{f_0} = \frac{v}{c} \Rightarrow \Delta f = v/\lambda$$

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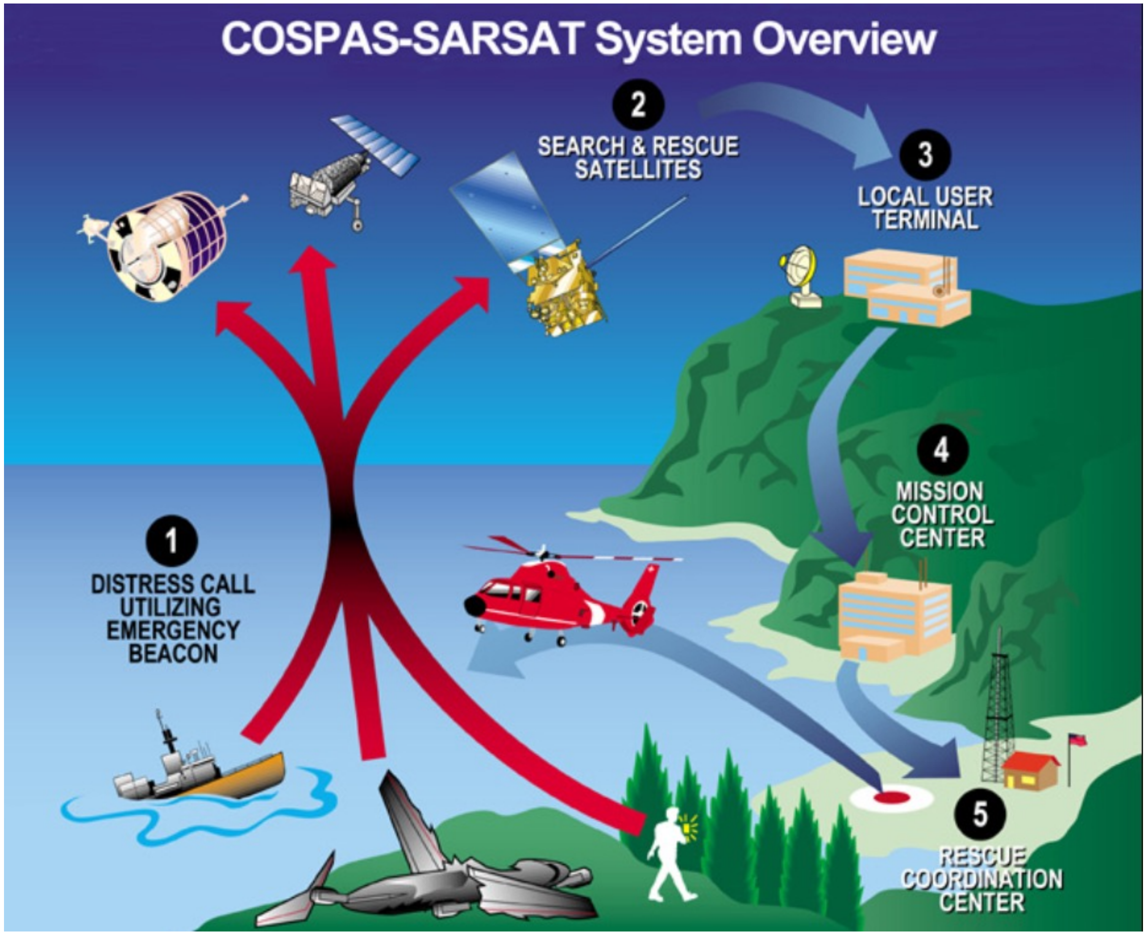
COSPAS-SARSAT

Search and Rescue satellite

Sponsored by Canada, France, Russia and US



Emergency Locator Radio Beams

http://www.equipped.com/cospas-sarsat_overview.htm,

https://www.sarsat.noaa.gov/

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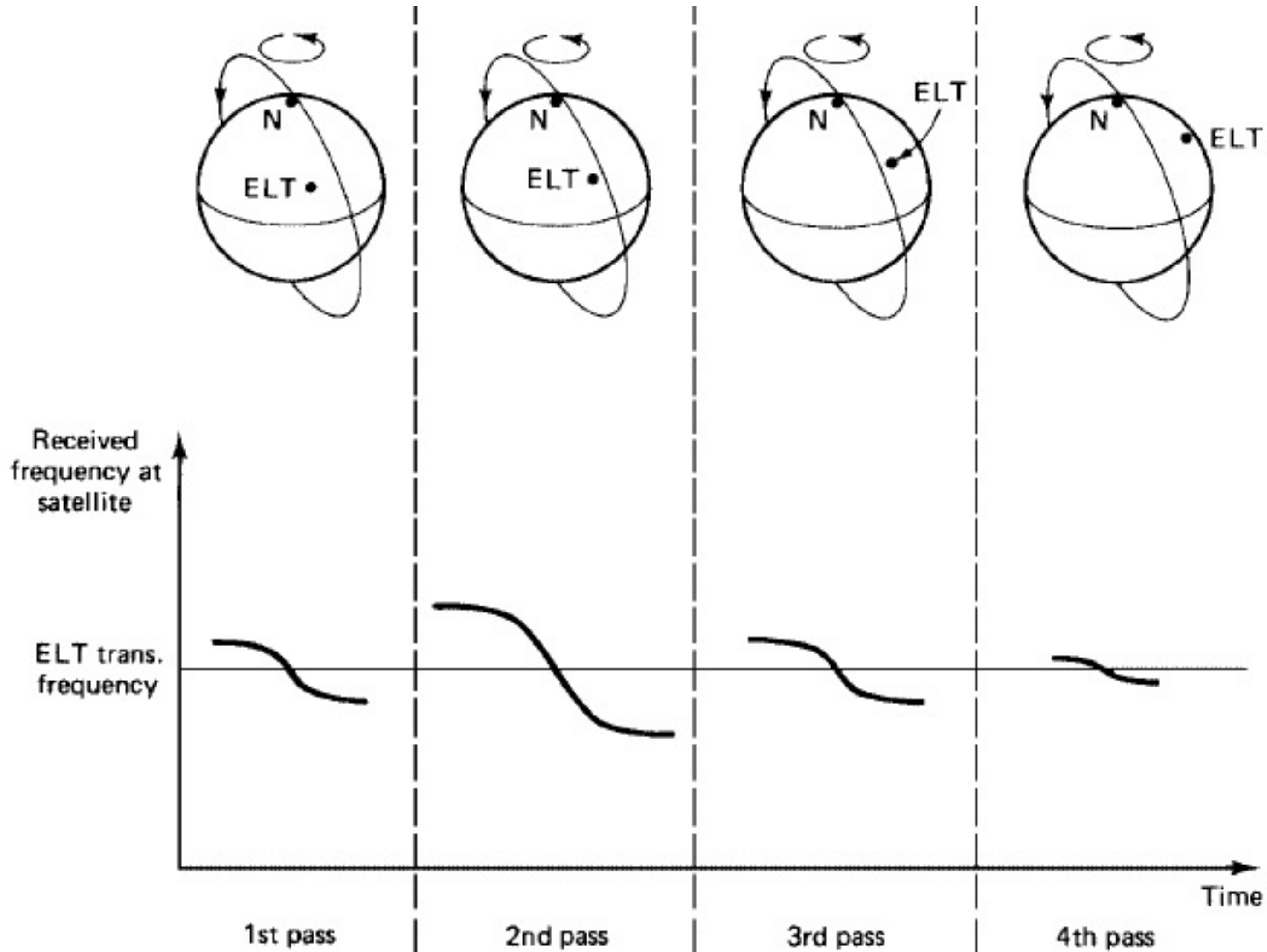
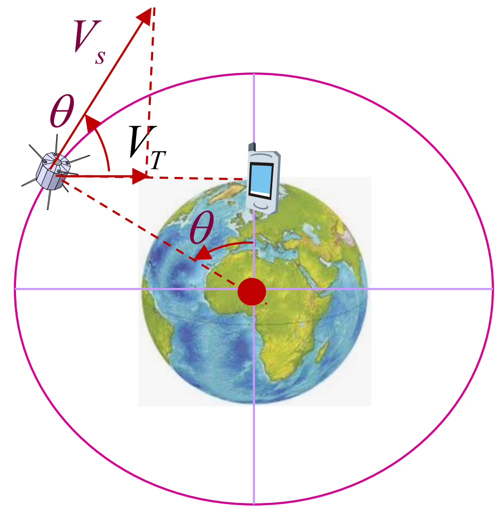
Location determination using Doppler processing



Example 2-1

Q: A SARSAT satellite is in a LEO at height 1450 km and has an orbital velocity of v_S =7.1358 km s^{-1}. Below the orbit is an emergency locator from a person in distress transmitting at f_0= 406 MHz. The projected velocity is v_T .What is the frequency the satellite receives at the time corresponding to the sketch in the Figure. The radius of Earth =6378.137 km.



A:

$$V_T = V_s \cos\theta$$
$$= 7.1358 \times \frac{6378.137}{6378.137 + 1450}$$

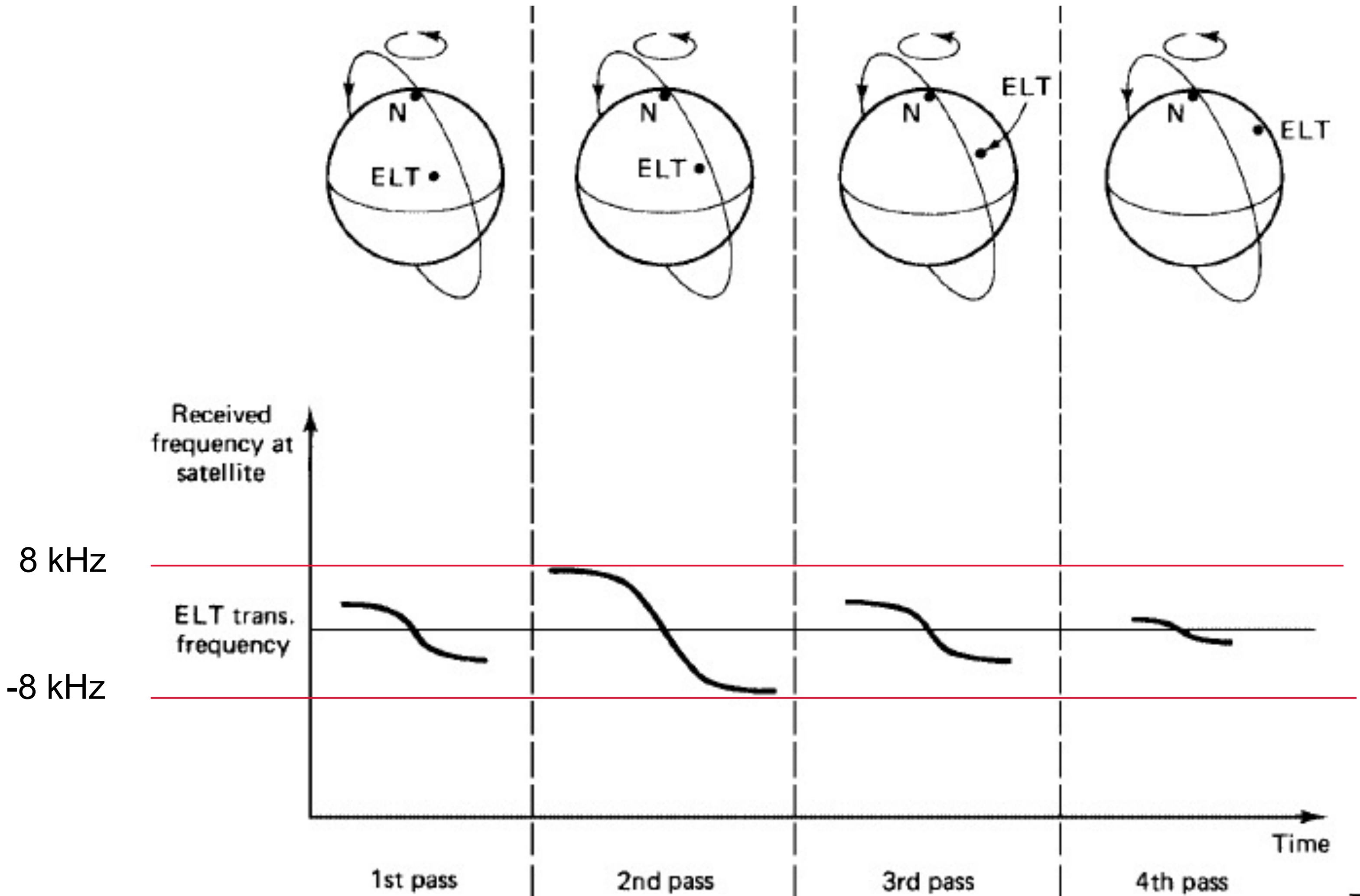
=5.8140 km s^{-1}

$$\frac{\Delta f}{f_0} = \frac{v}{c}$$ =5.8140/(3.0x10^5)

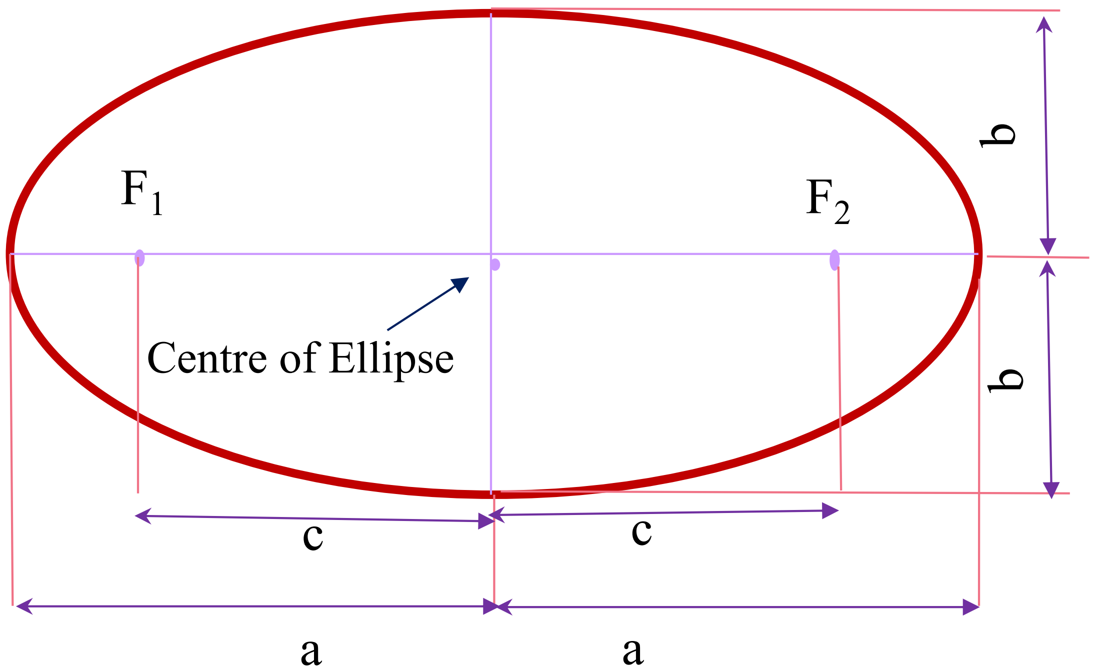
=0.00001938

Δf=7.868 kHz

Location determination using Doppler processing



Ellipse



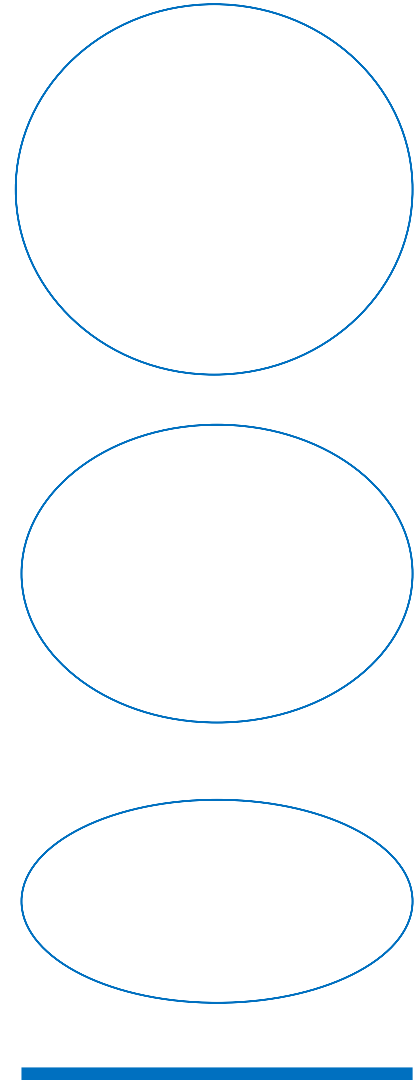
a: semimajor axis
b: seminor axis
e: eccentricity

$$c^2 = a^2 - b^2$$

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

Ellipses with different eccentricities

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$



$$e = 0, \quad a = b$$

$$e = 0.5, \quad \left(\frac{b}{a}\right)^2 = 1 - 0.25 = 0.75, \quad \frac{b}{a} \approx 0.85$$

$$e = 0.8, \quad \left(\frac{b}{a}\right)^2 = 1 - 0.64 = 0.36, \quad \frac{b}{a} \approx 0.6$$

$$e = 1, \quad b = 0$$

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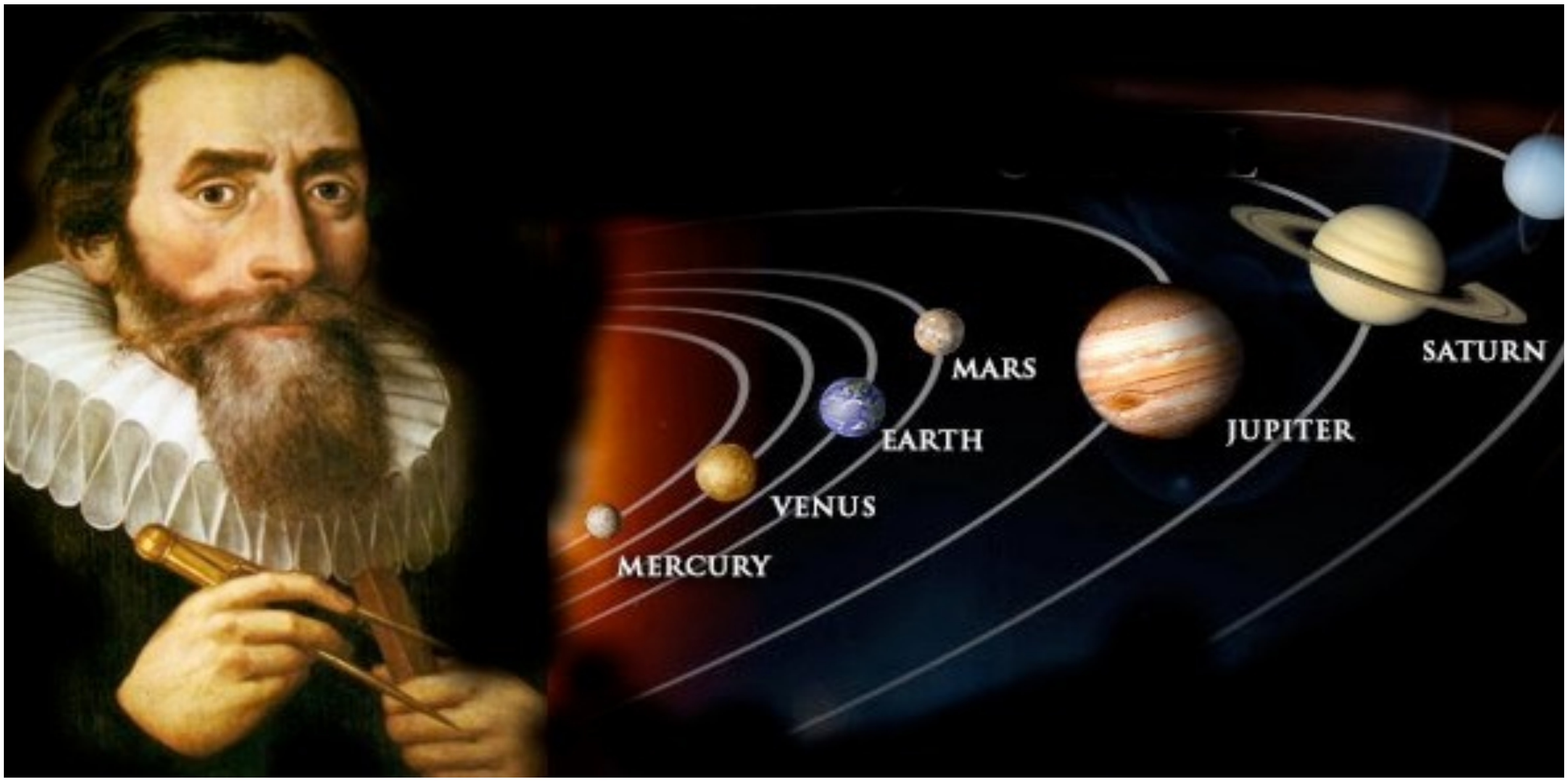
Johannes Kepler (1571-1630)

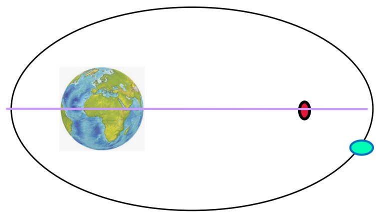


Wikipedia

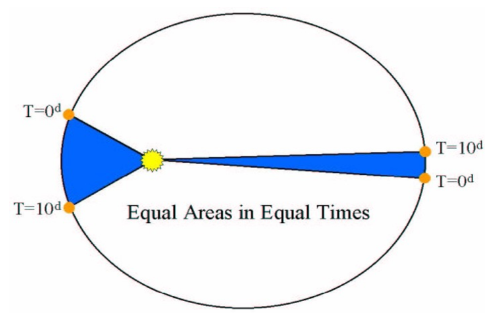
Kepler's Laws

as applied to satellites

- First Law: The orbit of each satellite is an ellipse with the Earth at one focus.



- Second Law: A satellite moves in such a way that a line drawn from the Earth to the satellite sweeps out equal areas in equal intervals of time.



- Third Law: The square of the orbital period, P, of a satellite is directly proportional to the cube of the semimajor axis of the orbit

$$P^2 = ka^3 \qquad k = \text{const}$$

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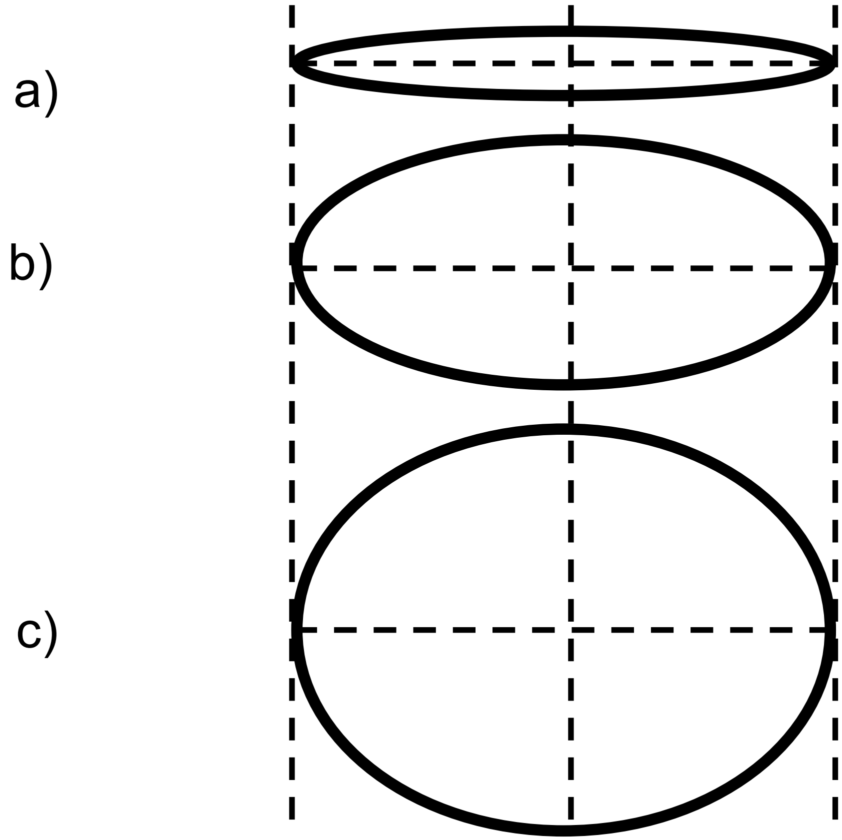
Example 2-2

- Q: The satellite on which orbit has the longest orbital period?



Isaac Newton (1642-1727)

Wikipedia

During the Great Plague of London,
That began in 1665,
Newton started his groundbreaking discoveries.

Newton's Universal Law of Gravitation

$$\vec{F} = G\frac{Mm}{r^2}\frac{\vec{r}}{r}$$

Gravitational constant
$G = 6.6726 \times 10^{-11}$ N m^2 kg^{-1}

Newton's 2nd Law of Motion

$$\vec{F} = m\ddot{\vec{r}}$$

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Weight and escape velocities

$\vec{F} = G\frac{Mm}{r^2}\frac{\vec{r}}{r}$ ⇒ **Weight: $W = mg$, $g = G\frac{M}{r^2}$**

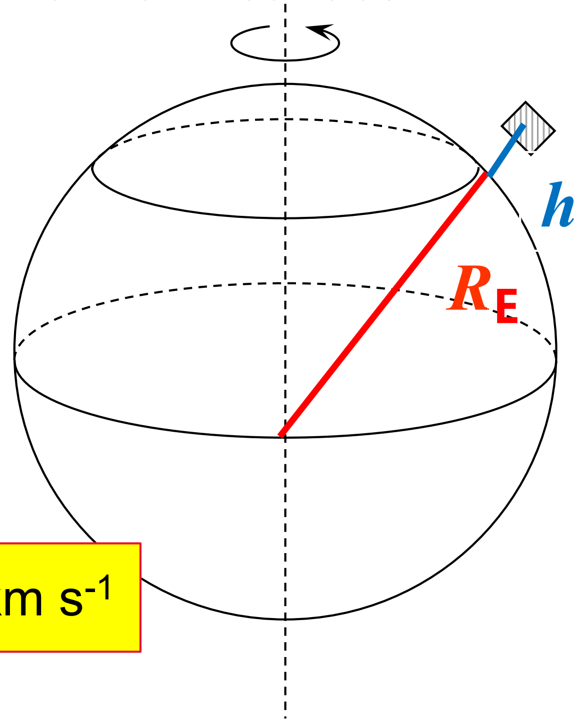
$M = 5.98\ 10^{24}$ kg

$GM = 3.986005 \cdot 10^{14}\ m^3\ s^{-2}$

$= 3.986005 \cdot 10^{5}\ km^3\ s^{-2}$

$R_e = 6.37814 \cdot 10^{3}$ km (at equator)

$g = 9.81\ m\ s^{-2}$ for Earth surface



$F_c = \frac{mv^2}{r}$ (centrifugal force)

$F_g = \frac{GMm}{r^2}$ (gravitational force)

$F_c = F_g$ ⇒ orbital velocity $v^2 = \frac{GM}{r}$

Orbital velocity around Earth at $r=R_E$: $v_{orb} = (\frac{GM}{R_e})^{1/2} = 7.905\ km\ s^{-1}$

$(E_{kin} + E_{pot})_{init} = (E_{kin} + E_{pot})_{fin}$

$$\frac{mv^2}{2} - \frac{GMm}{r} = 0+0$$

Escape velocity from Earth: $v_{esc} = (\frac{2GM}{R_e})^{1/2} = 11.180\ km\ s^{--1}$

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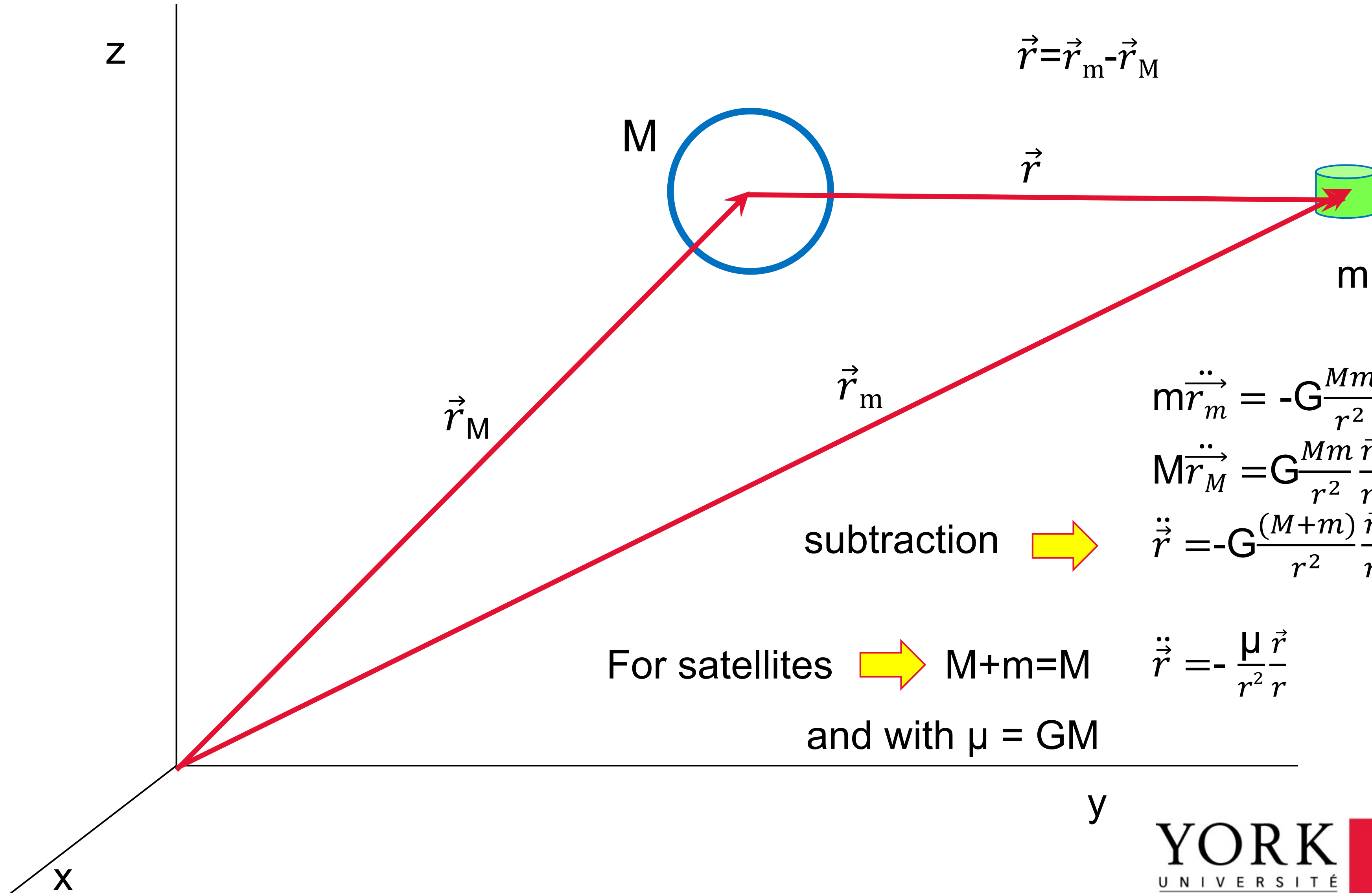
Two-body problem



$$\vec{r} = \vec{r}_m - \vec{r}_M$$

$$m\ddot{\vec{r}}_m = -G\frac{Mm}{r^2}\frac{\vec{r}}{r}$$

$$M\ddot{\vec{r}}_M = G\frac{Mm}{r^2}\frac{\vec{r}}{r}$$

subtraction ⇨ $\ddot{\vec{r}} = -G\frac{(M+m)}{r^2}\frac{\vec{r}}{r}$

For satellites ⇨ M+m=M $\quad \ddot{\vec{r}} = -\frac{\mu}{r^2}\frac{\vec{r}}{r}$

and with μ = GM

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This is a fundamental differential equation used in the study of artificial satellites.

- It is a 2nd order vector linear differential equation.
- The solution will involve 6 constants, 2 for each coordinate
- The constants are called ***orbital elements*** of ***Keplerian elements*** or ***Keplerian orbital elements***

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Characteristics of the solution

$\vec{r} \times \dot{\vec{v}} \quad = -\frac{\mu}{r^3}(\vec{r} \times \vec{r})$ ⇨ $= 0$

$\frac{d}{dt}(\vec{r} \times \vec{v}) = \vec{r} \times \dot{\vec{v}} + \dot{\vec{r}} \times \vec{v}$

$= \vec{r} \times \dot{\vec{v}} + \dot{\vec{r}} \times \dot{\vec{r}}$ ⇨ $= 0 + 0$

$\vec{r} \times \vec{v} \quad = \vec{h}$ = constant vector ⇨ $\vec{h} \perp$ to $\vec{r}$ *and* $\vec{v}$

Taking the scalar product of both sides with $\vec{r}$, we get:

$(\vec{r} \times \vec{v}) \cdot \vec{r}$ ⇨ $= 0$

since $\vec{r} \times \vec{v}$ is perpendicular to $\vec{r}$ and since the scalar product of two perpendicular vectors =0

⇨ $\vec{h} \cdot \vec{r} \quad = 0$

Conclusion

All the motion takes place:

- In a plane that is swept out by $\vec{r}$
- Through the origin
- Perpendicular to $\vec{h}$

⇨ Problem of motion in 3 dimensions reduces to a 2-dimensional problem of motion (motion in a plane) and to the problem of orienting the plane in space.

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Keplerian elements

The position of a satellite in space is given at any time by a set of six Keplerian elements:

Shape of the ellipse

a: semimajor axis

e: eccentricity

Timetable with which the satellite orbits Earth

ν : true anomaly at epoch, defines where the satellite is within the orbit with respect to the perigee. There are two other anomalies, M, mean anomaly and E, eccentric anomaly. For circular orbit => $M=\nu$.

Orientation of the ellipse in the orbital plane

ω: argument of perigee, i.e., the geocentric angle measured from the ascending node to the perigee in the orbital plane in the direction of the satellite's motion.

Orientation of the orbital plane in space

i: inclination of the orbital ellipse. It is the angle measured from the equatorial plane to the orbital plane at the ascending node going from east to north.

Ω: right ascension (RA) of the ascending node, i.e., the geocentric angle measured from the vernal equinox to the ascending node in the equatorial plane eastward.

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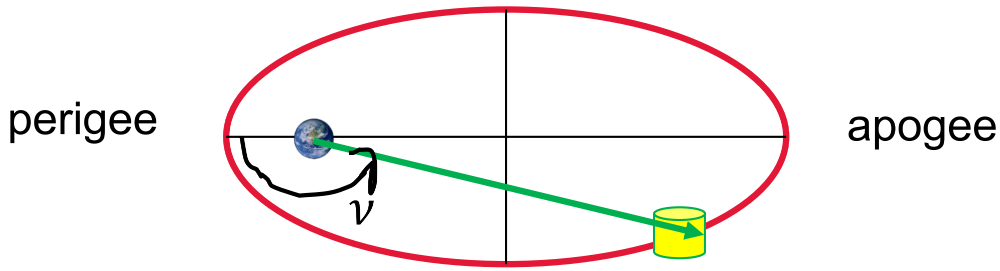
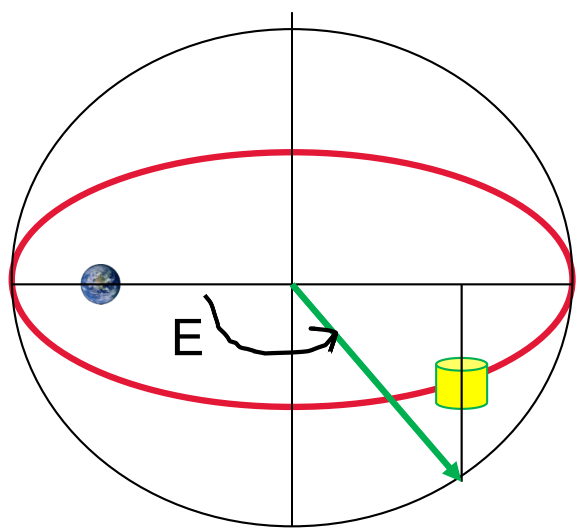
Three anomalies

- ν: true anomaly: geocentric angle measured from perigee to the satellite in the orbital plane in the direction of the satellite's motion.



- **M**: mean anomaly: $M = n(t - t_p)$; n: mean motion, t_p: time of perigee crossing

- **E**: Eccentric anomaly: angle measured at the orbit center from perigee to the satellites projection on a circle with radius a



All three anomalies are related through Kepler's and Gauss' equations

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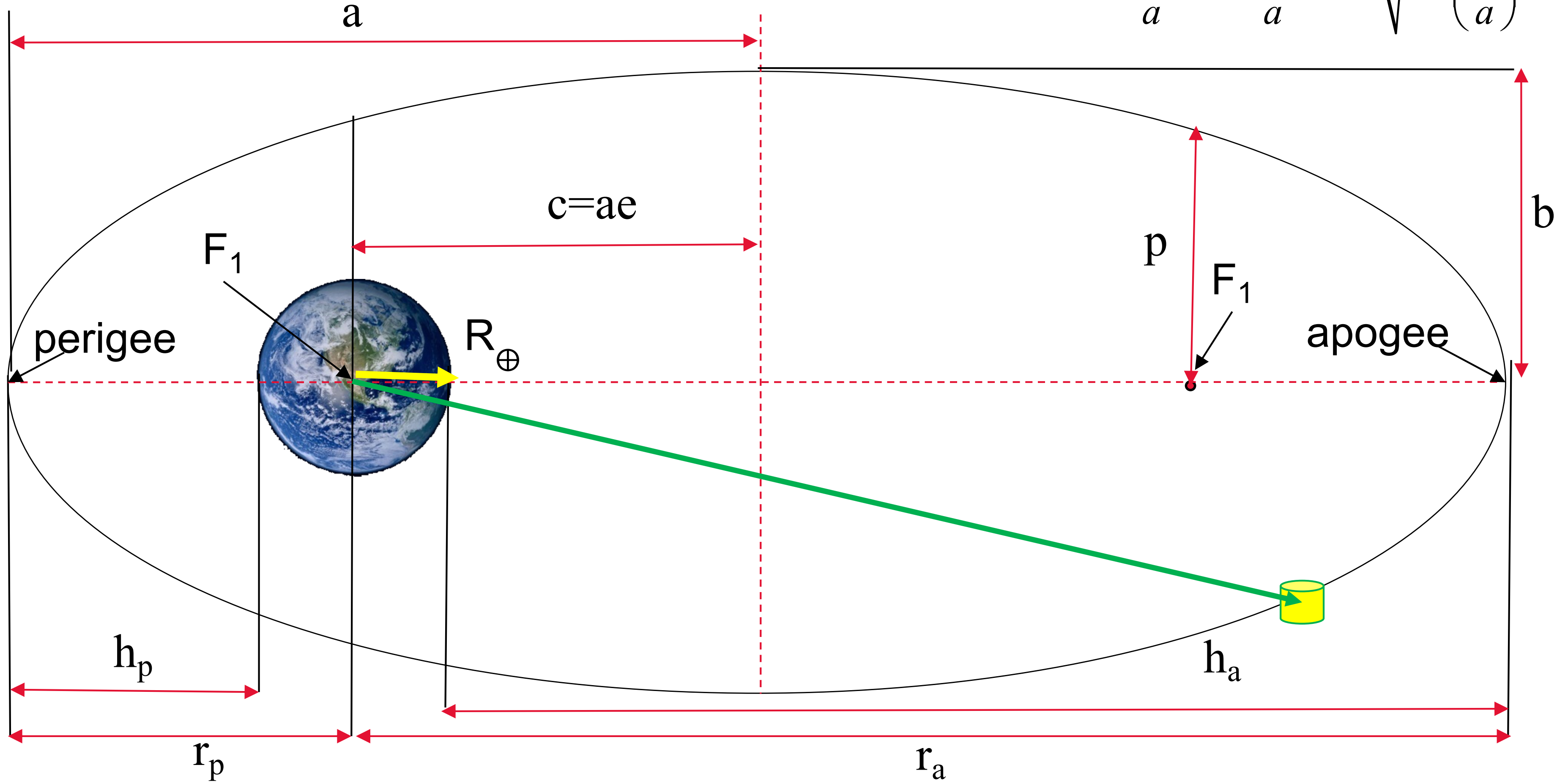
Graphical description of the ellipse

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$



$h_a = r_a - R_\oplus$ apogee height

$h_p = r_p - R_\oplus$ perigee height

$r_a = a(1+e)$ length of radius vector to apogee

$r_p = a(1-e)$ length of radius vector to perigee

$r = \frac{a(1-e^2)}{1+e\cos\nu}$ polar equation of the ellipse

$p = a(1 - e^2)$ semilatus rectum

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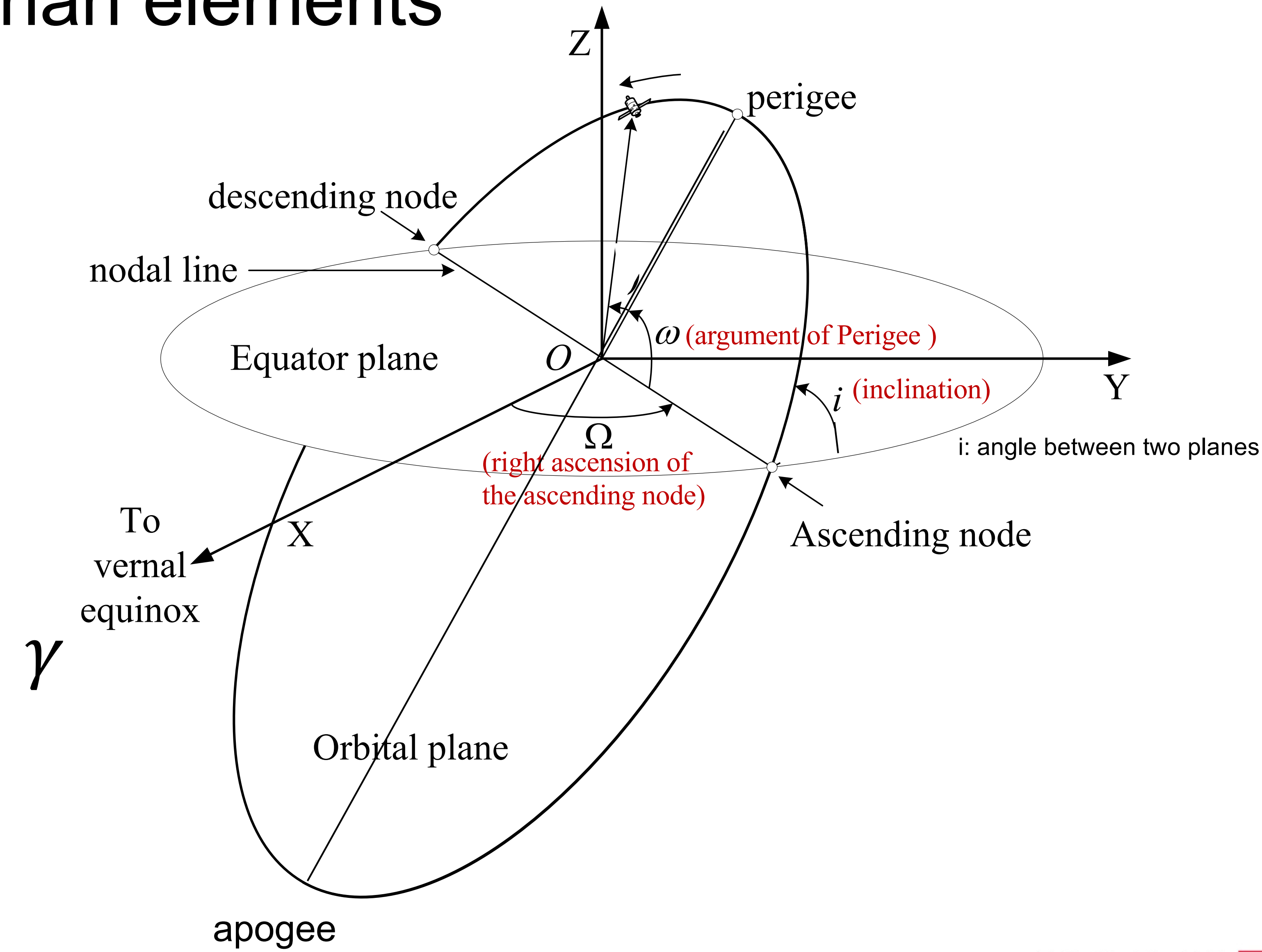
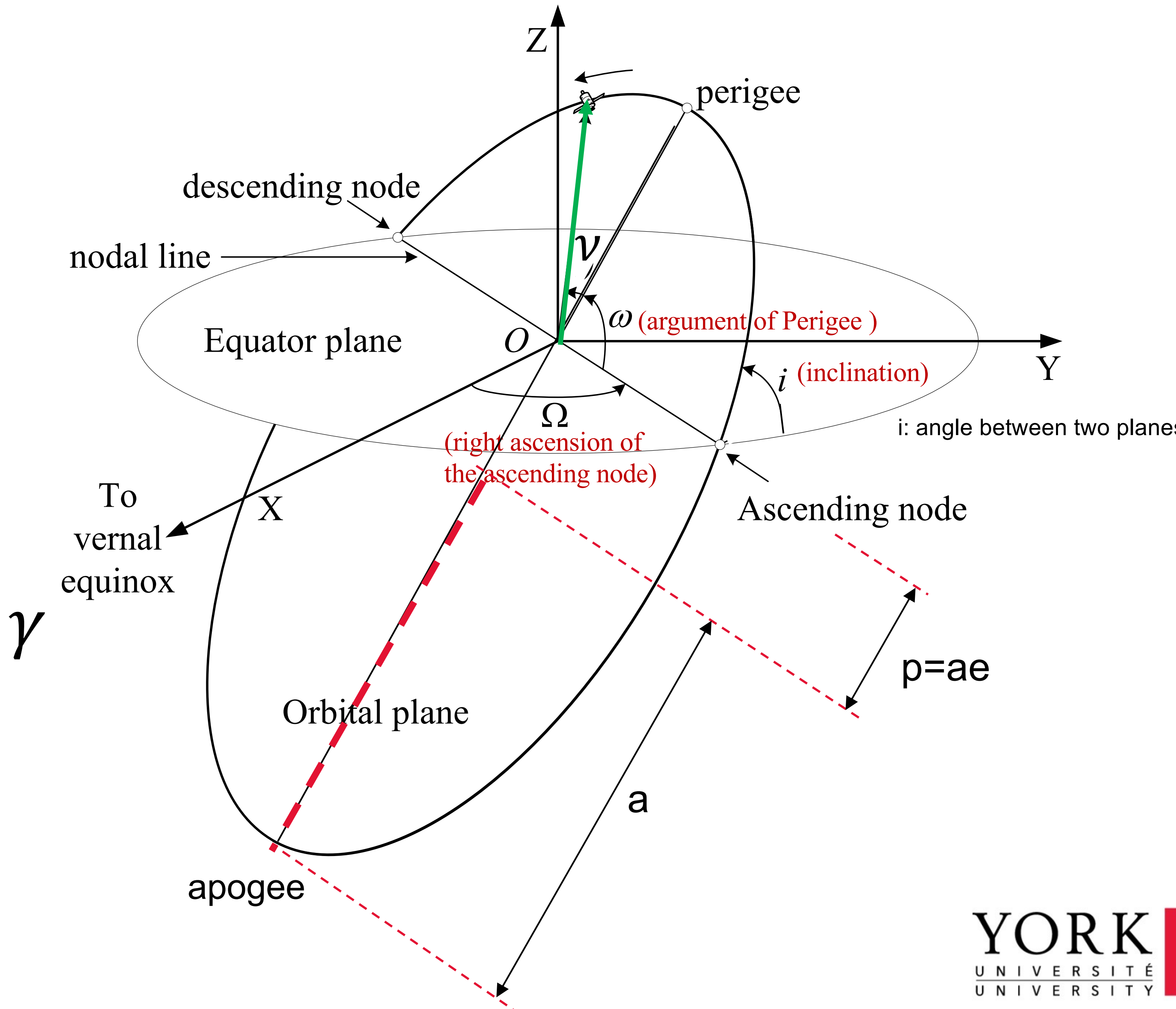
Keplerian elements



Keplerian elements

a, e, ν, i, Ω, ω



Other parameters used instead of "a"

- **P:** orbital period

$$v^2 = r^2\omega^2$$

$$G\frac{Mm}{r^2} = \mathrm{ma_c} = \mathrm{m}\left(\frac{v^2}{r}\right) = \mathrm{m}r\omega^2$$

a_c : acceleration in this case

$$G\frac{M}{r^3} = \omega^2 = \left(\frac{2\pi}{P}\right)^2 \qquad \frac{P^2}{r^3} = \frac{4\pi^2}{GM}$$

$$P^2 = \frac{4\pi^2}{GM}a^3$$

r=a (semimajor axis length)

- **n:** mean motion

$$n = \frac{2\pi}{P} = \sqrt{\frac{GM}{a^3}} = \sqrt{\frac{\mu}{a^3}}$$

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Example 2-2

Q: What is the period, P, velocity, v, and mean motion, n, of a geostationary satellite with a distance from the center of Earth of r=42164.17 km?

r=a

$P^2 = \frac{4\pi^2}{GM} a^3$ P= $\left(\frac{4\pi^2}{3.986005 \cdot 10^{14}} 4.216417 \cdot 10^7\right)^{1/2}$ = 86164.01 s

P= 23h 56m 04.0s

$v^2 = \frac{GM}{a}$ $v = \left(\frac{3.986005 \cdot 10^{14}}{4.216417 \cdot 10^7}\right)^{1/2}$ $= 3.07466$ km s^{-1}

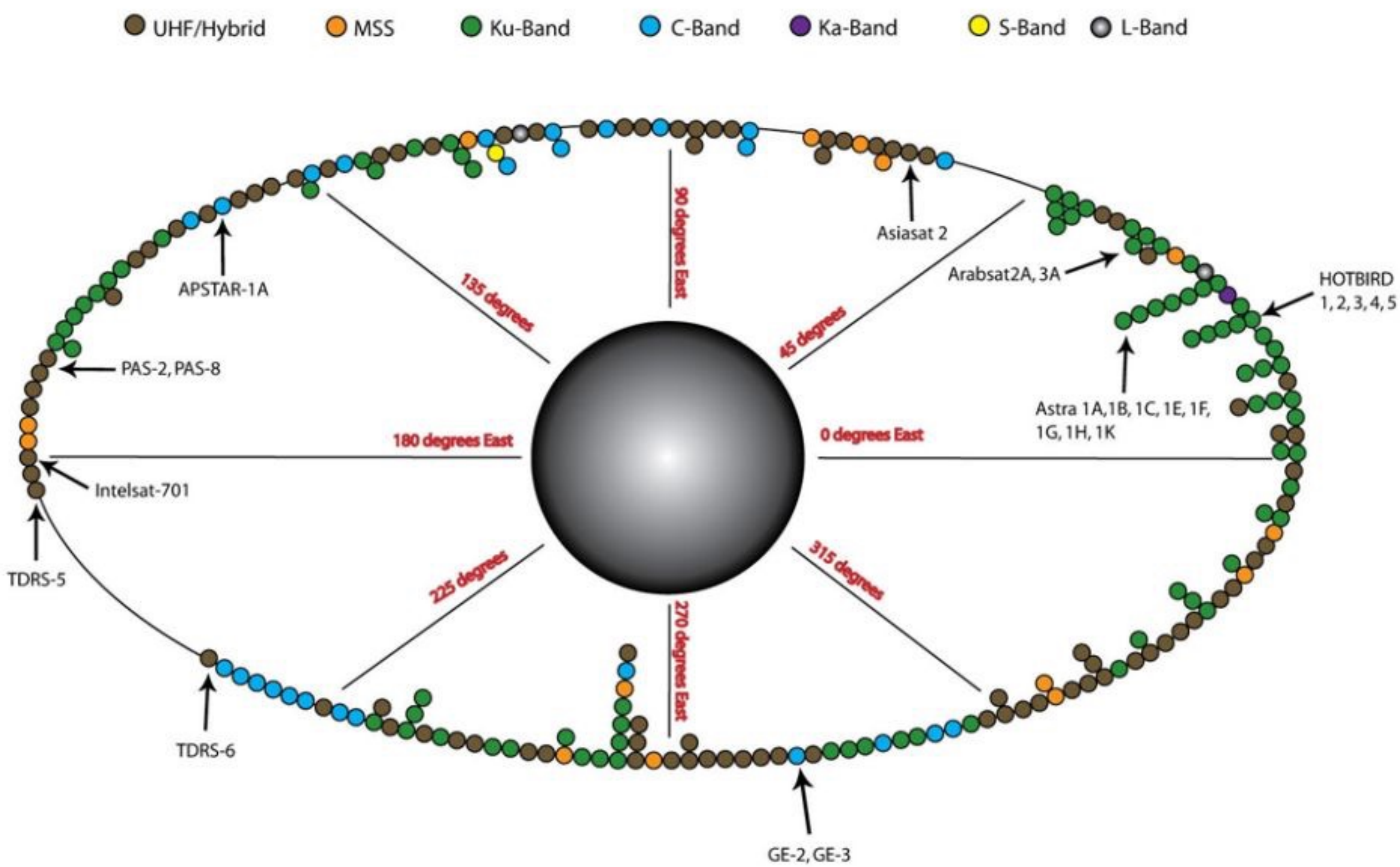
$n = \frac{2\pi}{P}$ $n = \frac{2\pi}{86164.01}$ $= 7.9212245 \cdot 10^{-5}$ rad s^{-1}

$n = \frac{1}{P}$ $n = \frac{1}{86164.01}$ $= 1.1605773 \cdot 10^{-5}$ revolutions s^{-1}

$= 1.0027388$ revolutions d^{-1}

Geostationary satellites on their orbit

present number ~400

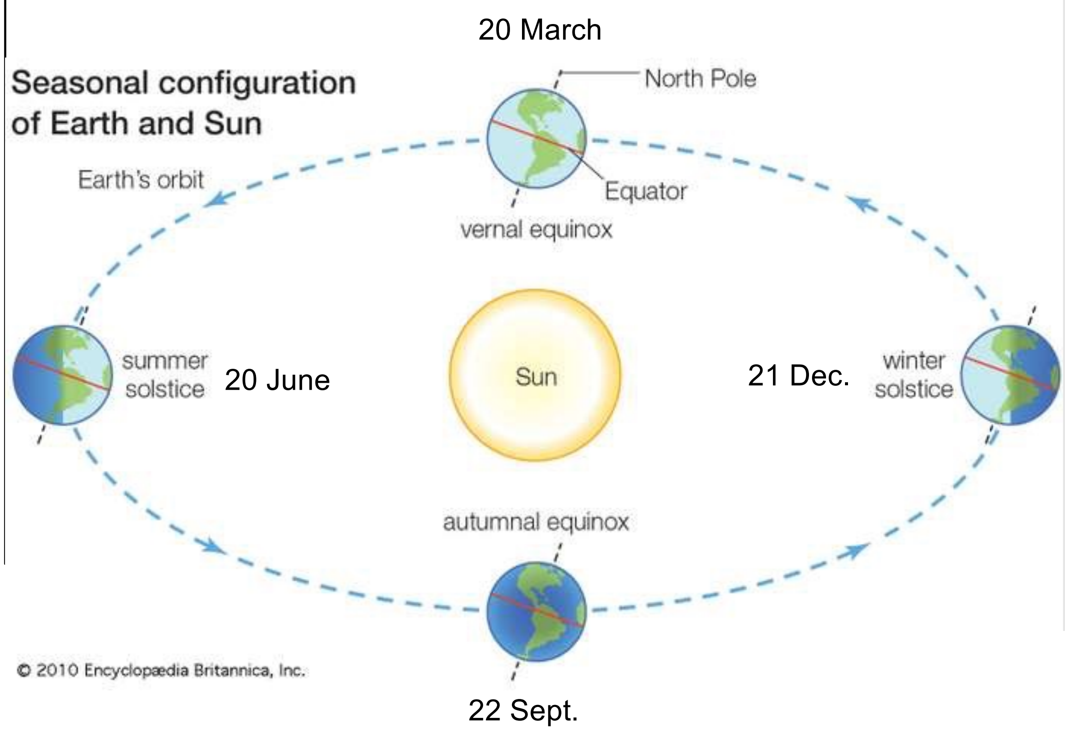


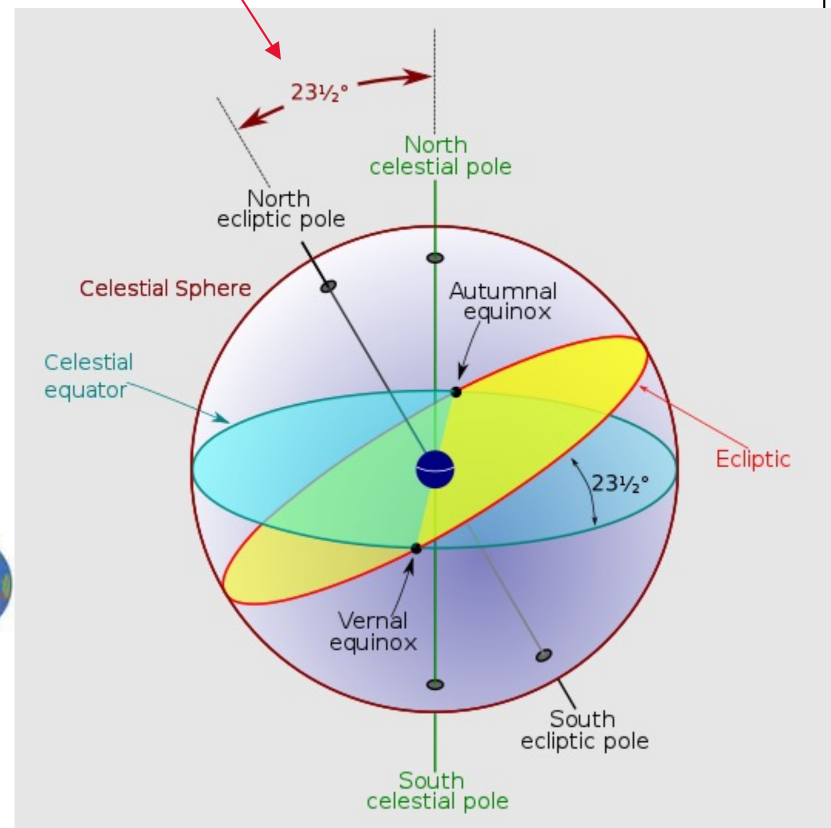
Space.com

Vernal equinox or First point of aries (γ)

Position on the sky where the Sun crosses the celestial equator to higher declinations

Obliquity of the ecliptic: 23.5 °





Seen from outside Sun and Earth

Seen from Earth

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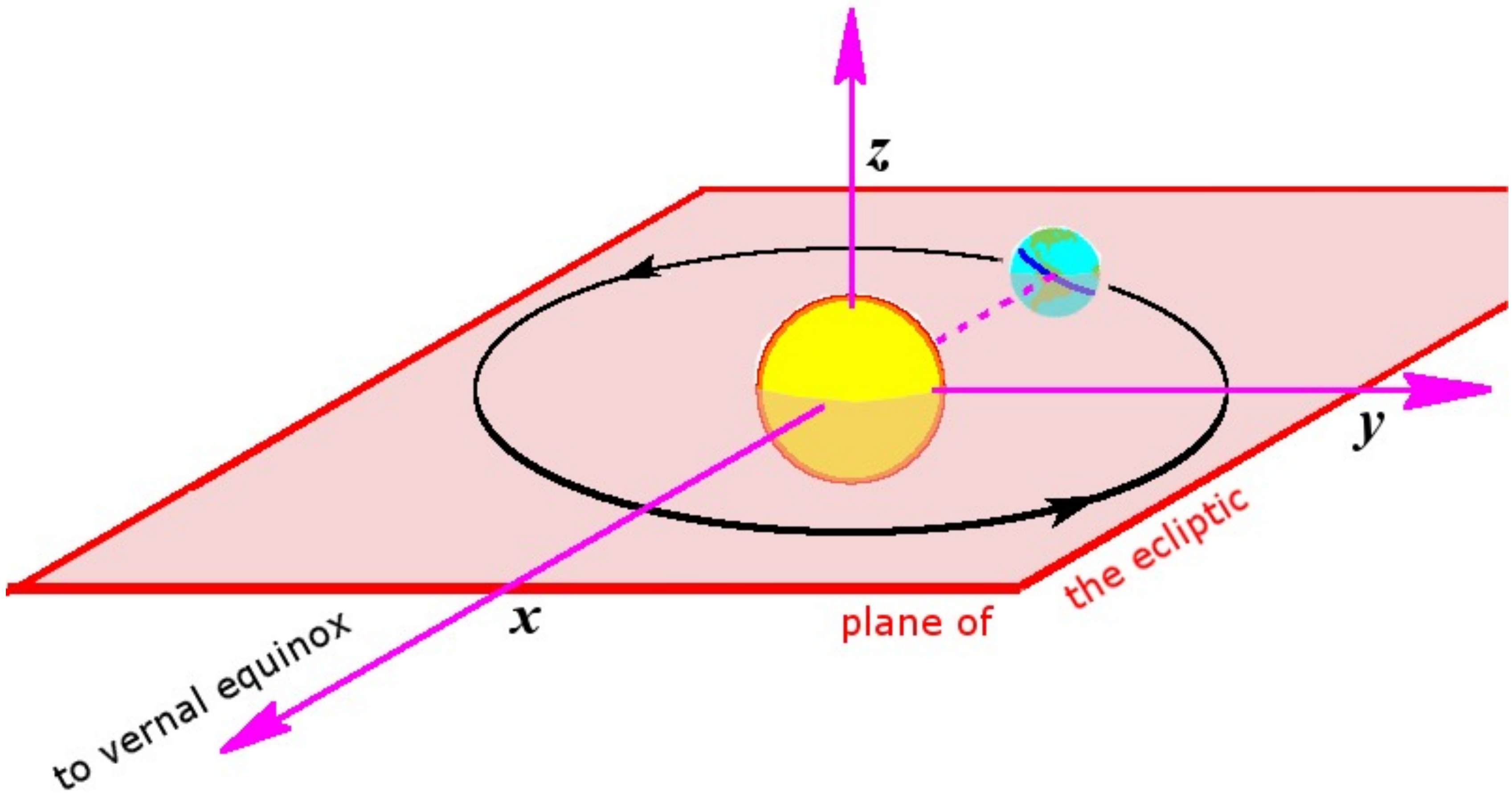

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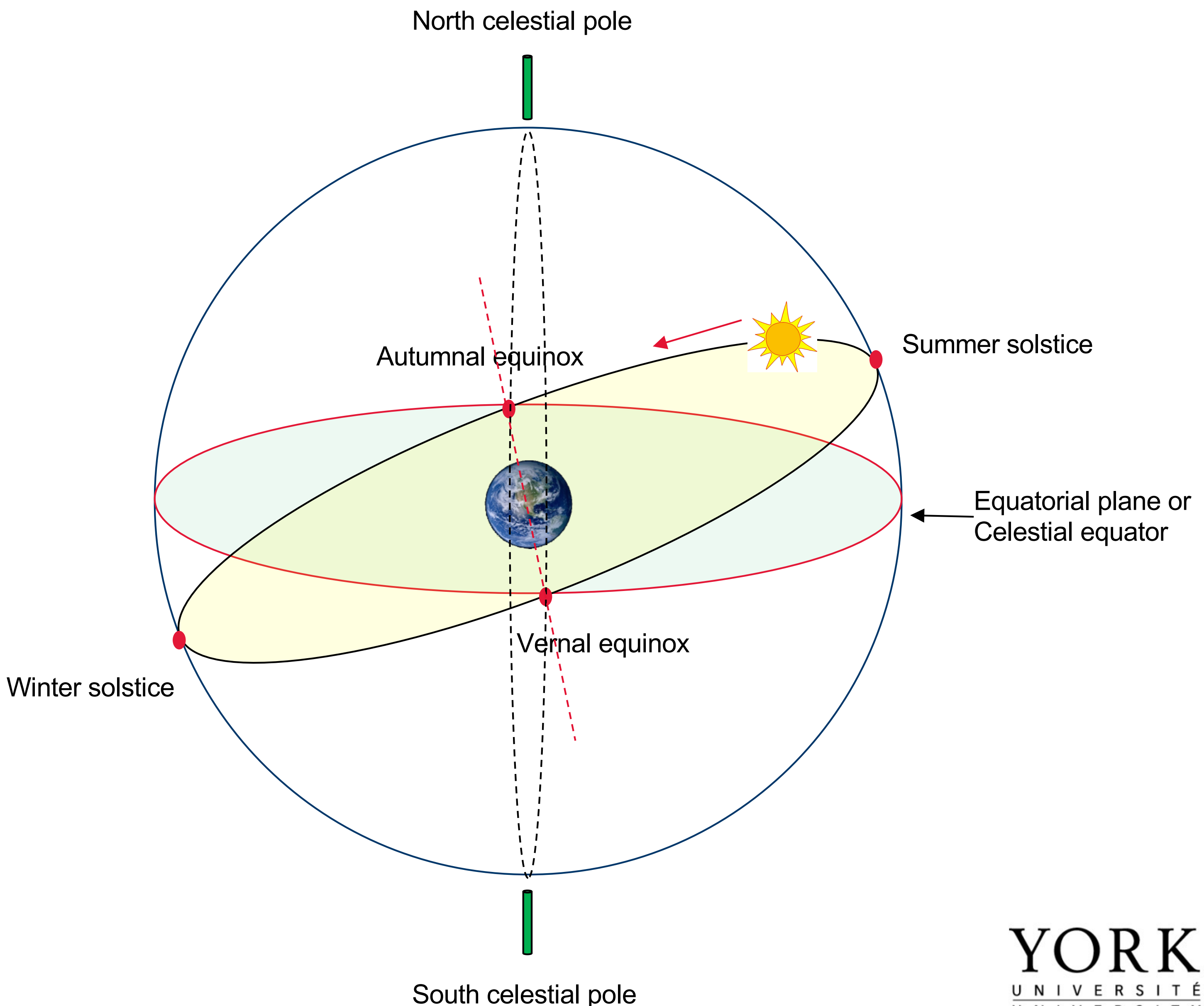
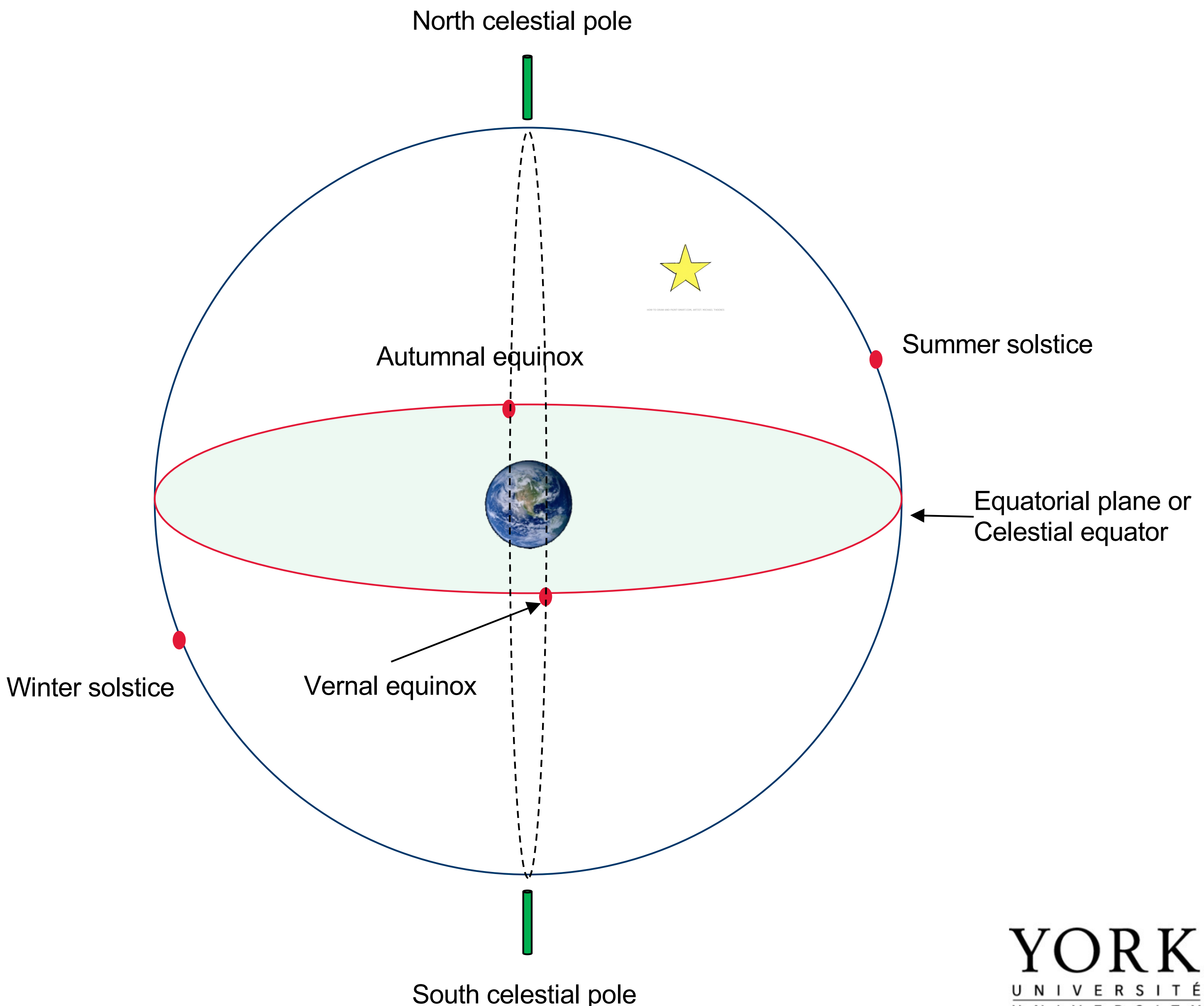
Celestial sphere



Geocentric equatorial coordinate system

what are the coordinates of the star?



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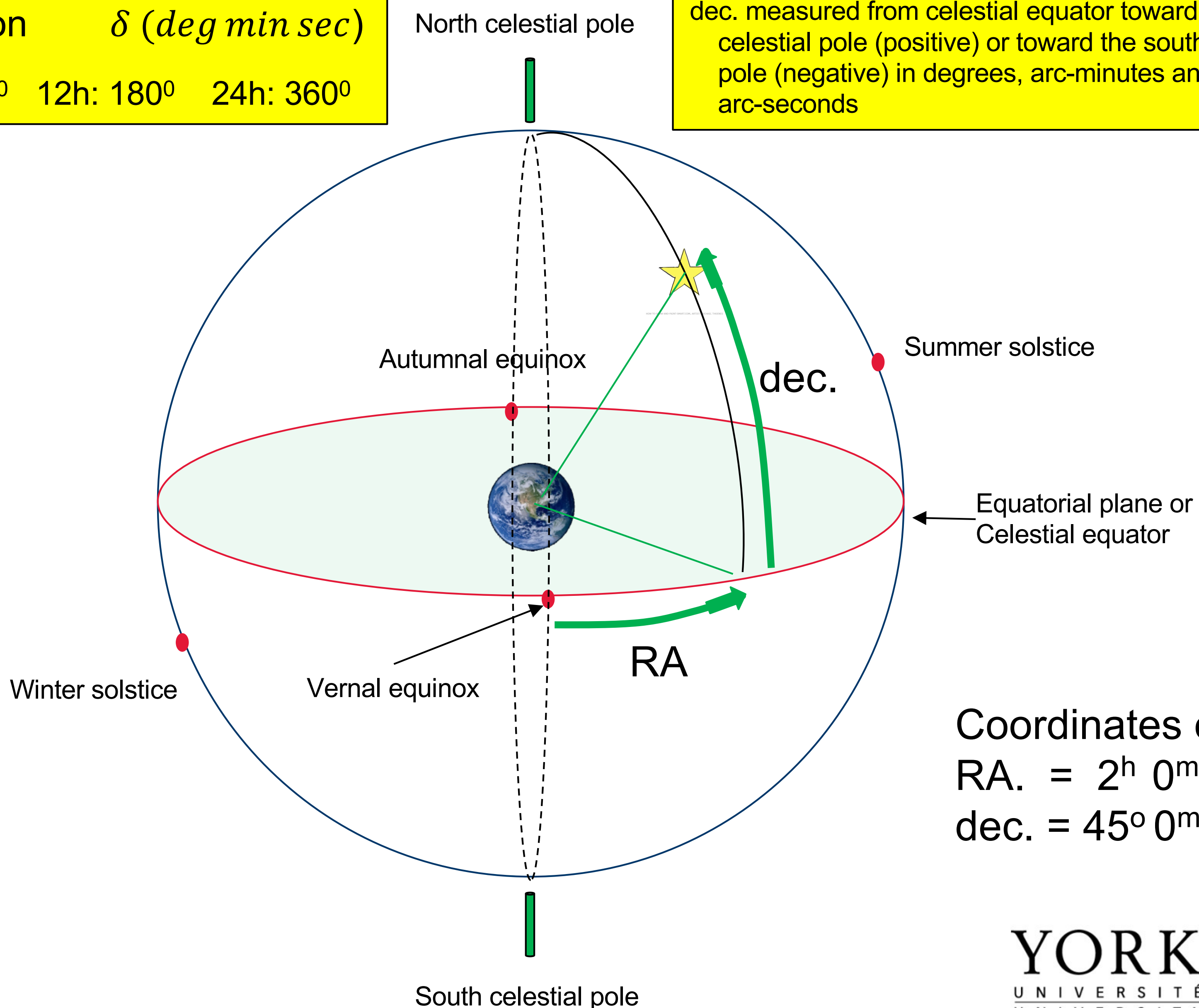
Geocentric equatorial coordinate system

RA: right ascension $\alpha\ (hh\ mm\ ss.s)$
dec.: declination $\delta\ (deg\ min\ sec)$

1h : 15^0 6h: 90^0 12h: 180^0 24h: 360^0

RA: measured eastward along the celestial equator in hours , minute, seconds
dec. measured from celestial equator toward the north celestial pole (positive) or toward the south celestial pole (negative) in degrees, arc-minutes and arc-seconds



Coordinates of star:
RA. = $2^h\ 0^m\ 0^s.000$
dec. = $45^o\ 0^m\ 0^s.000$

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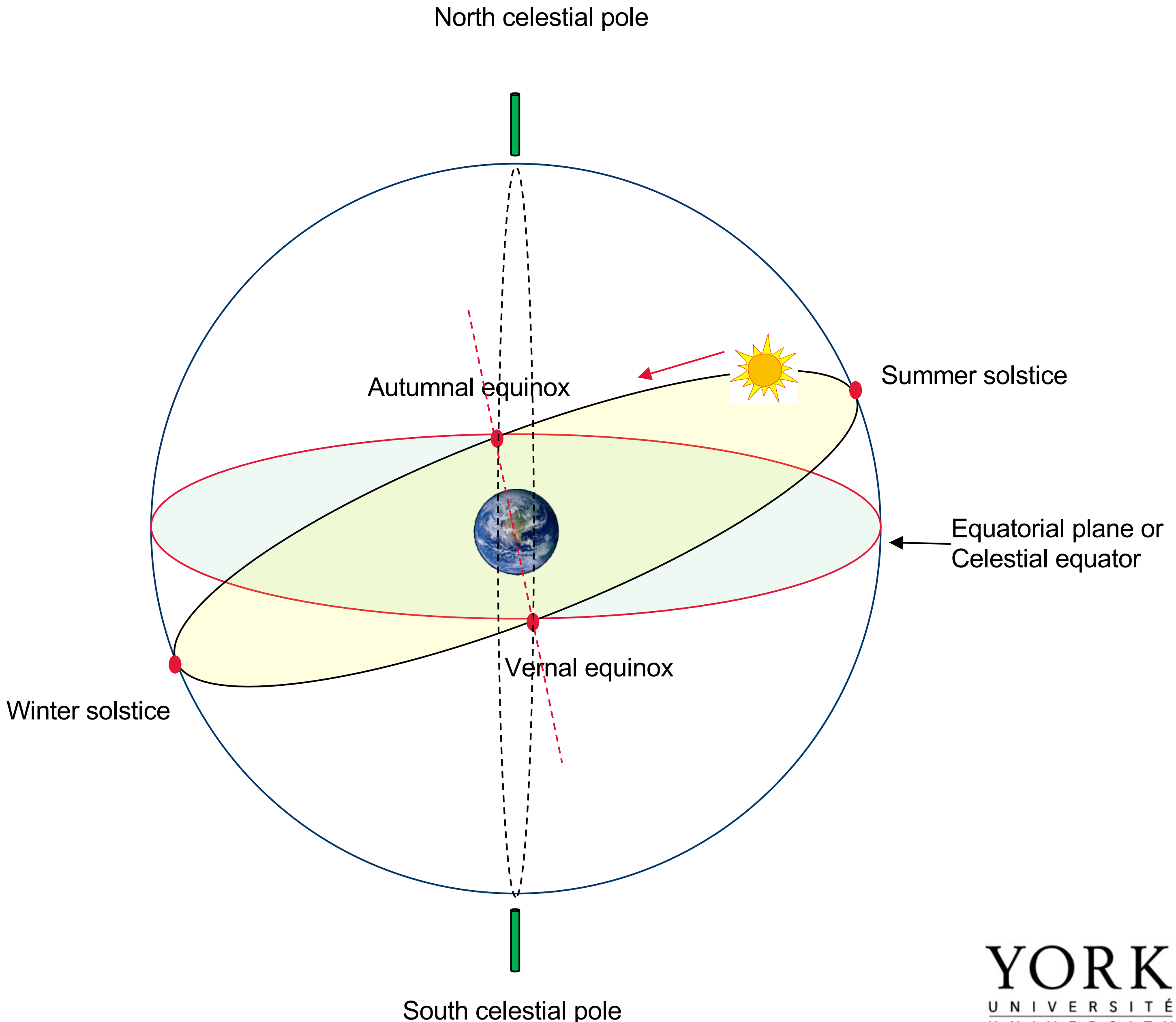
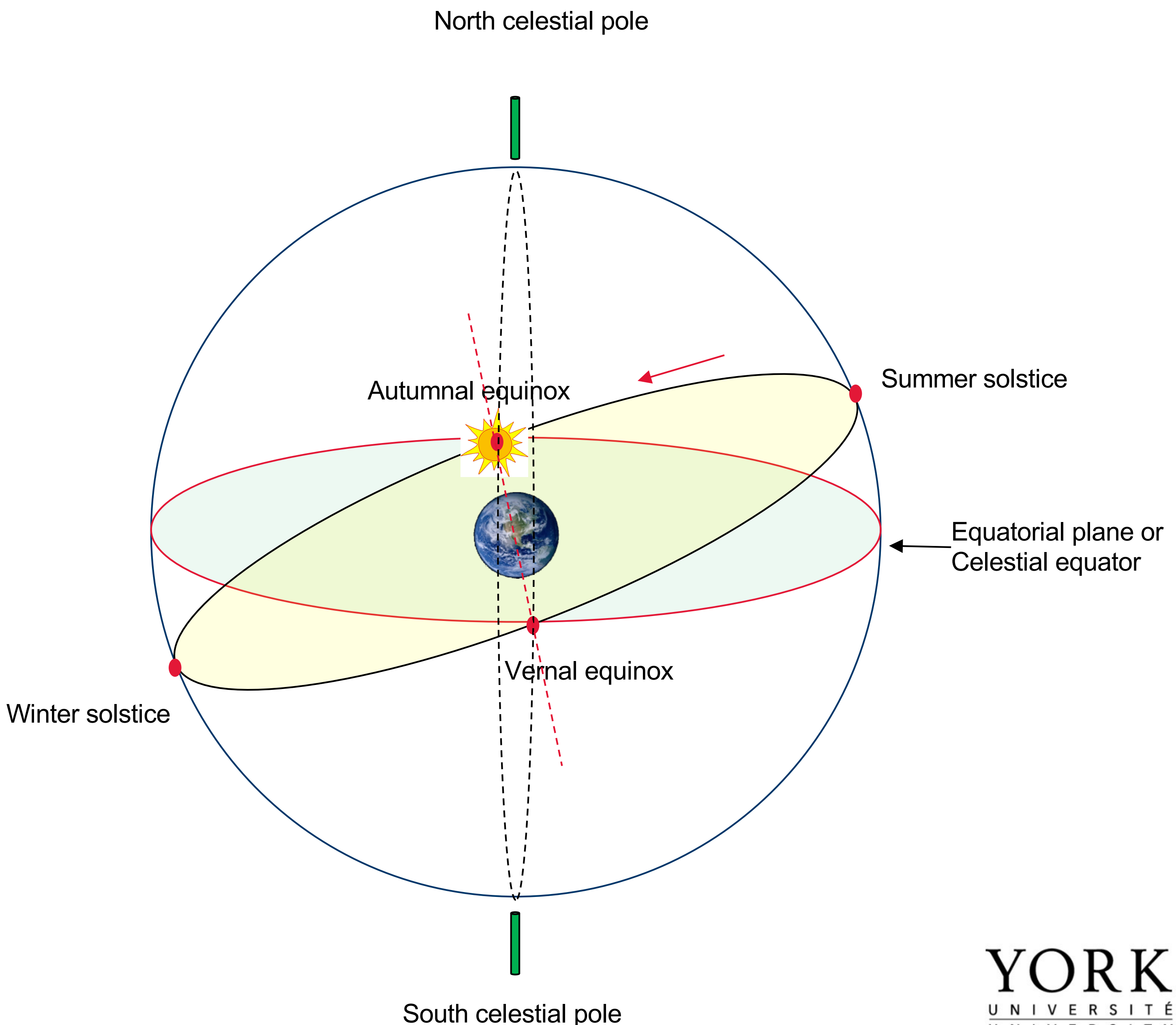
What are RA and dec. of Sun in sketch?



Where is the Sun today?



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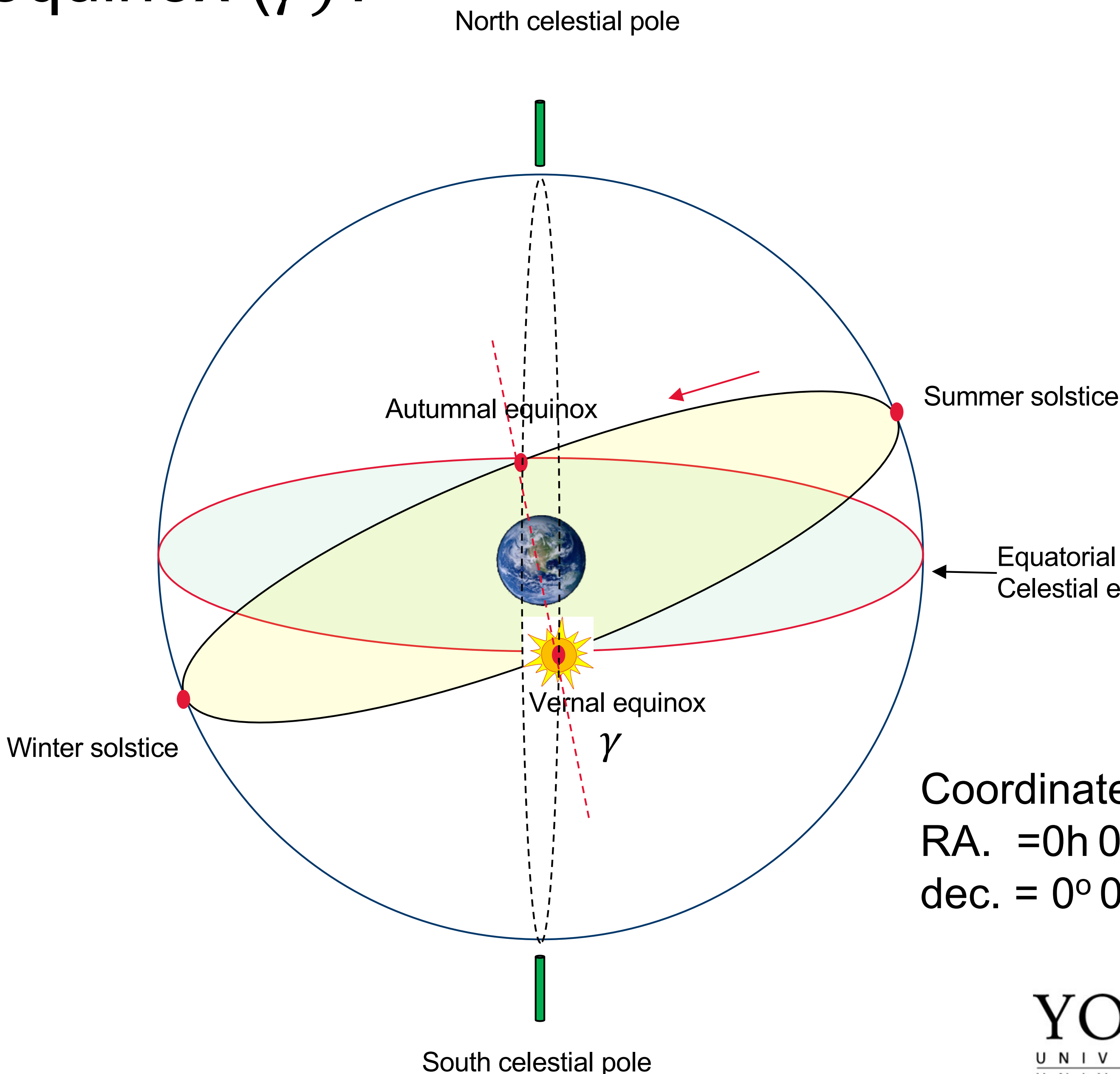
What are the RA and dec. coordinates of the vernal equinox (γ)?



Coordinates of γ :
RA. =0h 0m 0.000s
dec. = $0^o\, 0^m\, 0^s.000$

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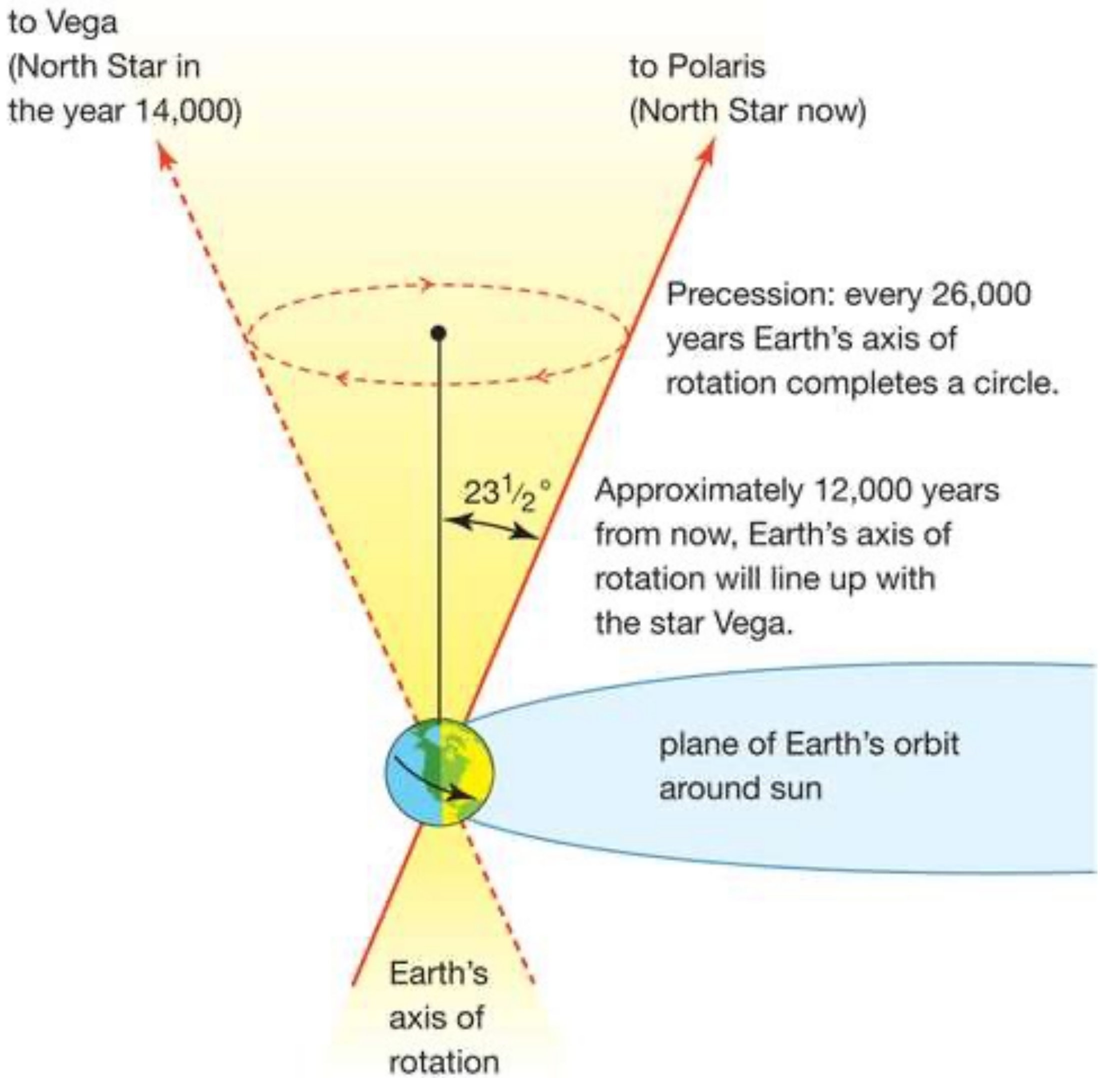
Precession of the equinoxes

motion of the equinoxes along the ecliptic (plane of the orbit of Earth)



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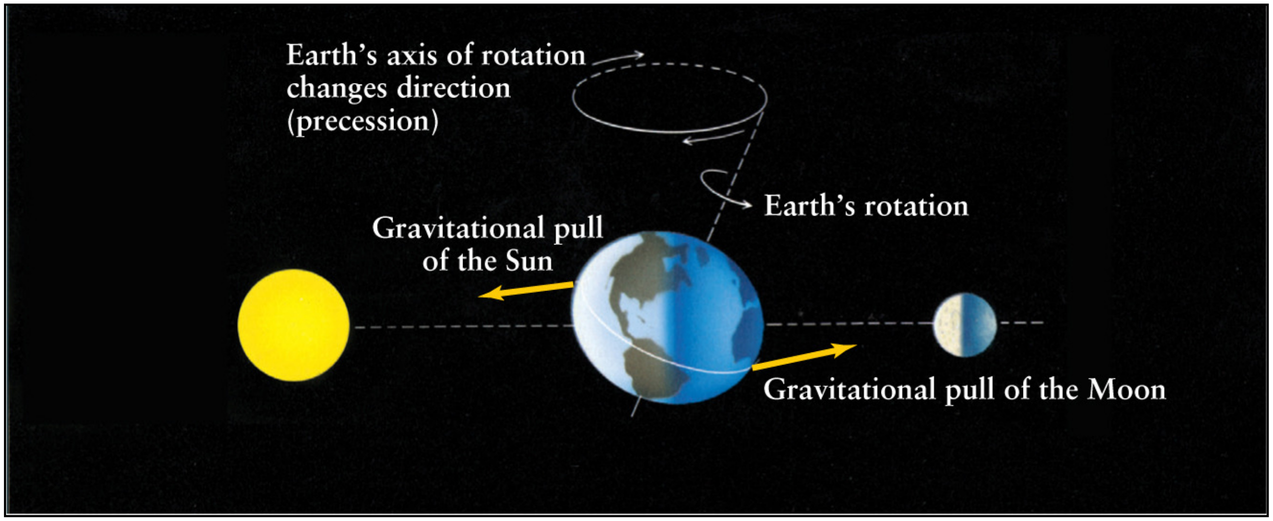
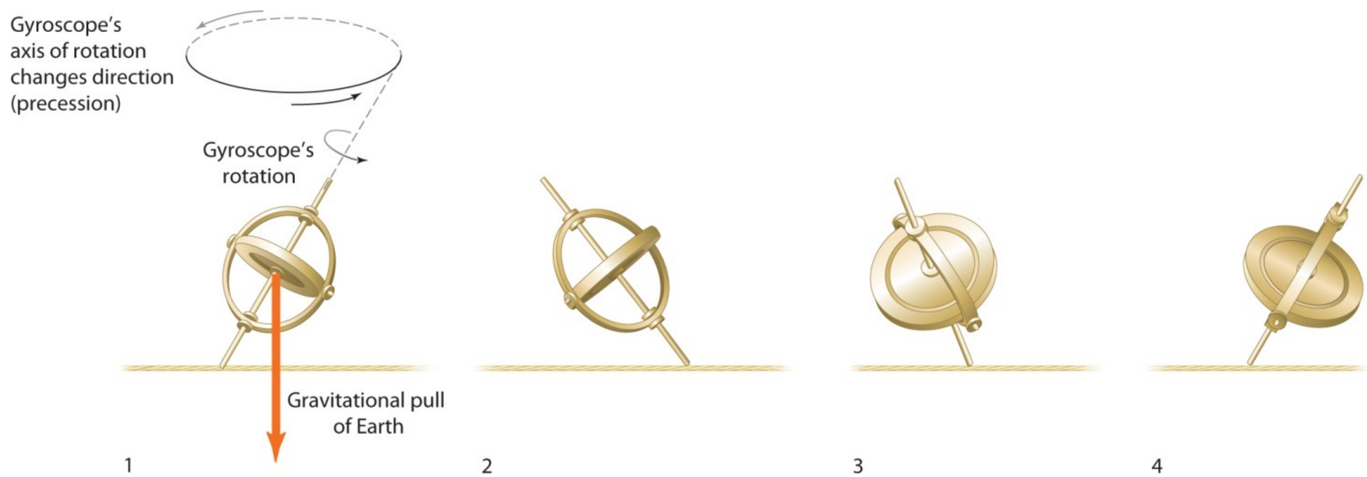
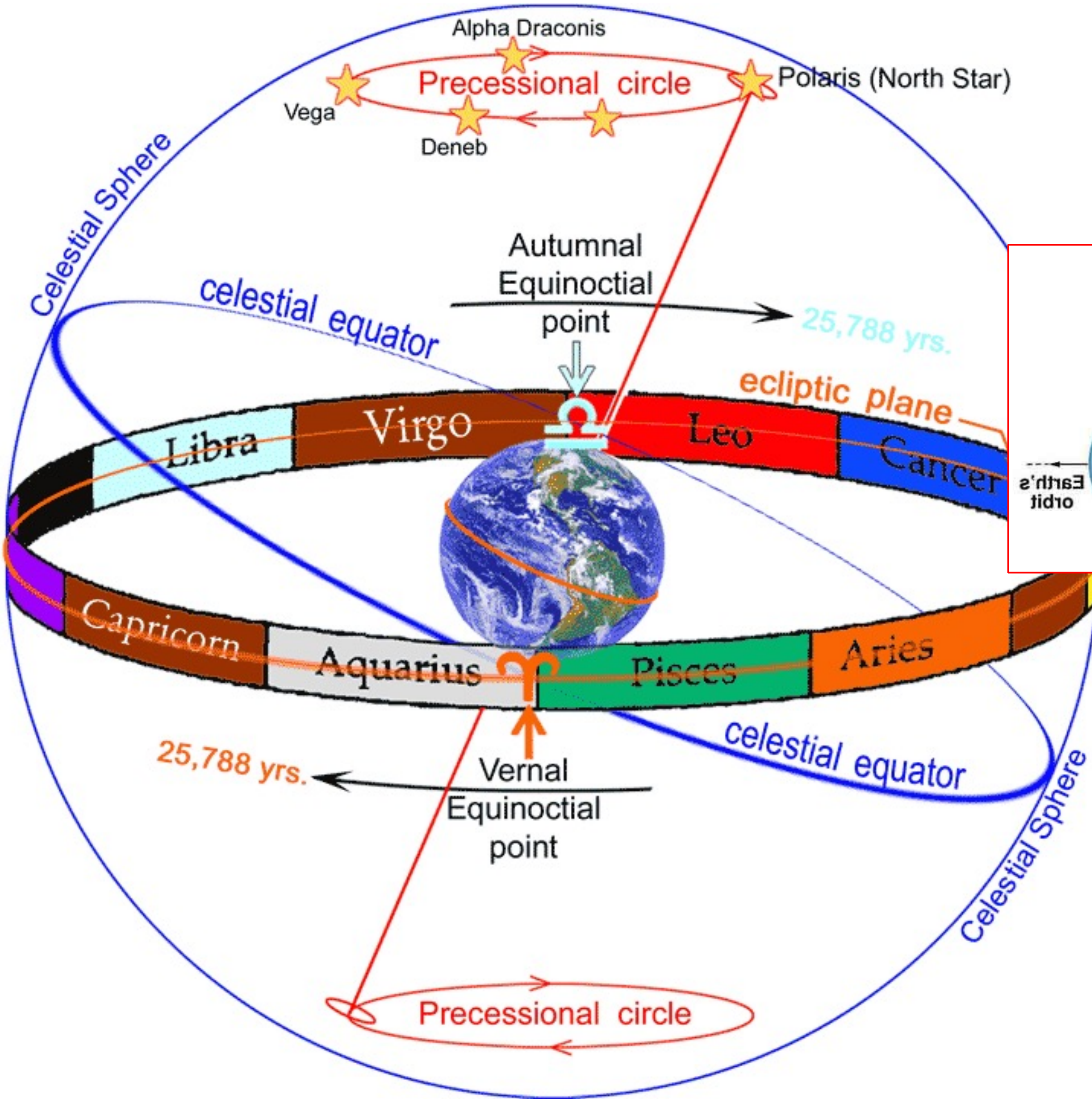
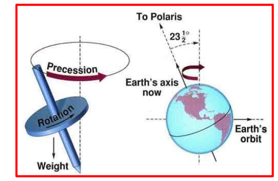
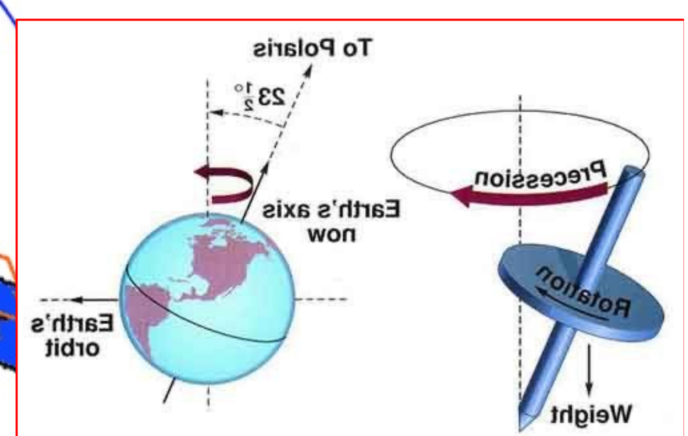




Gravitational forces of the Sun and the Moon pulling on Earth as it rotates cause Earth to undergo a top-like motion called precession. Over a period of 26,000 years, Earth's rotation axis slowly moves in a circular motion.


Alpha Draconis
Precessional circle
Polaris (North Star)
Vega
Deneb
Celestial Sphere
Autumnal
Equinoctial
point
celestial equator
25,788 yrs.
ecliptic plane
Libra
Virgo
Leo
Cancer
Capricorn
Aquarius
Pisces
Aries
celestial equator
25,788 yrs.
Vernal
Equinoctial
point
Celestial Sphere
Precessional circle

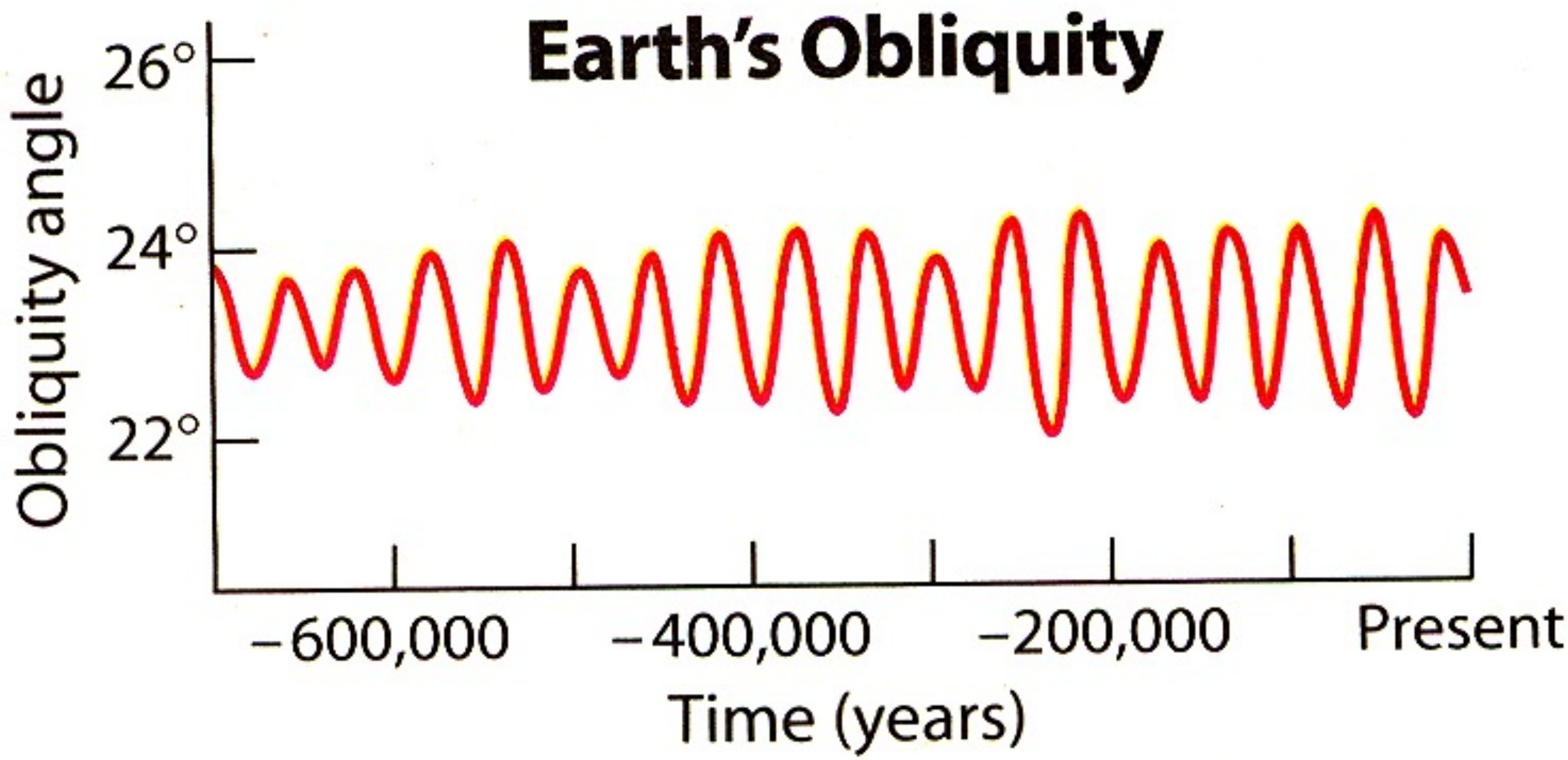

To Polaris
Precession
Rotation
Earth's axis now
Earth's orbit
Weight



This precession causes the position of the North Celestial Pole to slowly change over time. Today, the North Celestial Pole is near the star Polaris, which we call the "North Star." However, in 3000 BC, Thuban was close to the North Celestial Pole and in 14,000 AD, Vega will be in this location.

Precession also causes the vernal equinox to move along the celestial equator by 360^0 in 26,000 years. That means that the RA and dec changes slowly due to precession. In astronomy we therefore need to refer to a date for RA and dec. That date is the start of the year 2000. The coordinates are then in J2000.

Changes of the obliquity of the ecliptic



http://astro.wsu.edu/worthey/astro/html/lec-precession.html

2.2 Orbit perturbations

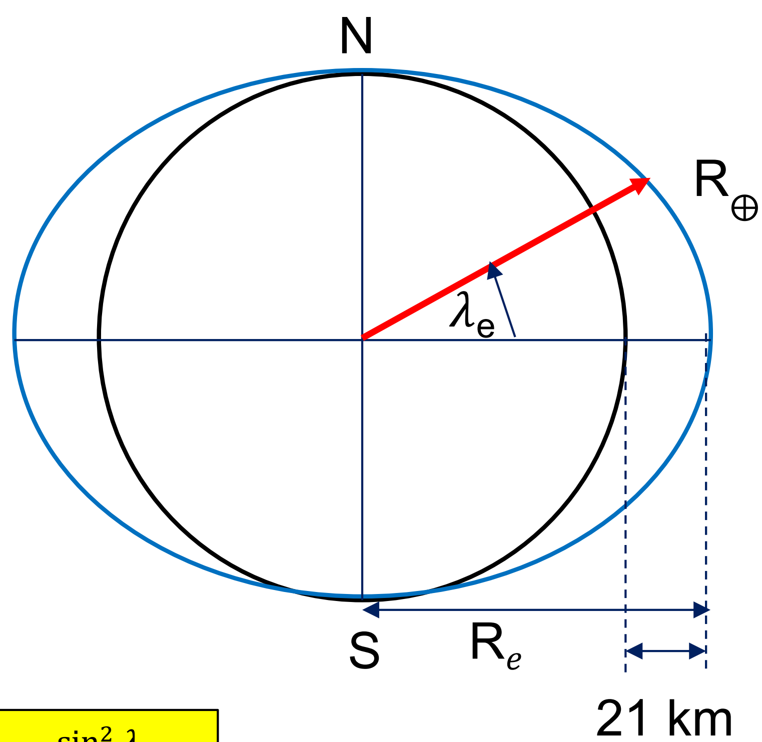
The Keplerian orbit is ideal. It is assumed that:

- The Earth is a sperically symmetric body with a uniformy distributed mass.
- Only forces present are:
  - The gravitational forces of the Earth with a $1/r^2$ dependence.
  - The centrifugal force from the satellite motion.
- The satellite is a point-like body with zero cross-section.

However, there are several effects that cause perturbations of the ideal orbit.

1. Effects of the non-spherical Earth

a) Effects of the equatorial bulge (effect on: n, $\boldsymbol{\Omega}$, $\boldsymbol{\omega}$)



$$R_\oplus = Re\left(1 - \frac{\sin^2 \lambda_e}{298.257}\right)$$

R_e = 6378.14 km   Radius of Earth at λ_e=0°

λ_e : latitude

i) Mean motion (n)

$$n = n_0\left[1 + \frac{K_1(1 - 1.5\sin^2 i)}{a^2\ (1 - e^2)^{1.5}}\right] \quad \text{rad s}^{-1}$$

$$n_0 = \frac{2\pi}{P_0} = \sqrt{\frac{\mu}{a^3}} \quad \text{rad s}^{-1}$$

K_1 = 66,063.1704 km^2 a in km, P_0 in s

Example 2-3

i=0, a=42,164 km, e=0

$$n = n_0\left[1 + \frac{K_1}{a^2}\right] = n_0 \cdot [1+3.708\cdot 10^{-5}]\ \text{rad s}^{-1}$$
$$= n_0 \cdot [1+0.002124]\ \text{deg d}^{-1}$$

$$P - P_0 = \frac{2\pi}{n} - \frac{2\pi}{n_0} = -3.2\ \text{s}$$

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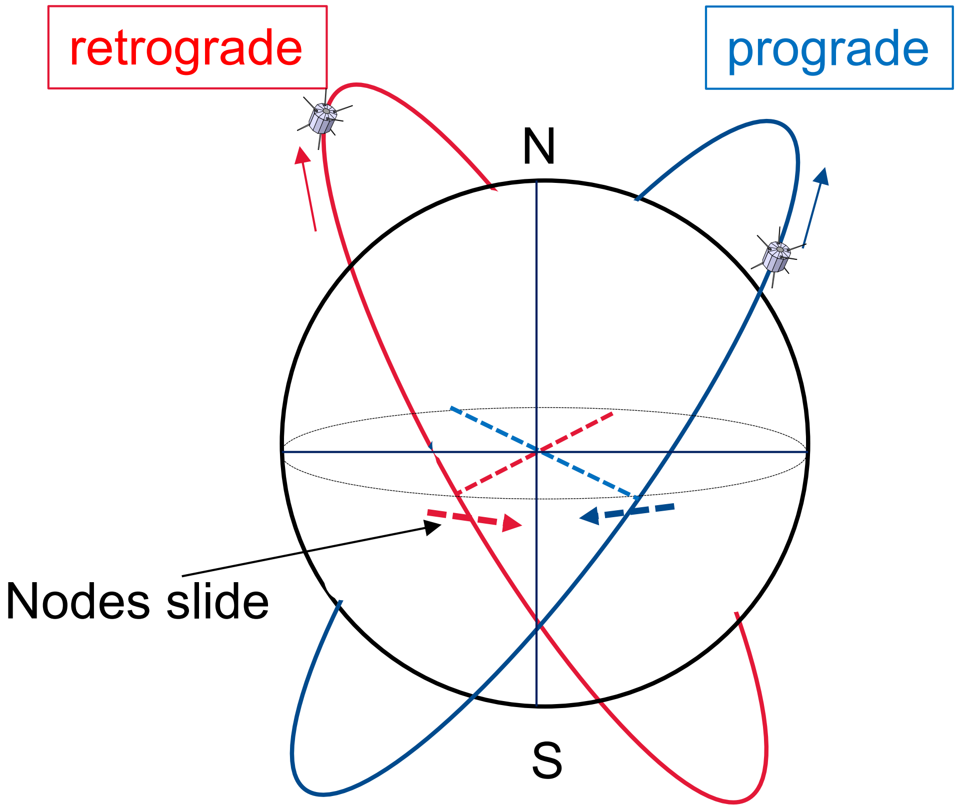
ii) *Regression of nodes – effect on Ω*

Nodes slide along the equator in a direction opposite to the satellite motion



$$\frac{d\Omega}{dt} = -K\cos i$$

$$K = \frac{nK_1}{a^2(1-e^2)^2}$$

K has the same units as n, for instance, rad d^{-1} or deg d^{-1}

Example 2-4

i=30°, a=7,500 km, e=0

$n \approx n_0 = 86400 \cdot \sqrt{\frac{\mu}{a^3}} = 86400 \cdot \sqrt{\frac{398600.5}{7500^3}}$ rad d^{-1} = 83.982 rad d^{-1}

K= 0.0984 rad d^{-1}

$\frac{d\Omega}{dt}$ = -0.0852 rad d^{-1}

$\frac{d\Omega}{dt}$ = -4.8832 deg d^{-1}

For prograde orbit: $\frac{d\Omega}{dt}$ = <0, westward slide of nodes

For retrograde orbit: $\frac{d\Omega}{dt}$ = >0, eastward slide of nodes

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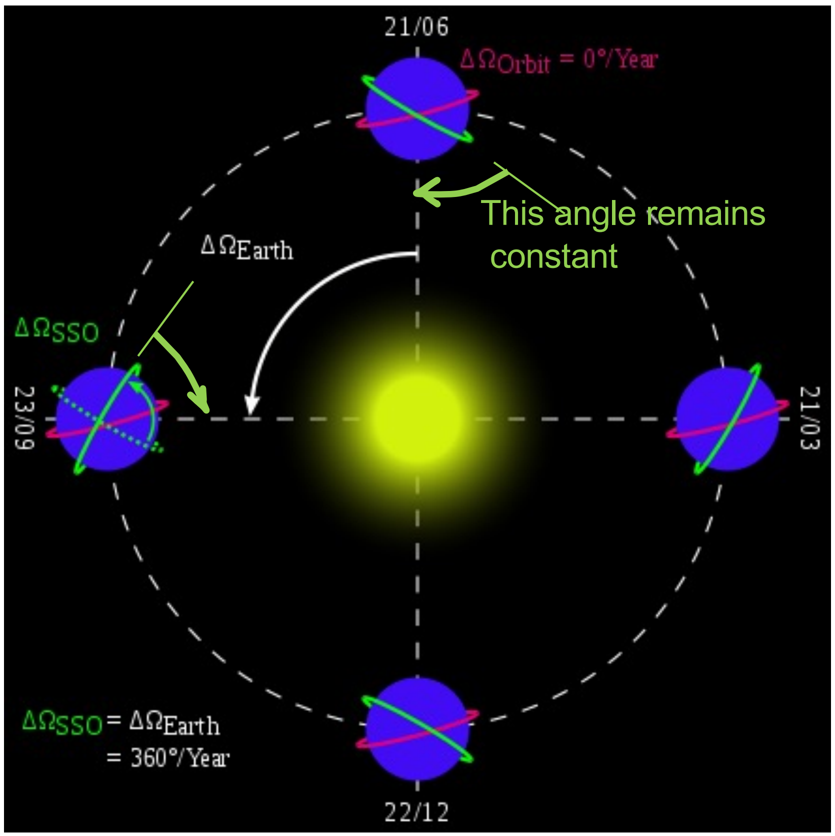
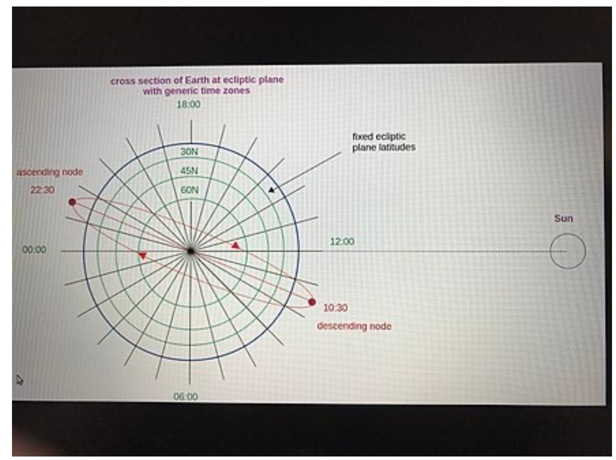
For a particular inclination, i, we get a sun synchronous orbit where the nodes slide eastward by exactly 2π rad or 360^0 in one year.

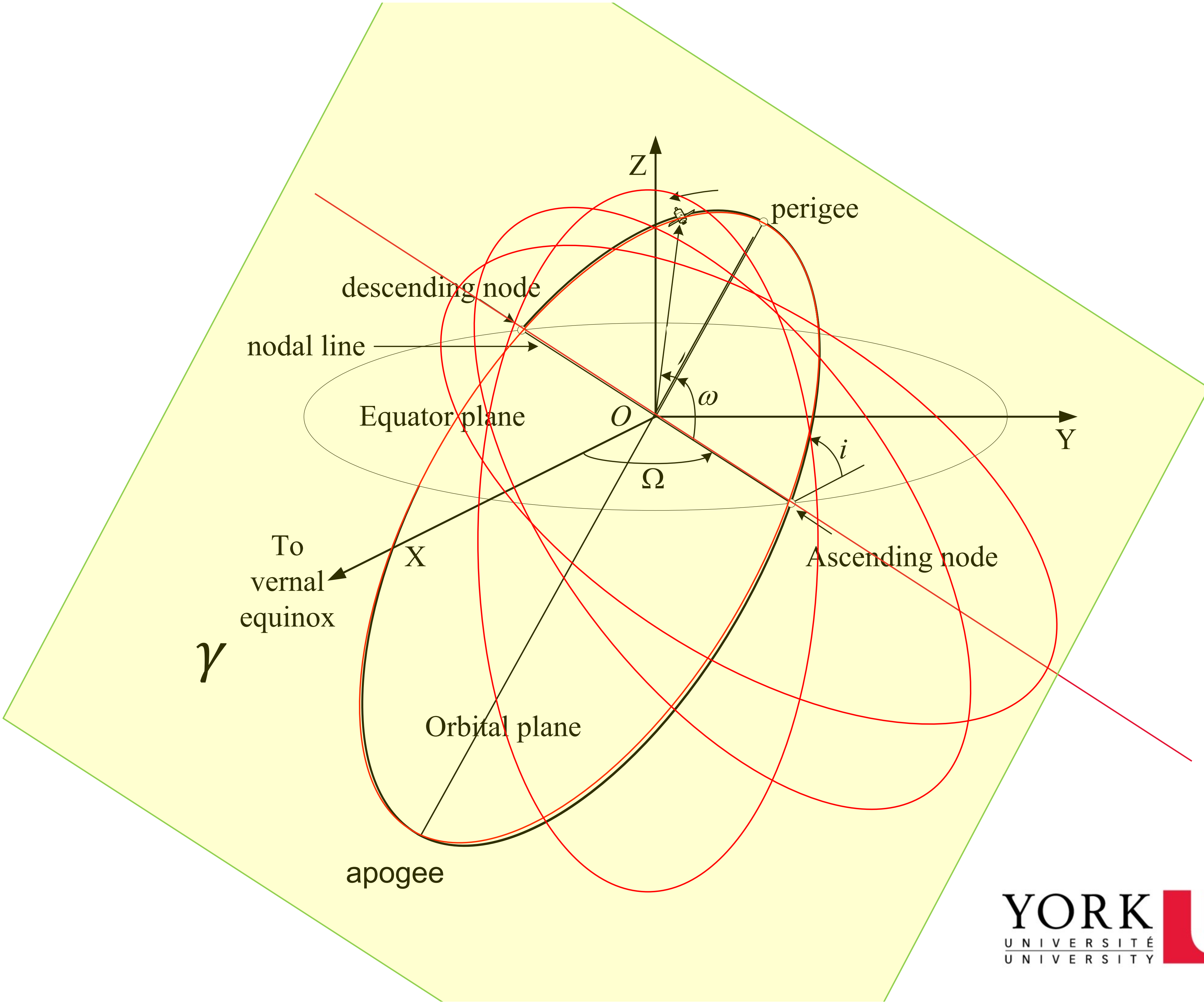
$$\frac{d\Omega}{dt} = \frac{2\pi}{365.24} \text{ rad d}^{-1} \quad \text{(for sun synchronous orbit)}$$



Wikipedia



Wikipedia



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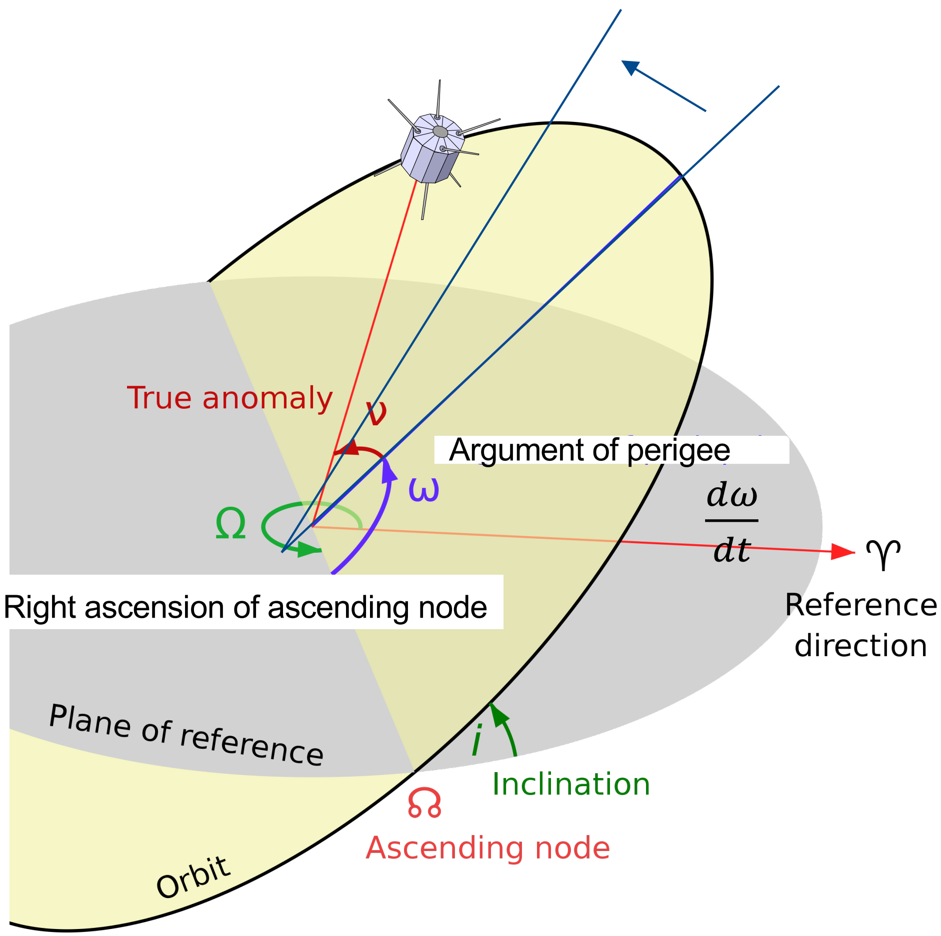
iii) *Rotation of line of apsides – effect on* ω

Orbital ellipse rotates in orbital plane around focus



$$\frac{d\omega}{dt} = K(2\text{-}2.5\sin^2 i)$$

$\frac{d\omega}{dt}$ has units as K and K has units as n

Example 2-5

i=30°, a=7,500 km, e=0

K= 0.0984 rad d^{-1} from previous example

$$\frac{d\omega}{dt} = 0.0984 \cdot (1.375) \text{ rad } d^{-1}$$

$$\frac{d\omega}{dt} = 0.1353 \text{ rad } d^{-1}$$

$$\frac{d\omega}{dt} = 7.752 \text{ deg } d^{-1}$$

There is an inclination, i, for which $\frac{d\omega}{dt}$ =0

Best example: Molniya communications satellites

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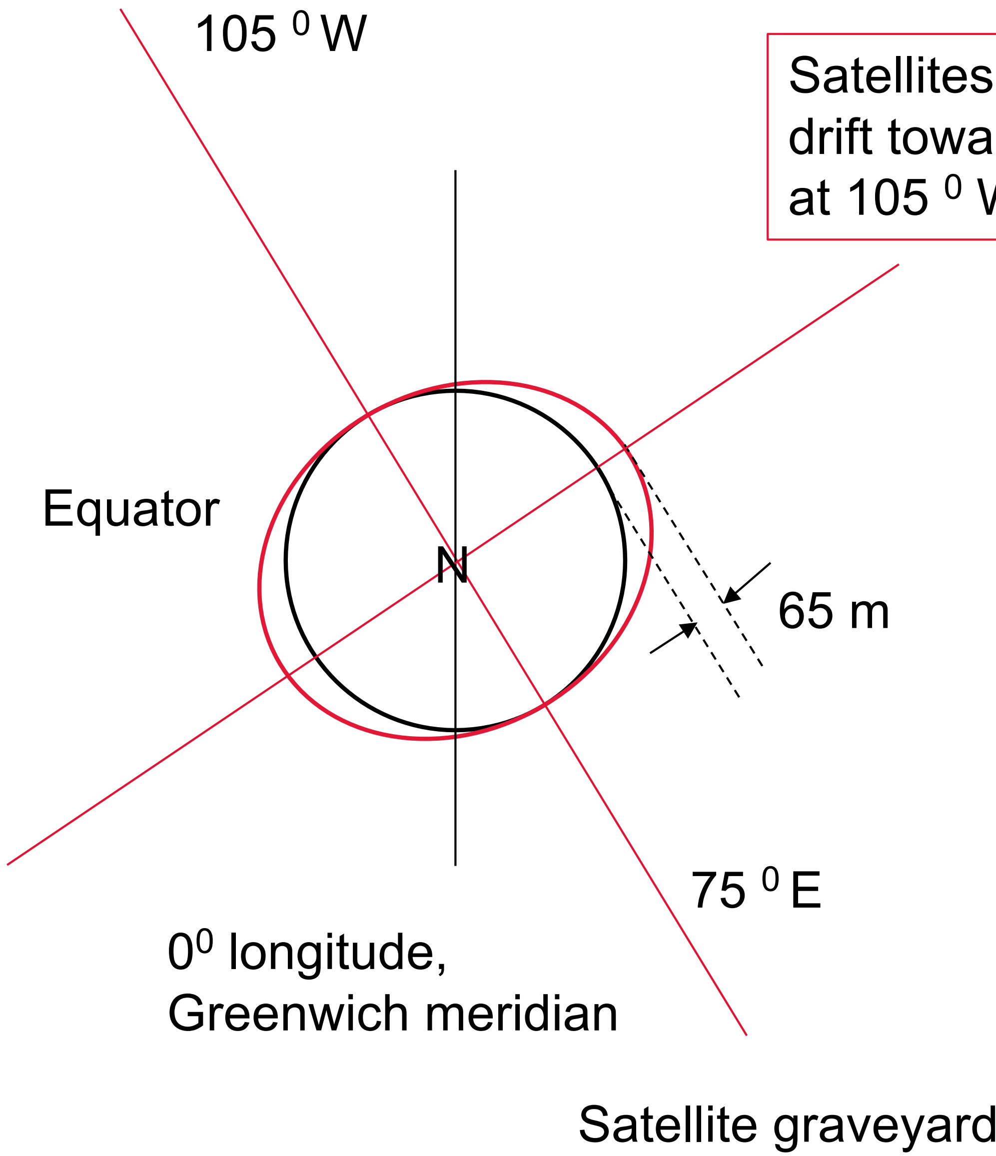
b) Effects of equatorial ellipticity

Satellites in geostationary orbit drift toward satellite graveyards at 105 ⁰ W and 75 ⁰ E

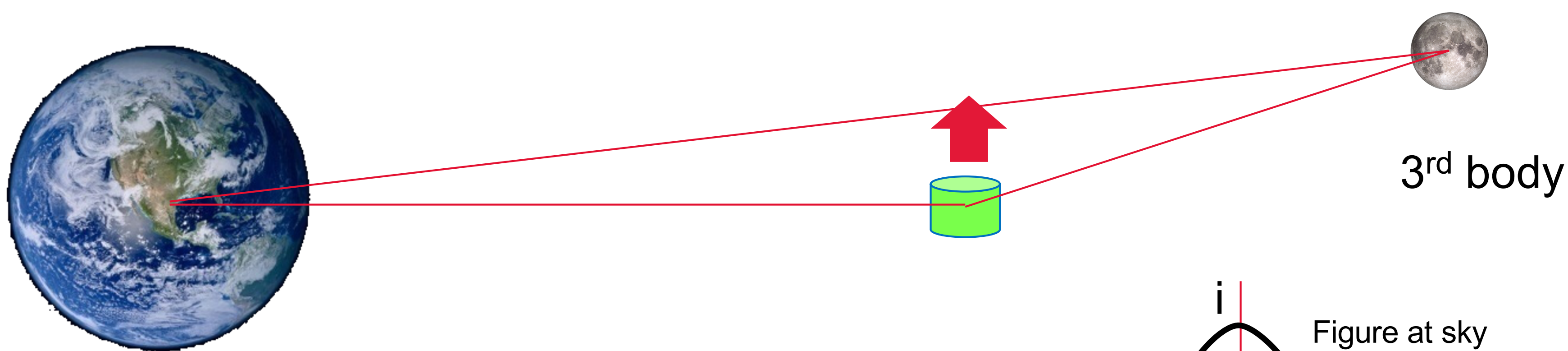


c) Effects of tides

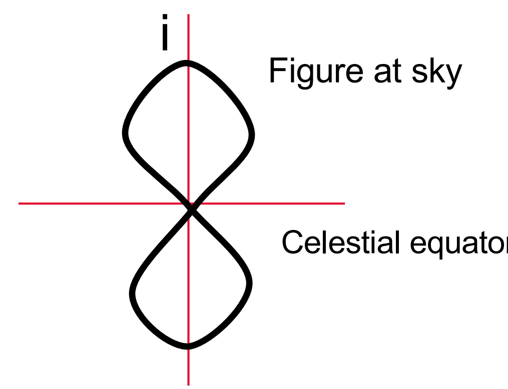
Tides change the mass distribution of the Earth
➔ Very small effect and only on LEO satellites

2. Direct third-body effects. (effect mostly on i)

Direct attractions of the Moon and the Sun are significant



For satellite in geostationary orbit:



$$\frac{di}{dt} = 0.8.\ \text{deg yr}^{-1}$$

I increases to a maximum of 15^{0} in 27 years and then decreases to $i=0^{0}$ again. For a geosyncronous orbit ($i\neq0^{0}$) the satellite appears to move along a figure "8" seen from the ground stations.

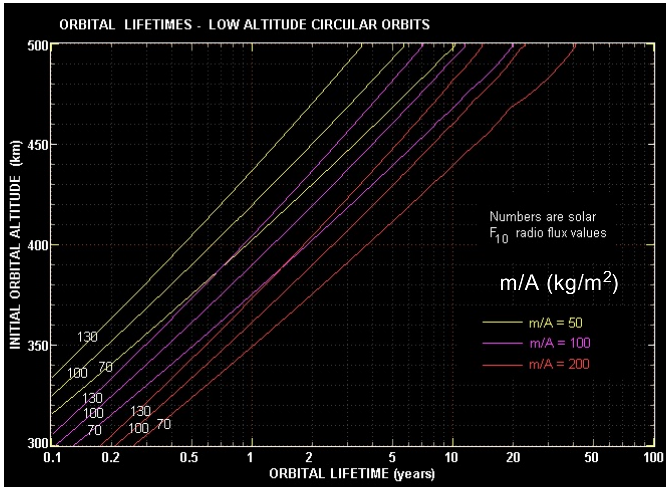
3. Atmospheric drag (effects mostly on a and e)

Important for satellites with perigee height < 1000 km.

Perturbation of orbit depends on:

- Atmospheric density
- Satellite cross section (m^2)
- Satellite mass (kg)
- Satellite speed

Satellite Altitude	Lifetime
200 km	1 day
300 km	1 month
400 km	1 year
500 km	10 years
700 km	100 years
900 km	1000 years



Spaceacademy.net.au

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4. Solar radiation pressure

Acceleration on satellite depends on:

- Solar radiation at satellite
- Satellite mass
- Satellite surface area exposed to Sun
- Albedo of satellite depending on material

2.3 Visibility

There are three different planes and coordinate systems:

- Orbital plane -- perifocal coordinate system
- Equatorial plane -- geocentric equatorial coordinate system
- Plane tangential to surface of earth -- topocentric horizon coordinate system

➔ coordinate transformations necessary (definitions of time necessary)

Problem:
How to determine from the Keplerian elements the
look angles (azimuth and elevation) of a satellite and
the range to the satellite for any point on earth.

Solution path:

1) Describe locations of satellite and earth station in same non-rotating coordinate system that travels with Earth through space (geocentric equatorial coordinate system).

2) Then determine range vector and express it in topocentric horizon coordinate system.

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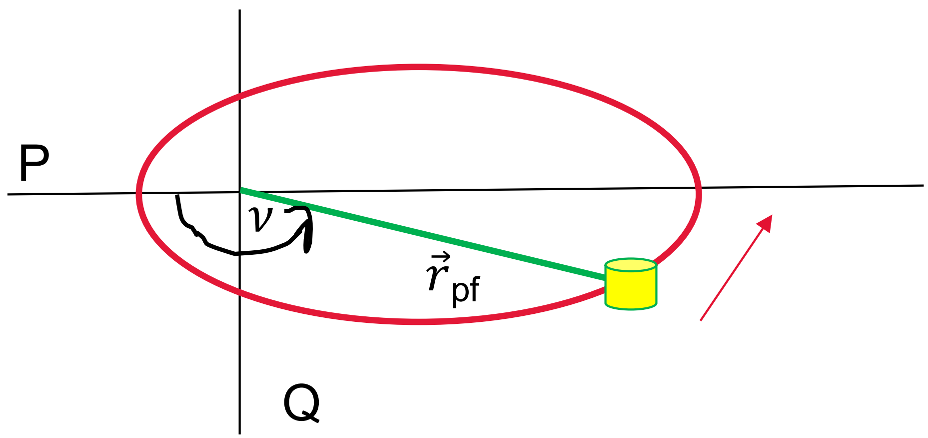
Steps to solve the problem:

1. Locate satellite in perifocal coordinate system

$$\vec{r}_{\text{pf}} = \begin{bmatrix} r\cos\nu \\ r\sin\nu \end{bmatrix} = \begin{bmatrix} r_p \\ r_q \end{bmatrix}$$



2. Locate satellite in geocentric equatorial coordinate system

$$\vec{r}_{g.\,eq} = \tilde{R}\begin{bmatrix} r_p \\ r_q \end{bmatrix} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}$$

$\tilde{R}$ matrix elements are functions of , i, Ω, ω

$R_{11} = \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i$

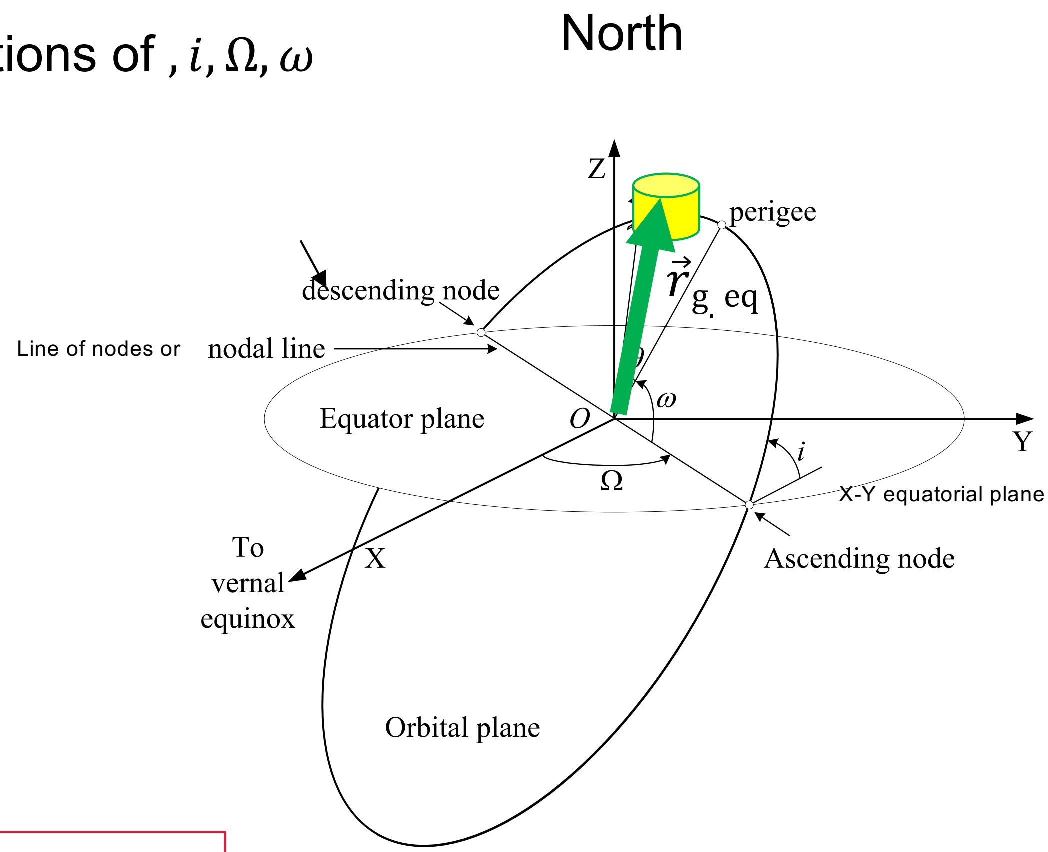
$R_{12} = \cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i$

$R_{21} = \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i$

$R_{22} =$ ....

$R_{31} =$ ....

$R_{32} =$ ....



This is a non-rotating coordinate system which travels with Earth through space.

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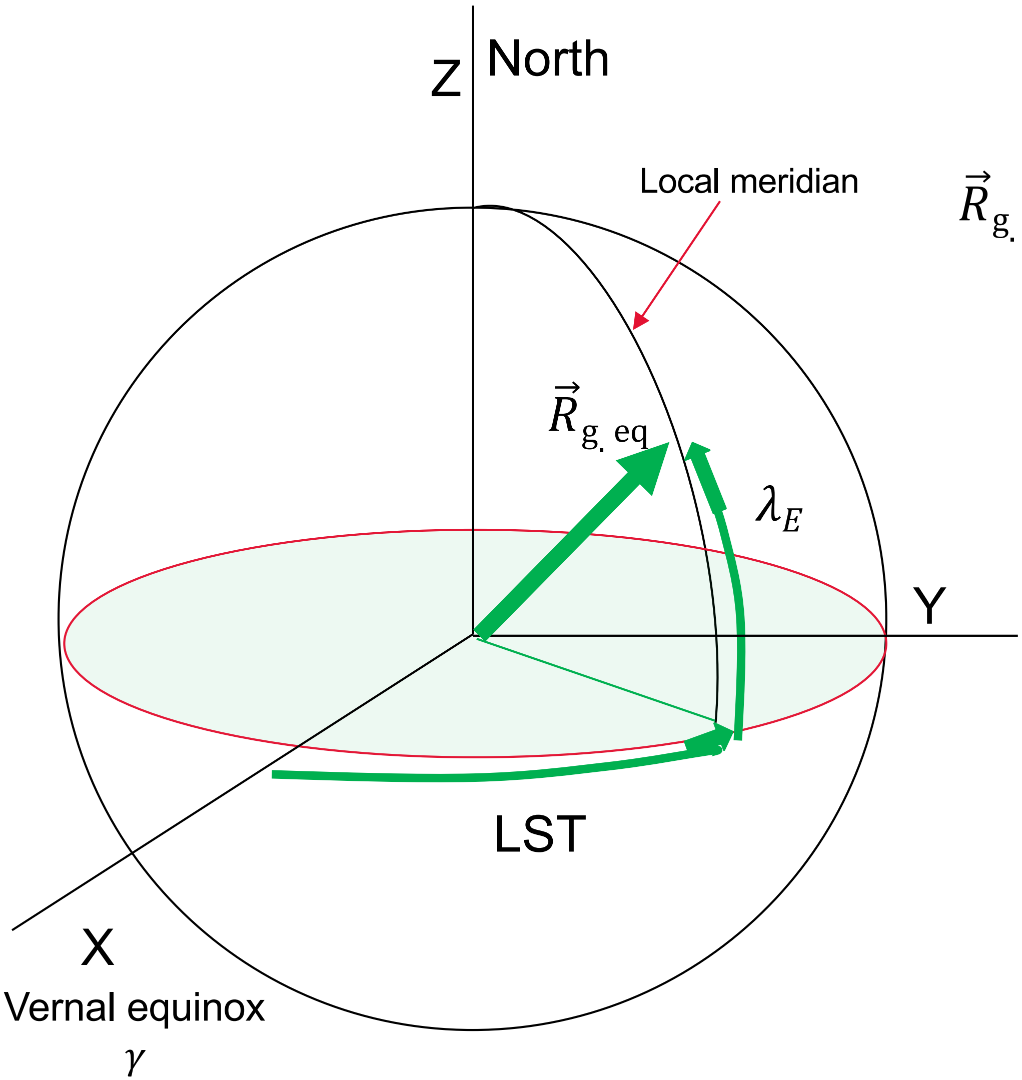
3. Locate Earth station in geocentric equatorial coordinate system



$$\vec{R}_{g.\ eq} = \begin{bmatrix} |R_\oplus + H| \cos\lambda_E \cos(LST) \\ |R_\oplus + H| \cos\lambda_E \sin(LST) \\ |R_\oplus + H| \sin\lambda_E \end{bmatrix} = \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix}$$

$R_\oplus$: Earth radius at λ_E
H : height above mean sea level
λ_E : Latitude of Earth station
LST Local sidereal time. (24h 360°)

LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

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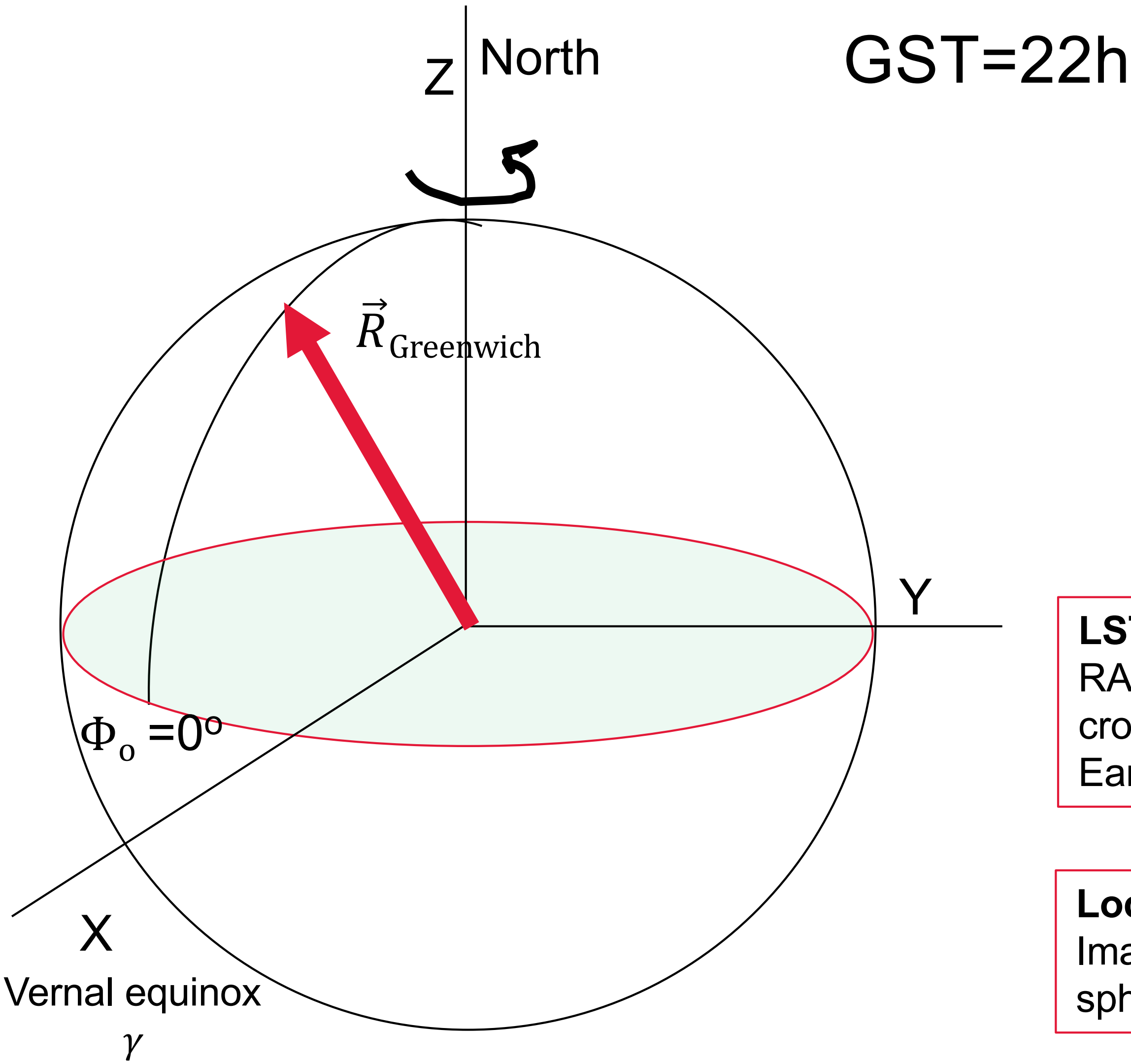
How is LST related to standard time?

LST ⟷ GST ⟷ UT ⟷ standard time

GST and LST

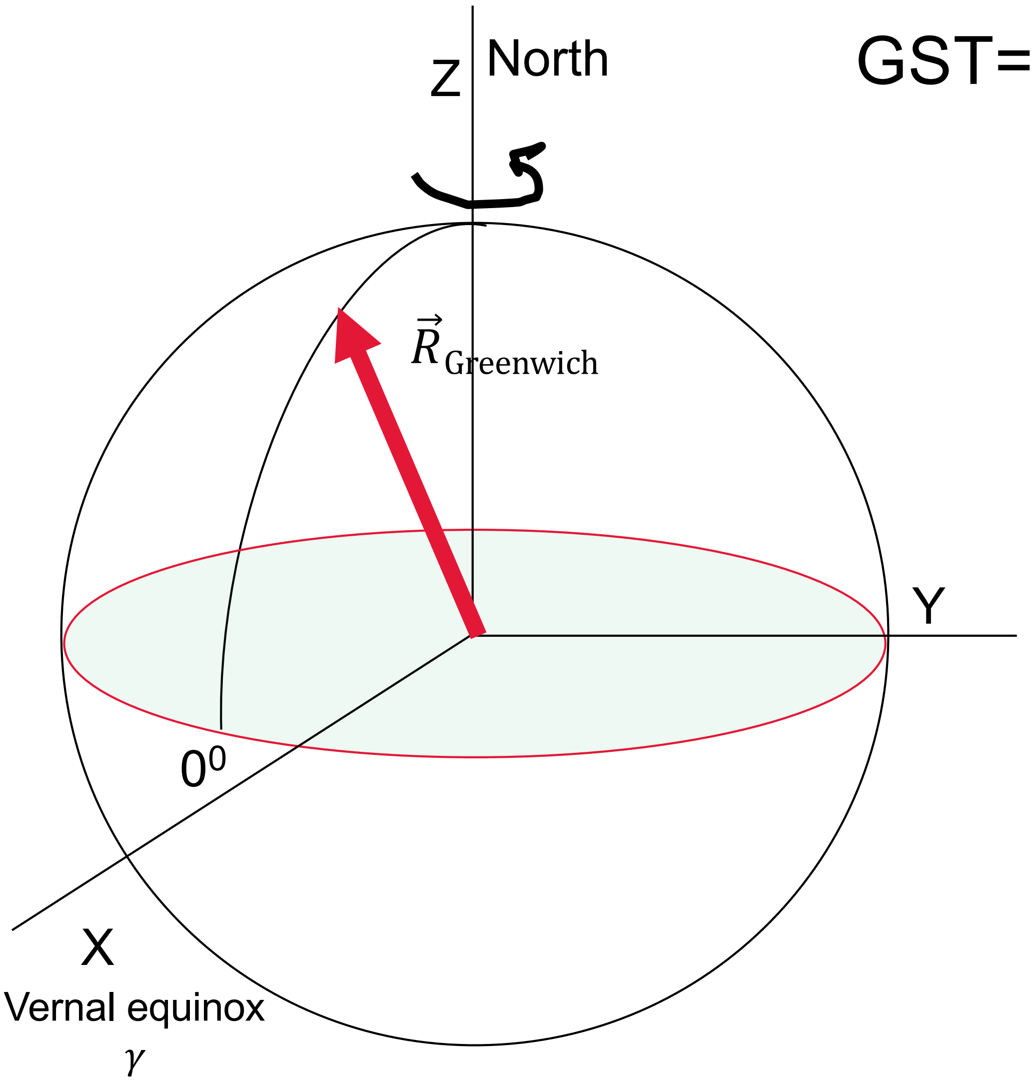


LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

$\Phi_o = 0^\circ$ Longitude of Greenwich

GST and LST



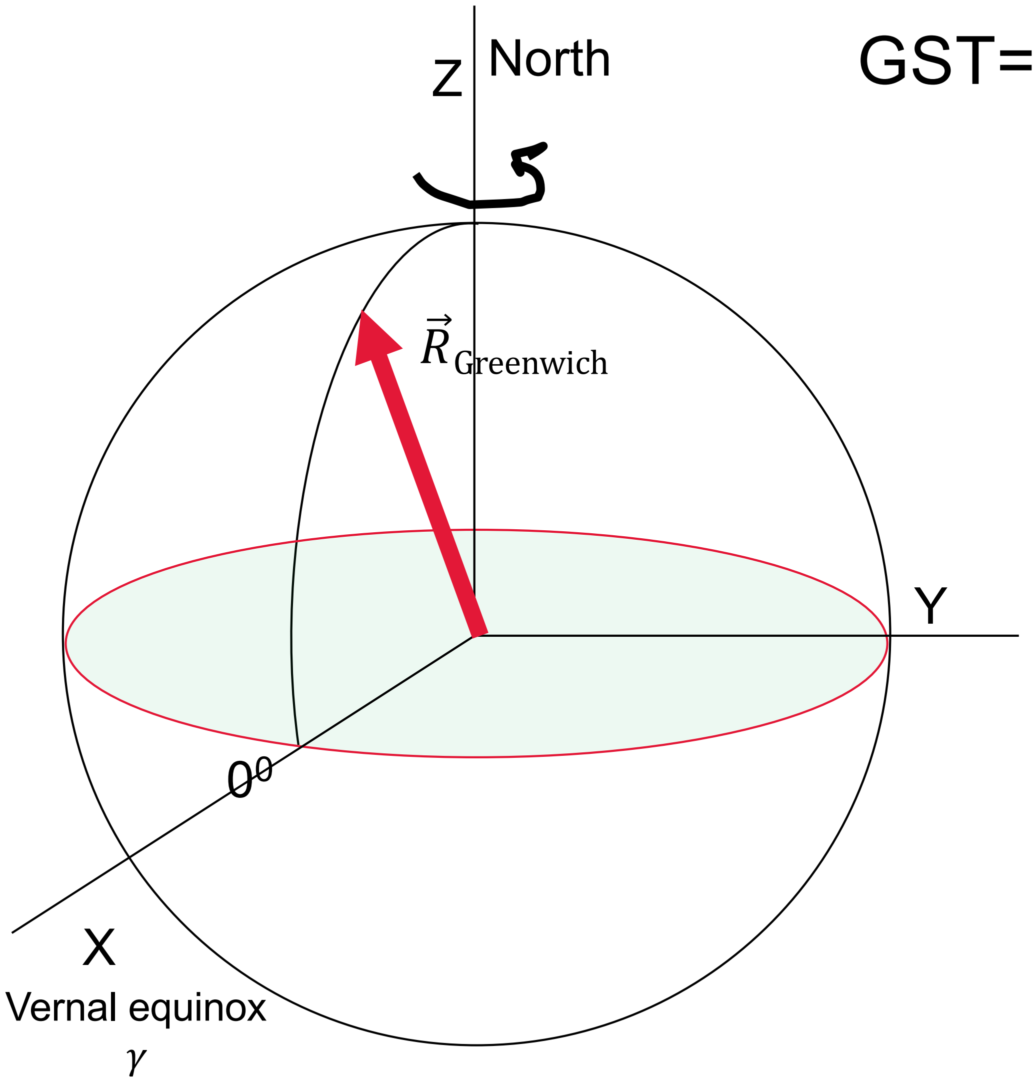
GST=23h

LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

$\Phi_o = 0^o$ Longitude of Greenwich

GST and LST



GST= 0h

LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

$\Phi_o = 0^o$ Longitude of Greenwich

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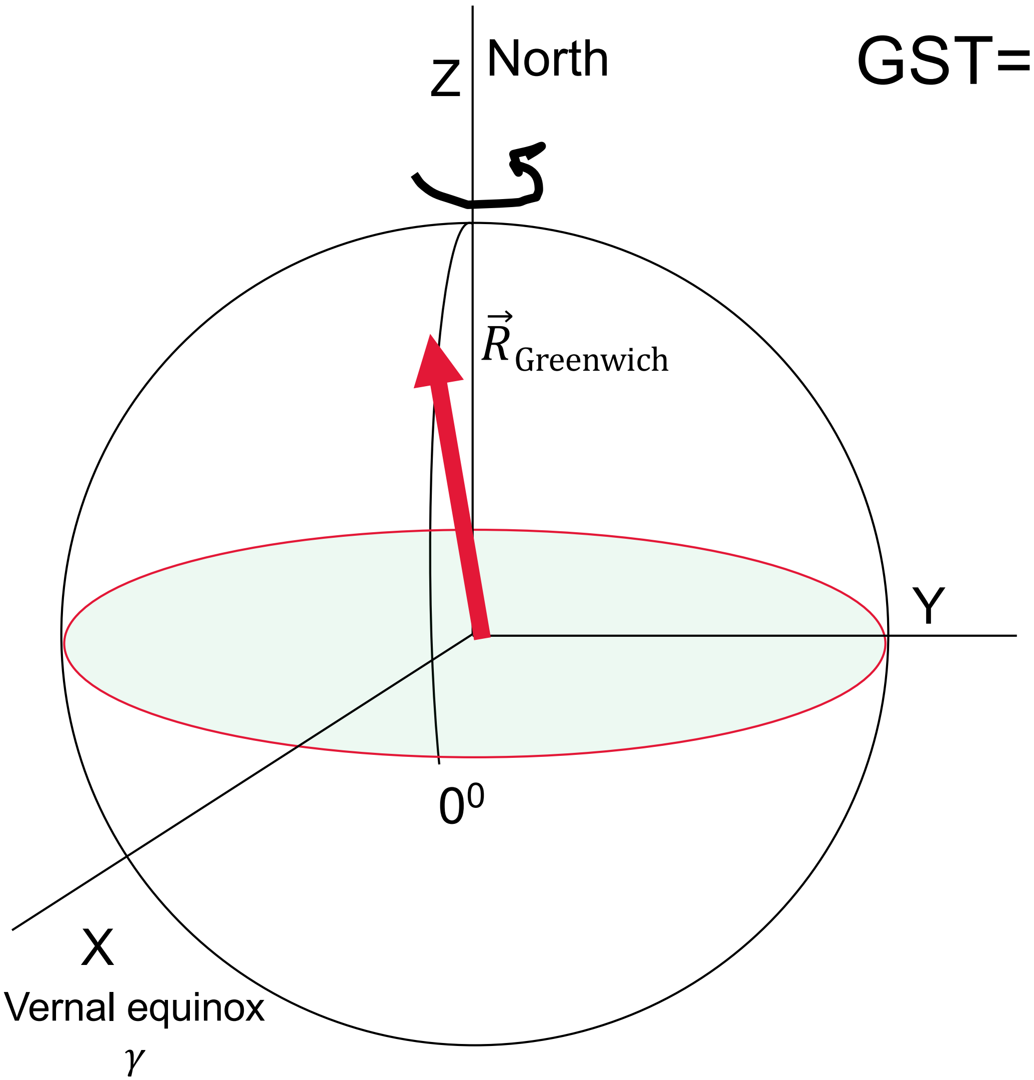
GST and LST



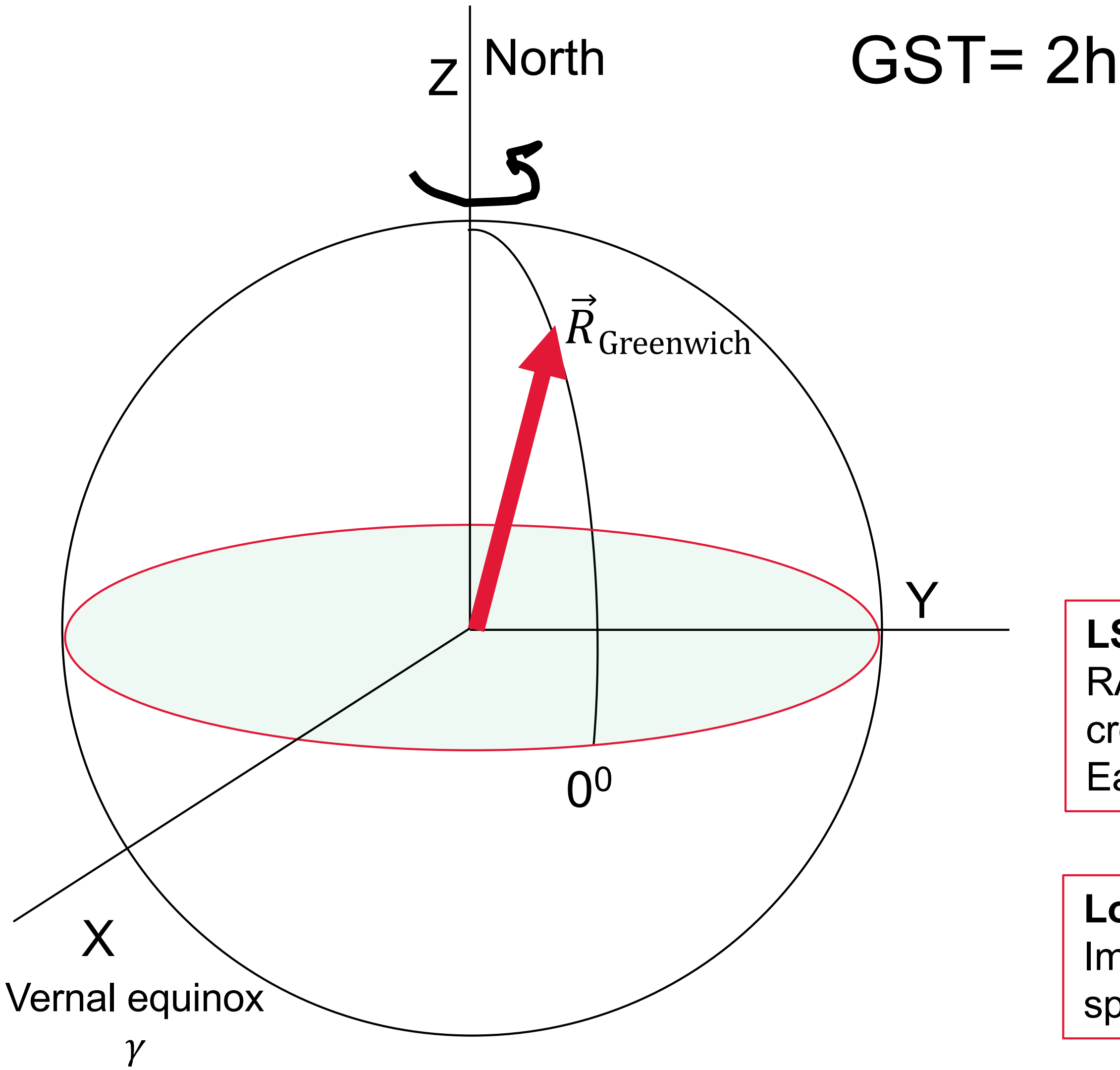
GST= 1h

LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

$\Phi_o = 0^o$ Longitude of Greenwich

GST and LST

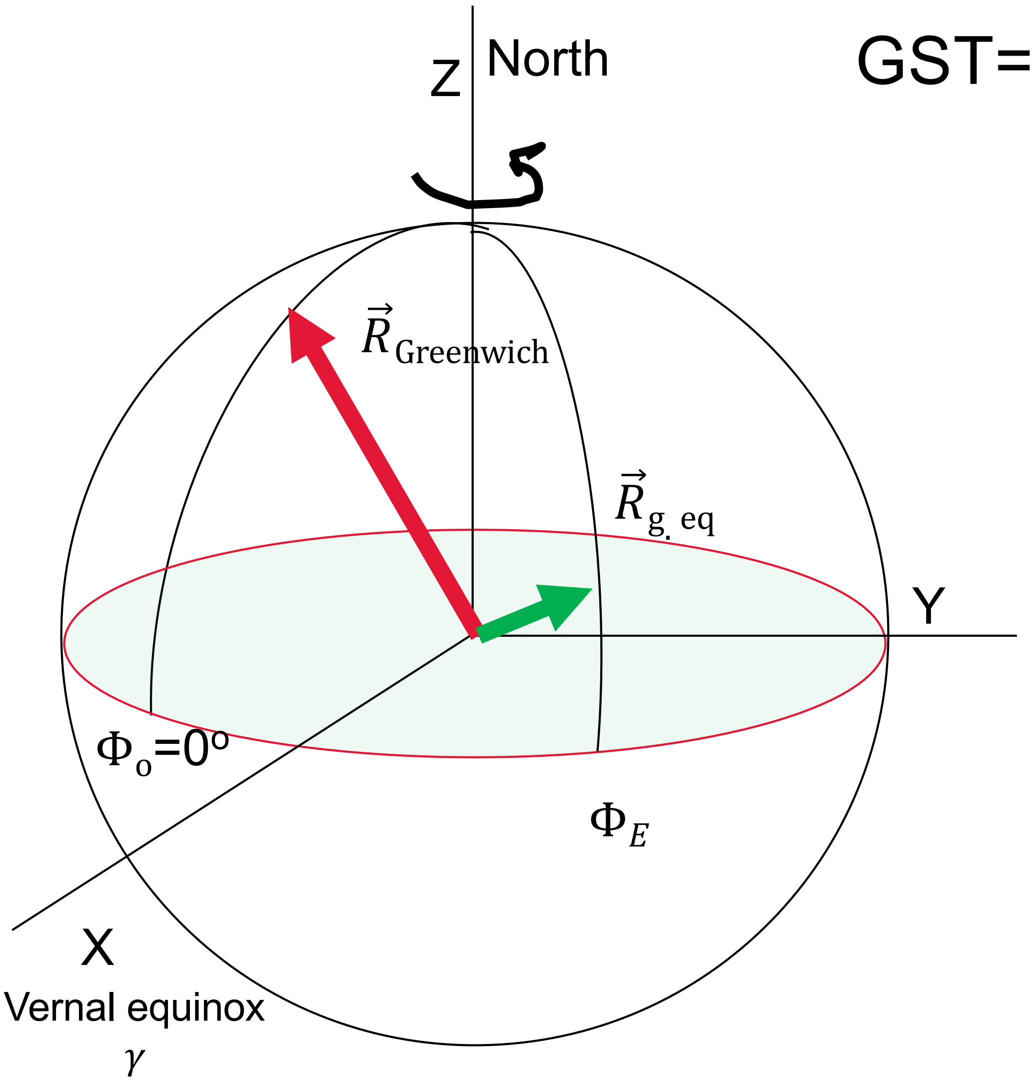


LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

$\Phi_o = 0^o$ Longitude of Greenwich

GST and LST



GST=22h

LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

Φ_o =0°	Longitude of Greenwich
Φ_E	Longitude of Earth station

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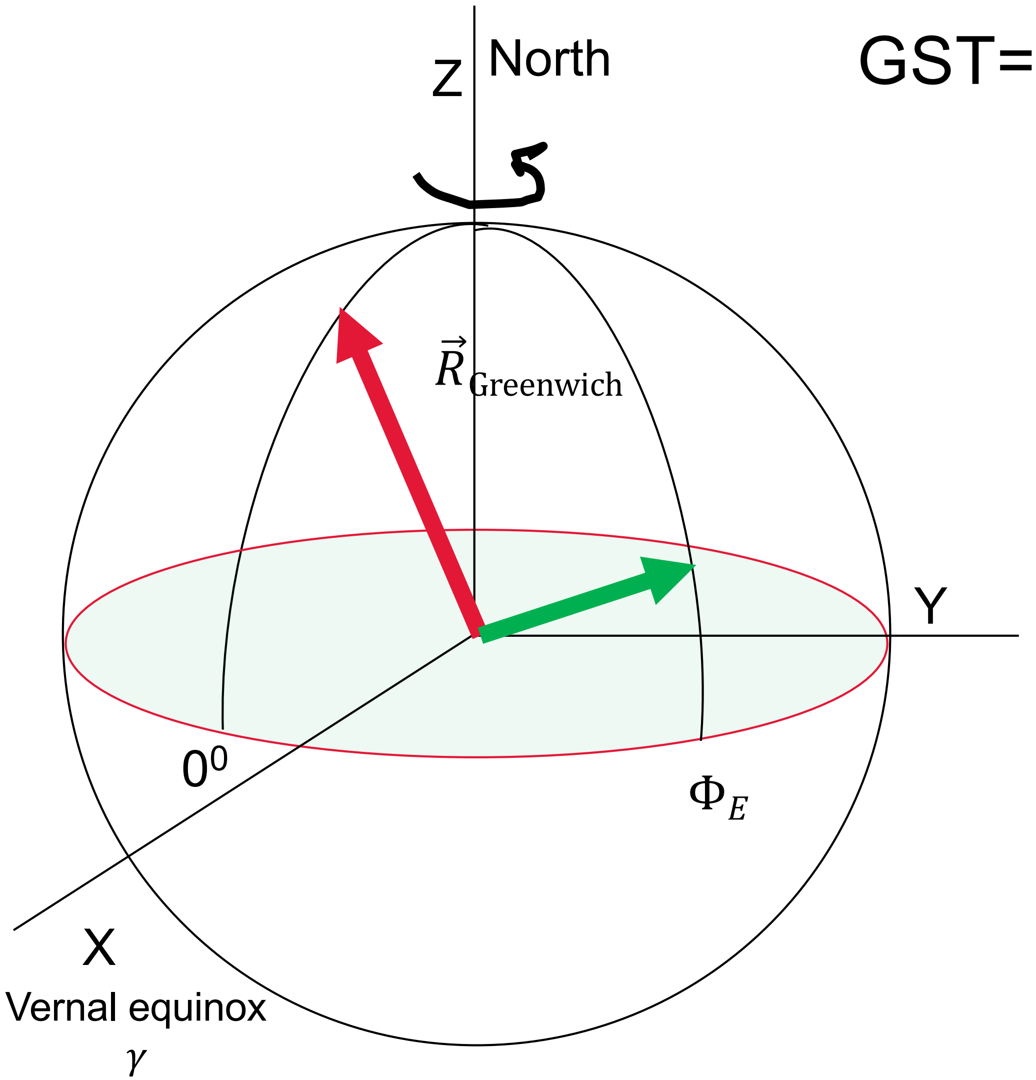
GST and LST

GST=23h

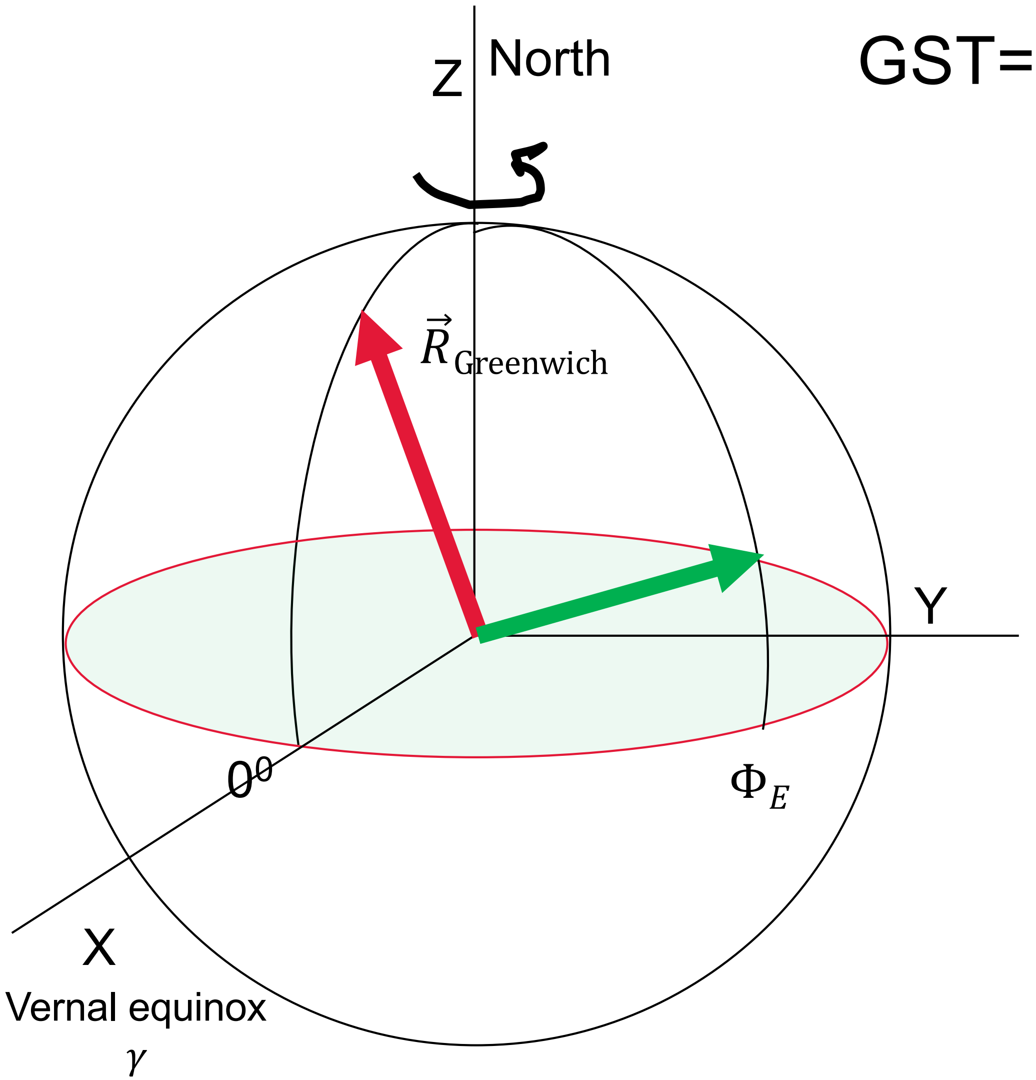


LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

$\Phi_o = 0^o$ Longitude of Greenwich

GST and LST



GST= 0h

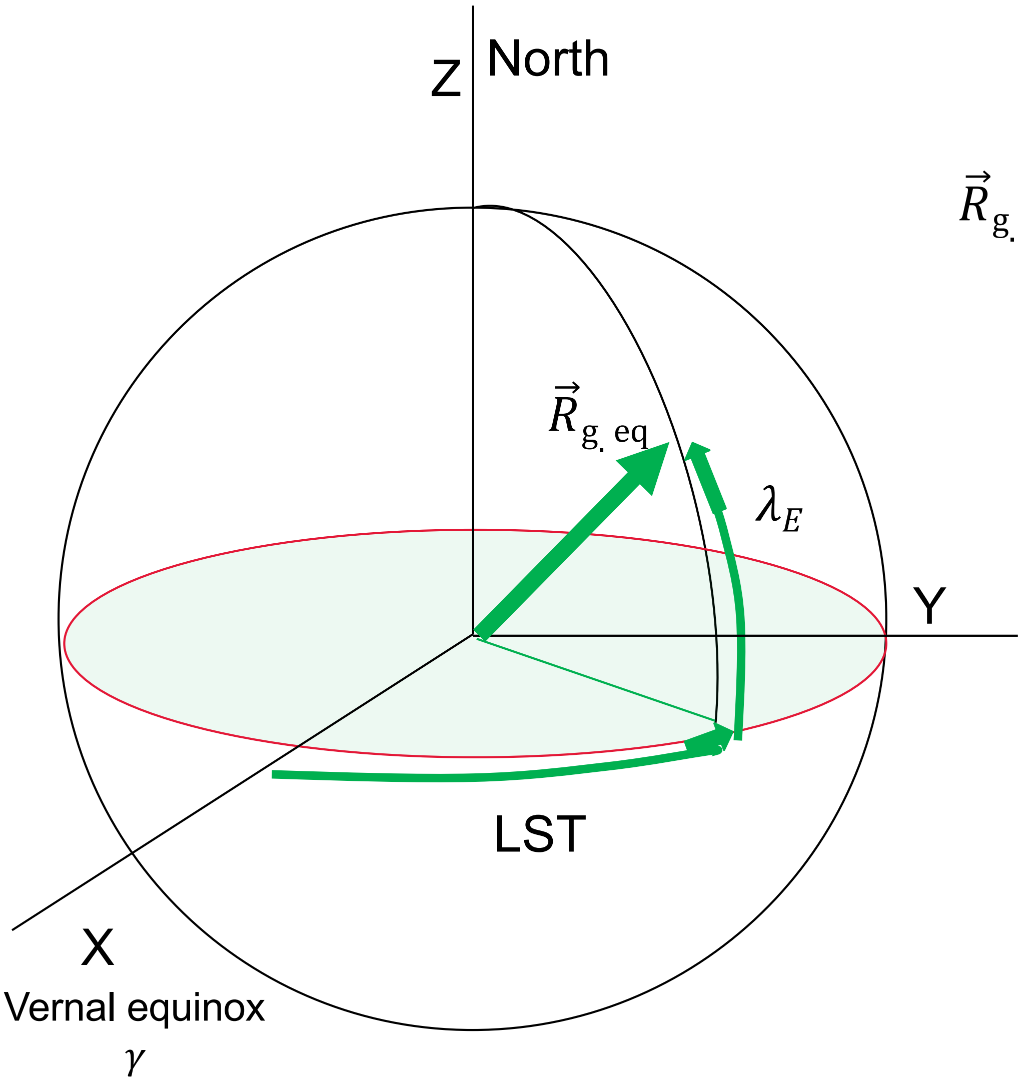
LST of Earth station:
RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
Imaginary great circle on the celestial sphere from north through the zenith to south.

$\Phi_o = 0^o$ Longitude of Greenwich


US Dept of State Geographer
© 2020 Google
© 2020 ORION-ME
Image Landsat / Copernicus
Google Earth
24°27'02.01" N 60°01'04.01" E eye alt 11119.97 km

3. Locate Earth station in geocentric equatorial coordinate system



$$\vec{R}_{g.\ eq} = \begin{bmatrix} |R_\oplus + H| \cos\lambda_E \cos(LST) \\ |R_\oplus + H| \cos\lambda_E \sin(LST) \\ |R_\oplus + H| \sin\lambda_E \end{bmatrix} = \begin{bmatrix} R_x \\ R_y \\ R_z \end{bmatrix}$$

$R_\oplus$: Earth radius at λ_E
H : height above mean sea level
λ_E : Latitude of Earth station
LST Local sidereal time. (24h 360°)

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RA of a celestial object that is currently crossing the local meridian of the Earth station.

Local meridian:
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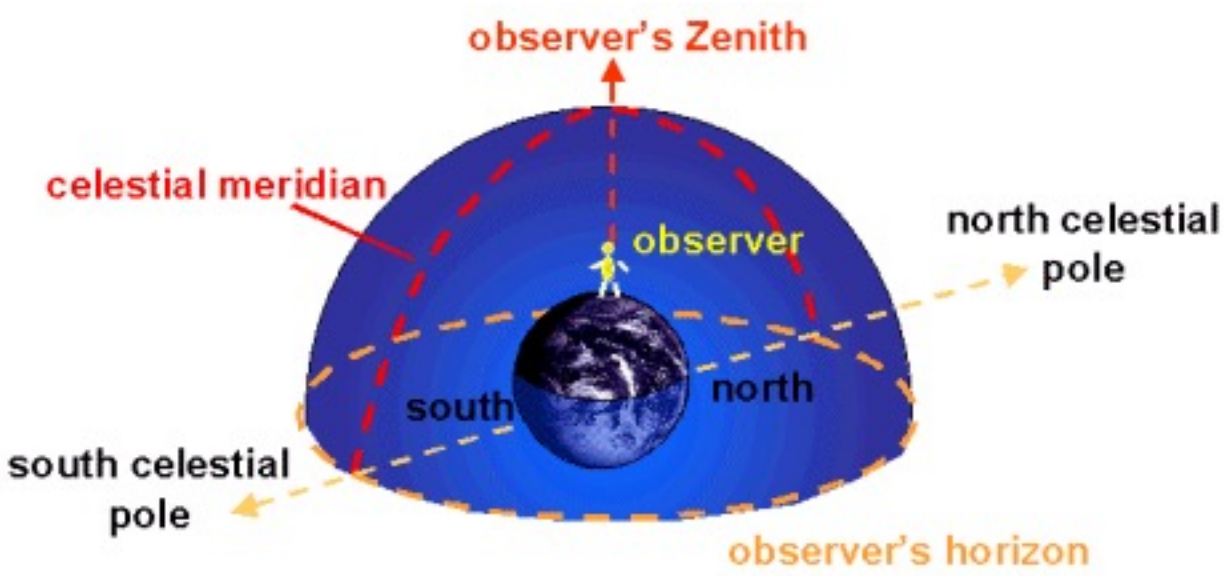
LST, RA and HA



Wikipedia

LST 0h when local meridian of Earth station cuts through the direction to the vernal equinox (RA= 0h) during the course of the day

HA: Hour angle

HA = LST - RA

Seen from Earth station, a celestial object
rises (HA < 0h)
culminate (HA = 0h)
sets (HA > 0h)

How is LST related to standard time?

LST ⟷ GST ⟷ UT ⟷ standard time

a) $LST = GST + \Phi_E$ Φ_E : Longitude of location

Example 2-6

York: longitude= 79^0 35' W

➔ $\Phi_E = -79^0\ 35'$

if $GST = 120^0$
$= 8^h$

➔ $LST = 40^0\ 25'$
$= 2^h\ 41^m\ 40^s$

Both LST and GST are measured relative to fixed stars
Unit: sidereal day which is < mean solar day.

James Cook 1728 - 1779

His goal was to find the Great South Land

He charted the east coast of Australia with a clock without a pendulum and claimed the land for Great Britain

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Mean sidereal day and mean solar day

$P_{\oplus, orbit.}$ = 366.2422 mean sidereal days
= 365.2422 mean solar days

Leap year: if year is divisible by 4, except if it is a centennial year. However, if the centennial year is divisible by 400, then it is also a leap year.

JD: Julian date: continuous count of days since the beginning of the Julian period on 1 January 4713 BC

GST: Greenwich sidereal time = hour angle (HA) of the(average position of the vernal equinox

GST[deg] = 99.6909833+36000.7689 T_c + 0.00038708 T_c^2 +UT[deg]

T_c = (JD-2415020)/36525 Julian centuries
= elapsed time in Julian centuries between Julian day JD and noon UT on Jan 0, 1900 (Jan 0.5, 1900).

UT or UTC: Universal time coordinated (based on atomic time given by Cesium clocks; the atomic time is broadcasted).

UT[deg] = 360 [1/24(h + min/60 + sec/60)] [deg]

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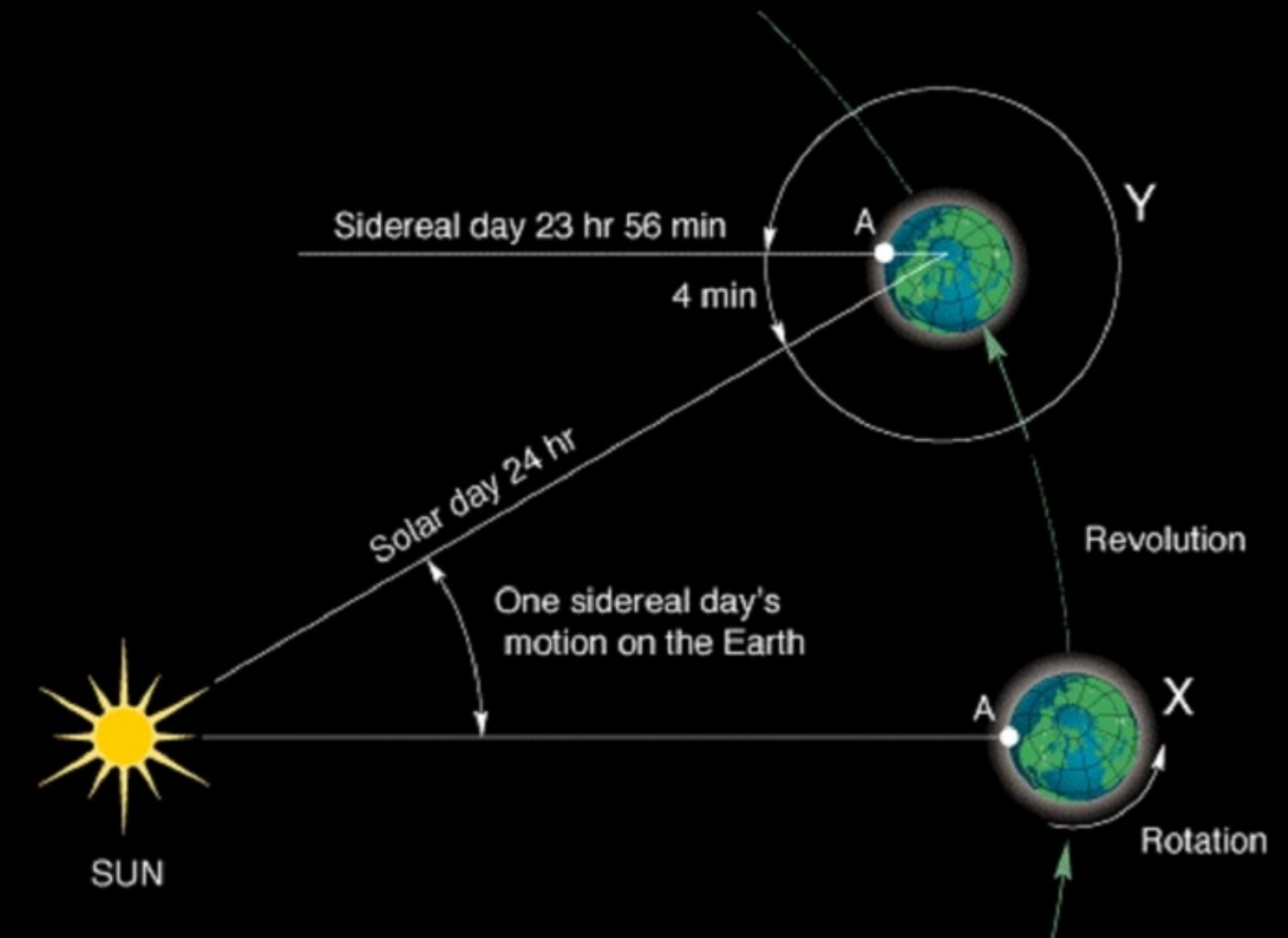
Sidereal Day vs. Solar Day



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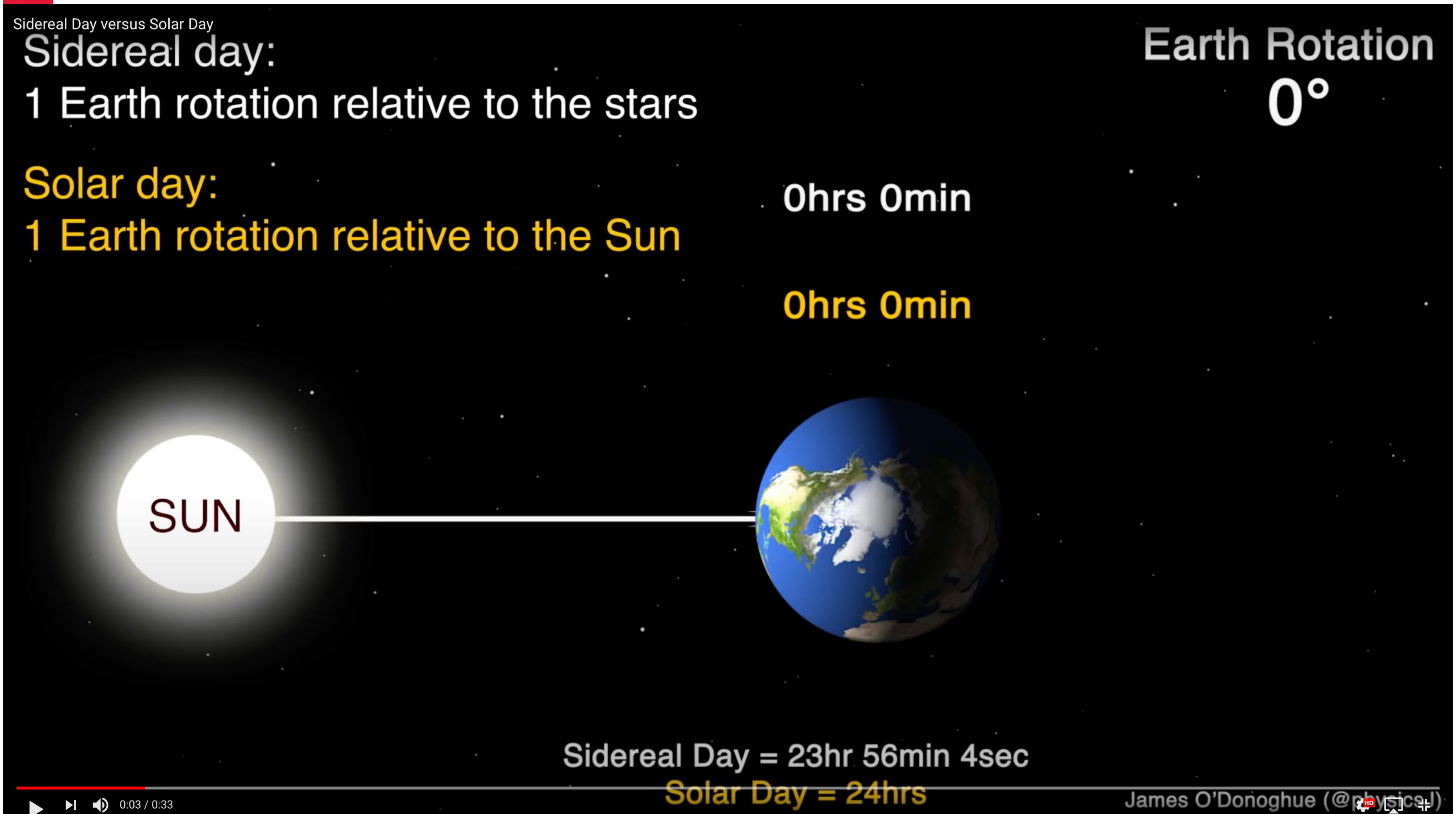

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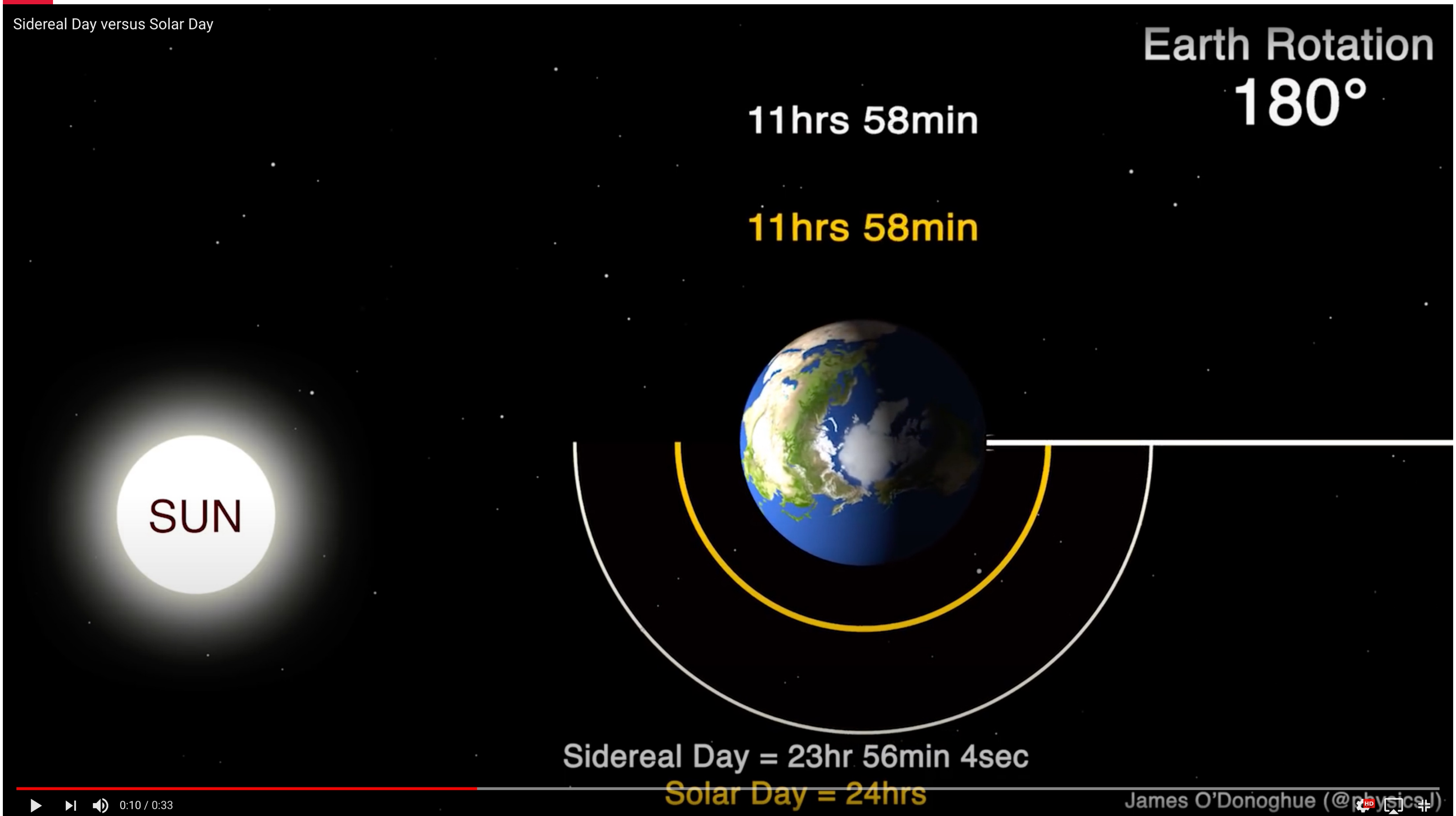
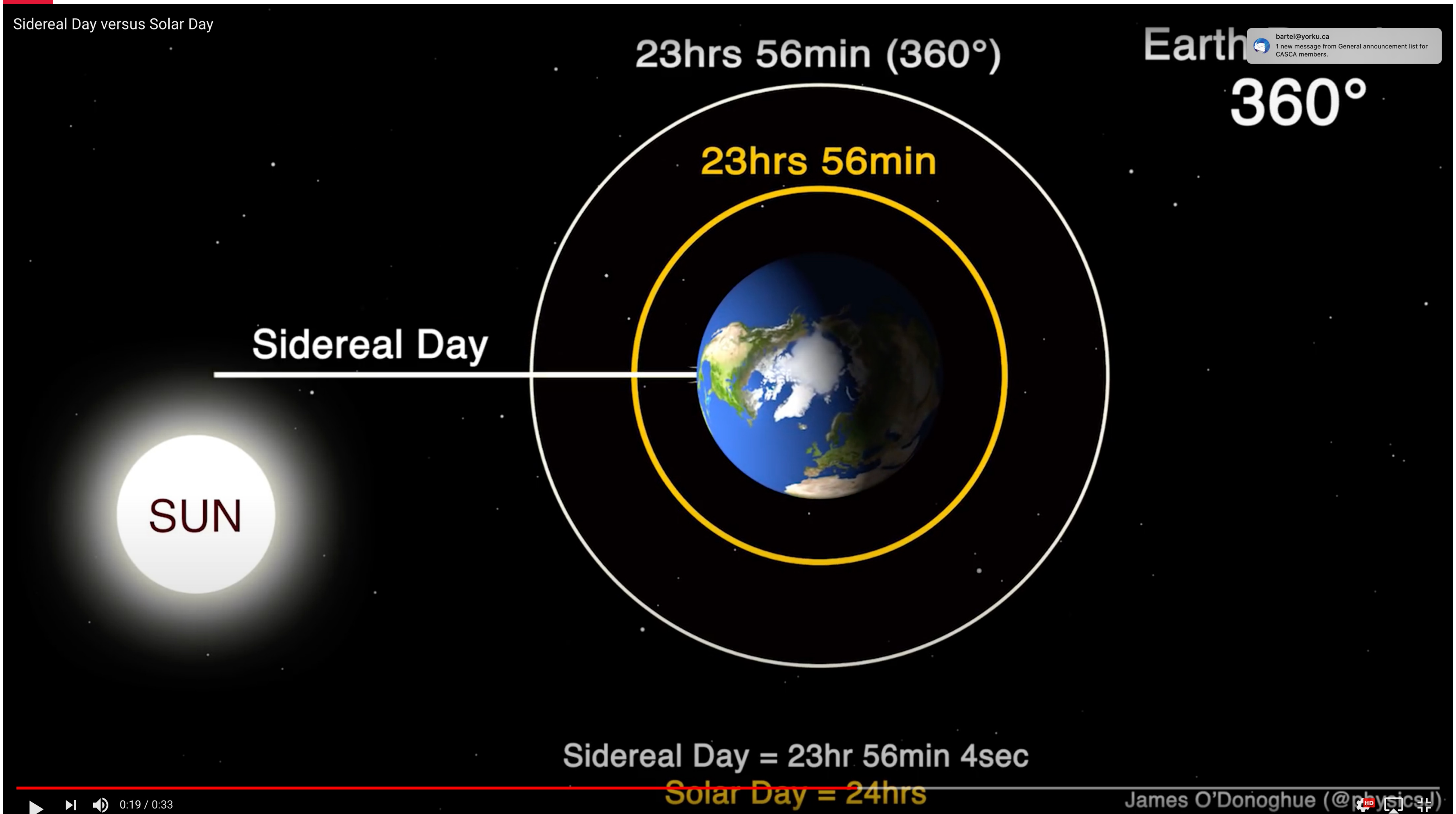
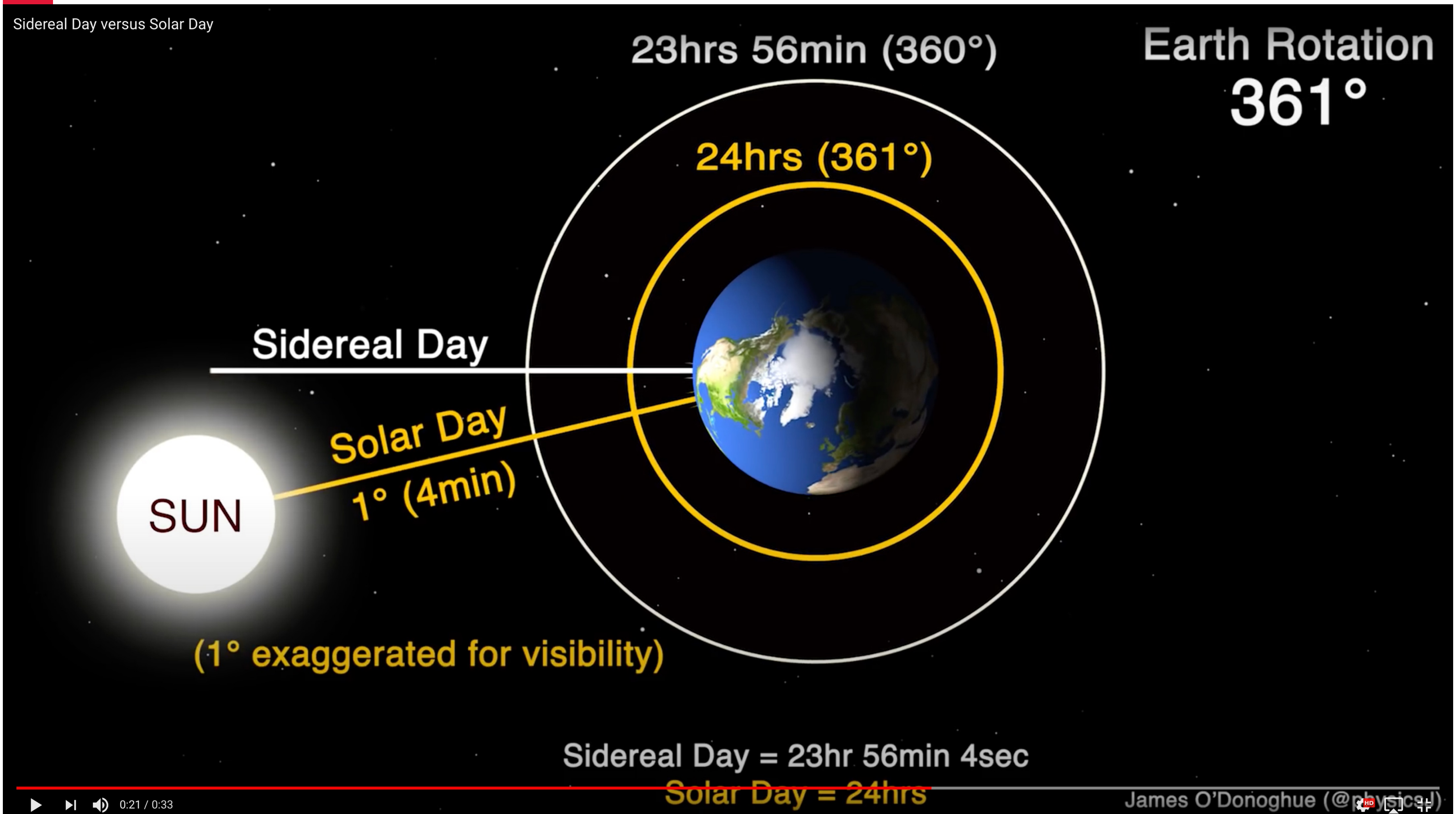


Sidereal Day versus Solar Day
Earth Rotation
180°
11hrs 58min
11hrs 58min
SUN
Sidereal Day = 23hr 56min 4sec
Solar Day = 24hrs
0:10 / 0:33


Sidereal Day versus Solar Day
23hrs 56min (360°)
Earth
bartel@yorku.ca
1 new message from General announcement list for CASCA members.
360°
23hrs 56min
Sidereal Day
SUN
Sidereal Day = 23hr 56min 4sec
Solar Day = 24hrs
0:19 / 0:33
James O'Donoghue

J. O'Donoghue


Sidereal Day versus Solar Day
23hrs 56min (360°)
Earth Rotation
361°
24hrs (361°)
Sidereal Day
Solar Day
1° (4min)
SUN
(1° exaggerated for visibility)
Sidereal Day = 23hr 56min 4sec
Solar Day = 24hrs
0:21 / 0:33

J. O'Donoghue

Example 2-7

What is GST on 28 January 1994 at 12:00 UT?

0.0 Jan 1994: JD = 2449352.5
28.5 Jan 1994: + 28.5

2449381.0

T_c =(2449381.0 – 2415020.0)/36525
= 0.9407529

UT=180 deg

GST = 34,147.51907 [deg]
= 307.51907 [deg] (94 • 360 deg subtracted)
= 307^0 31' 8.652"
= $20^h\ 30^m\ 4.5768^s$

On 28 January 1994 at 12:00 UT, GST = $20^h\ 30^m\ 4.5768^s$

On 28 September 2020 at 09:00 UT, GST = $13^h\ 31^m\ 3.9^s$

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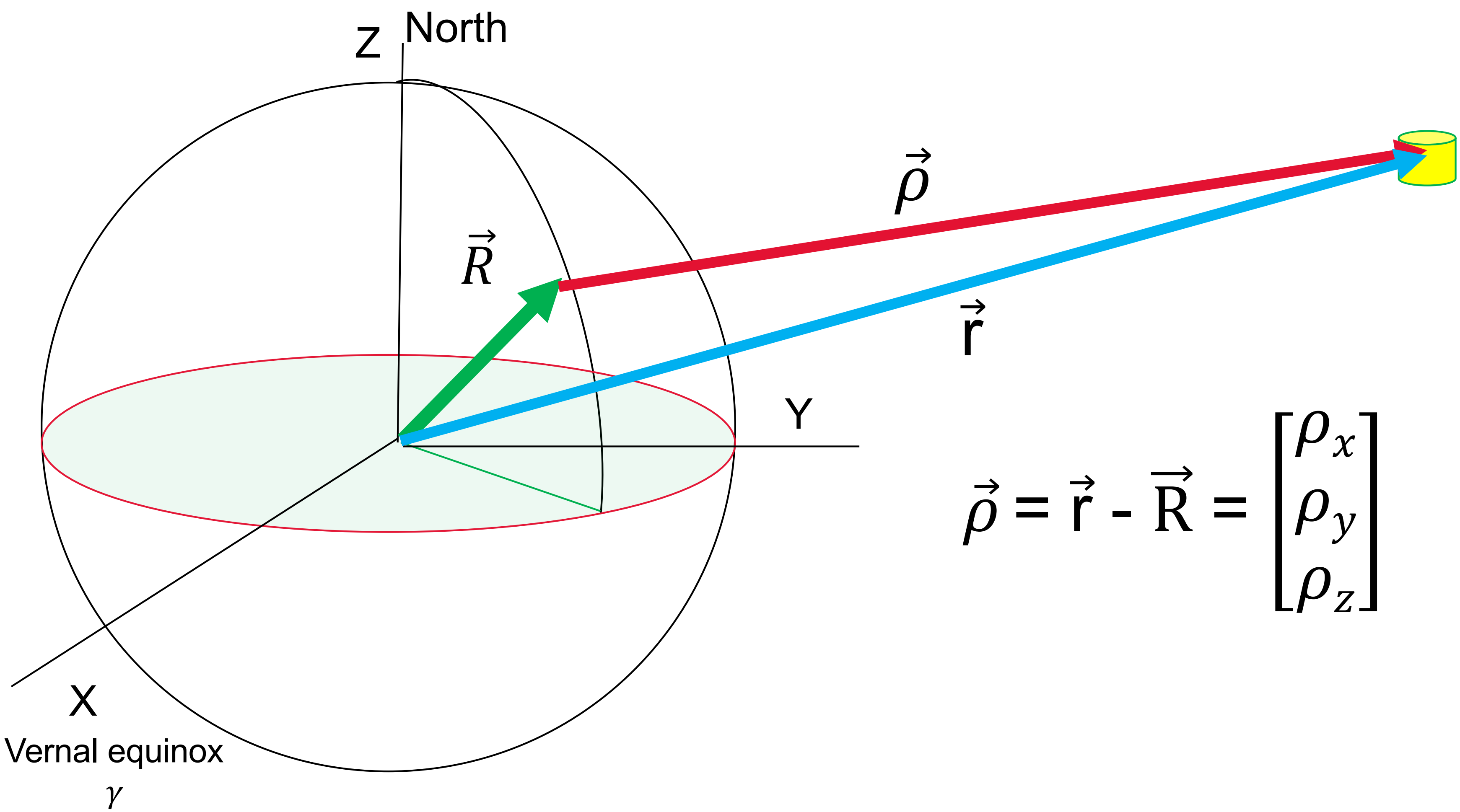
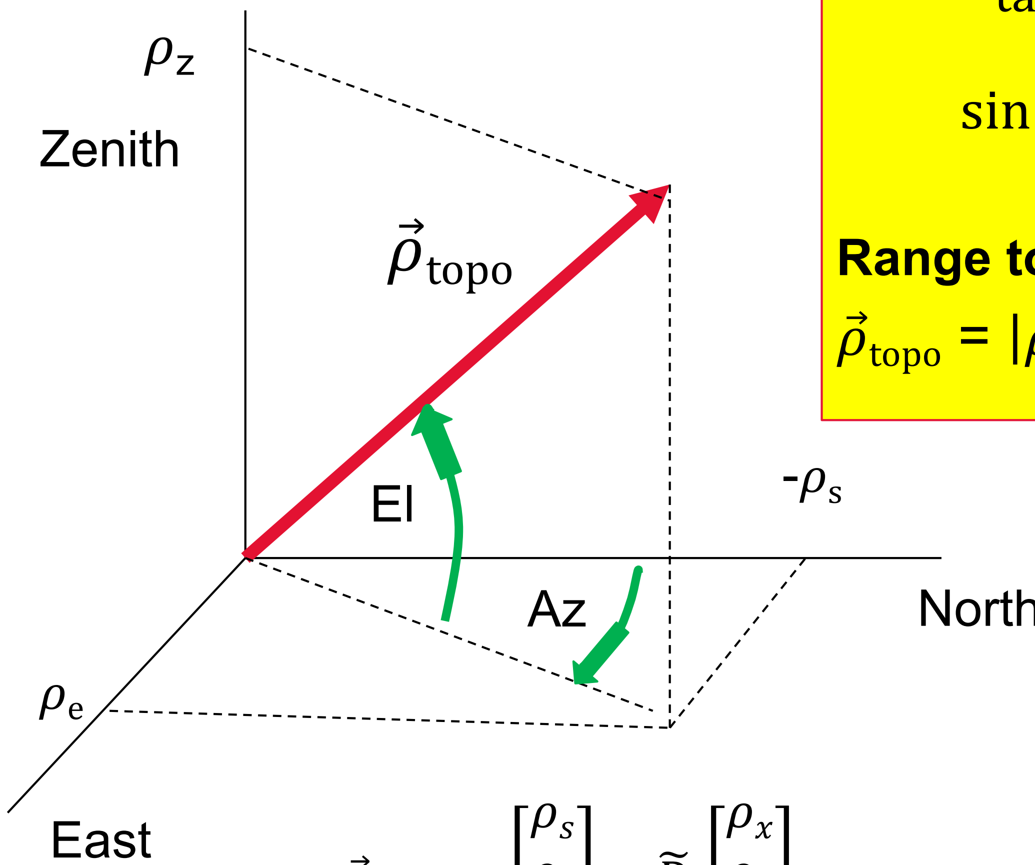
4. Locate satellite in topocentric-horizon coordinate system

a) Calculate range vector in geocentric equatorial coordinate system



$$\vec{\rho} = \vec{r} - \vec{R} = \begin{bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{bmatrix}$$

b) Make coordinate transformation to the topocentric horizon coordinate system



Antenna look angles: Az, El

$$\tan Az = -\frac{\rho_E}{\rho_s}$$

$$\sin El = -\frac{\rho_Z}{|\rho|}$$

Range to the satellite

$$\vec{\rho}_{\text{topo}} = |\rho| = \sqrt{\rho_S^2 + \rho_E^2 + \rho_Z^2}$$

$$\vec{\rho}_{\text{topo}} = \begin{bmatrix} \rho_s \\ \rho_e \\ \rho_z \end{bmatrix} = \widetilde{D} \begin{bmatrix} \rho_x \\ \rho_y \\ \rho_z \end{bmatrix}$$

$\widetilde{D}$ Matrix elements are functions of LST and λ_e

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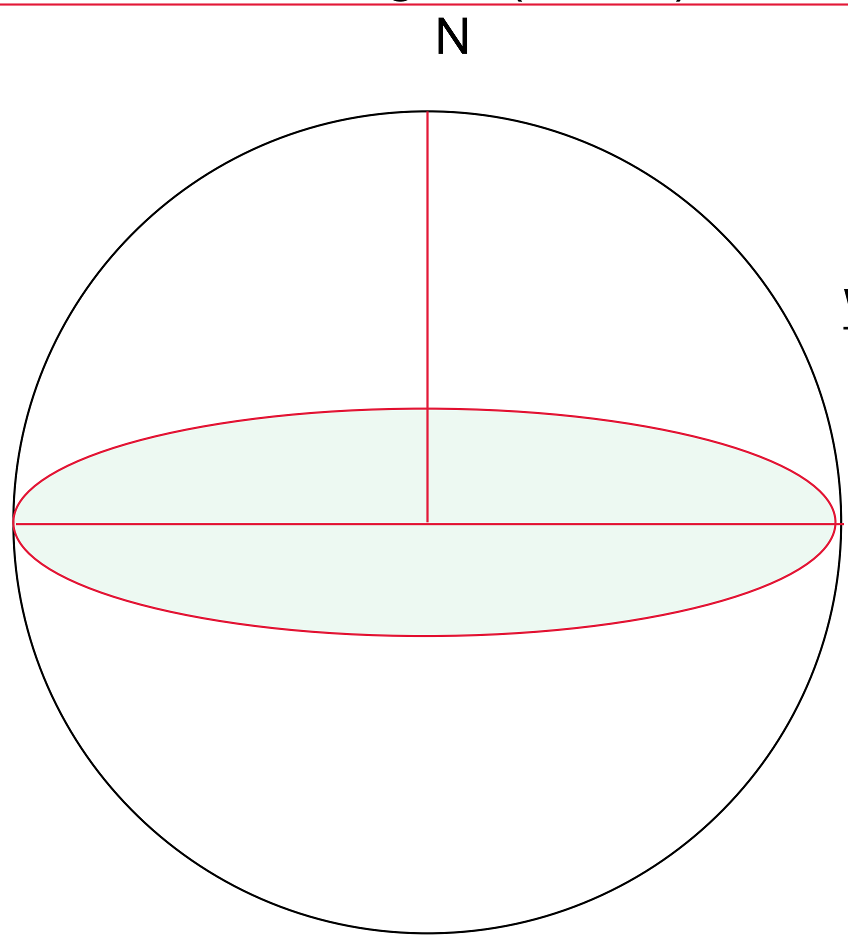
Special case:

Antenna look angles (Az, El) and range ($|\rho|$) for a geostationary satellite

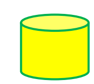


Given: Φ_E : longitude of Earth station

λ_E : latitude of earth station

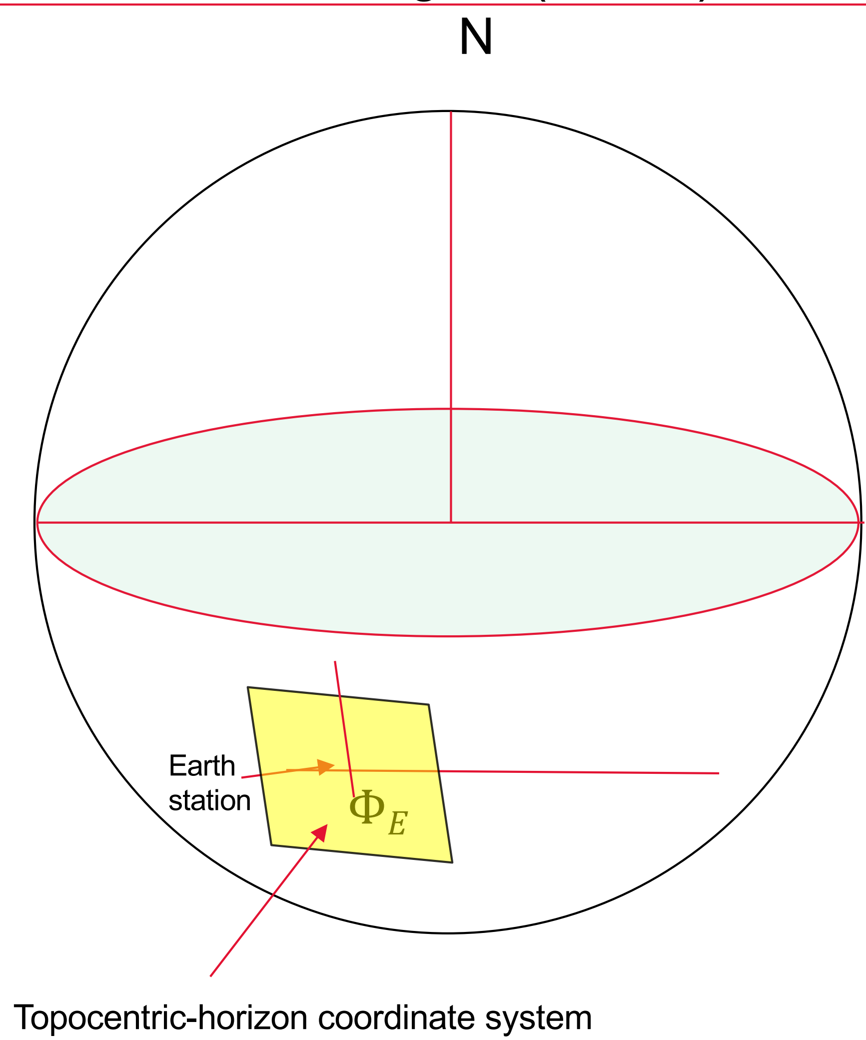
Φ_S : longitude of sub-satellite point

What are Az, El, $|\rho|$, $|\Phi_E - \Phi_S|_{\text{lim}}$ (visibility limits)?



Special case:

Antenna look angles (Az, El) and range ($|\rho|$) for a geostationary satellite



Given: Φ_E : longitude of Earth station

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What are Az, El, $|\rho|$, $|\Phi_E - \Phi_S|_{\text{lim}}$ (visibility limits)?

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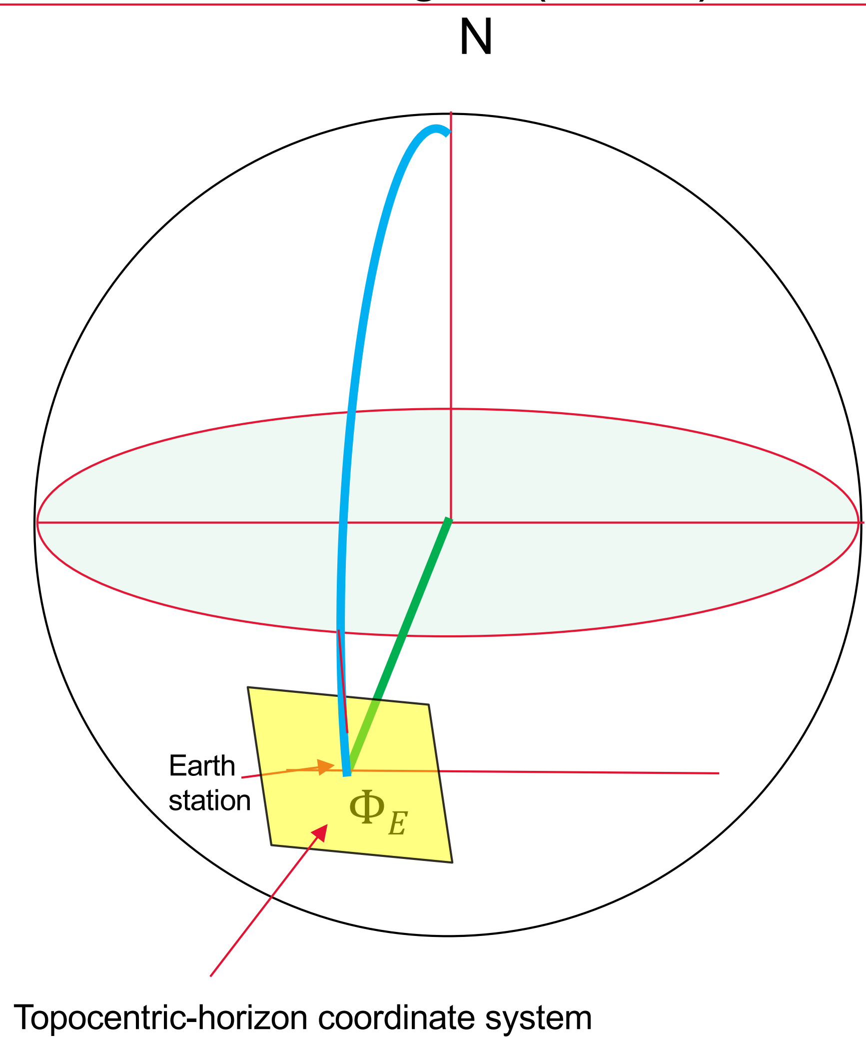
Antenna look angles (Az, El) and range ($|\rho|$) for a geostationary satellite



Given: Φ_E : longitude of Earth station

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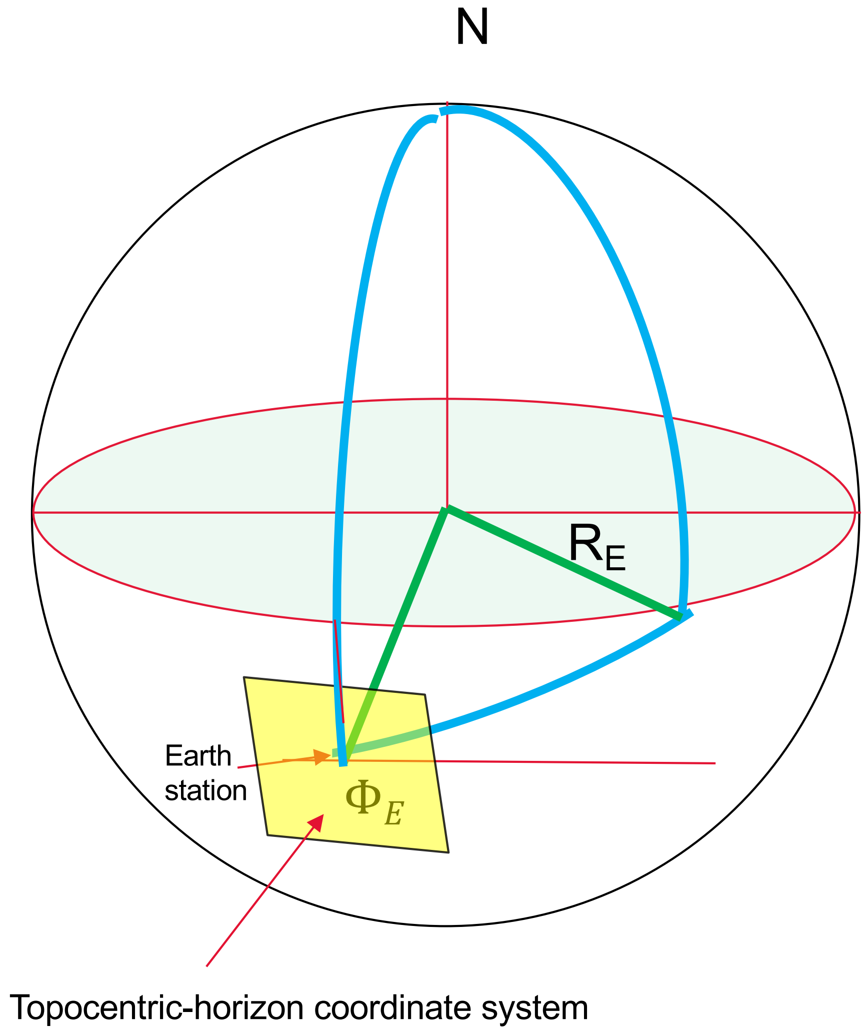
<u>What are Az, El, $|\rho|$, $|\Phi_E - \Phi_S|_{lim}$ (visibility limits)?</u>

Special case:

Antenna look angles (Az, El) and range ($|\rho|$) for a geostationary satellite



Given: Φ_E : longitude of Earth station

λ_E : latitude of earth station

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What are Az, El, $|\rho|$, $|\Phi_E - \Phi_S|_{\text{lim}}$ (visibility limits)?

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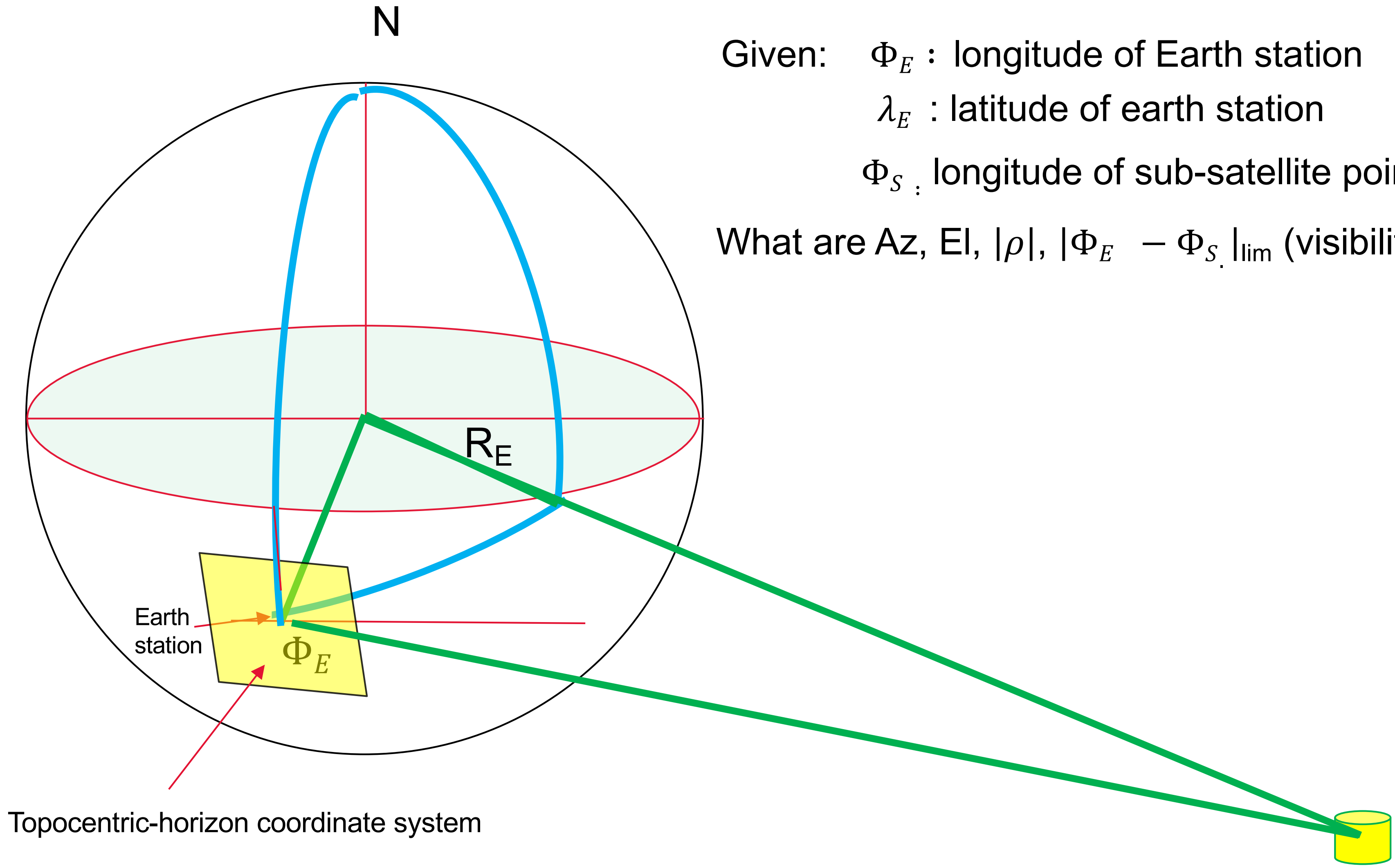

Given: Φ_E : longitude of Earth station

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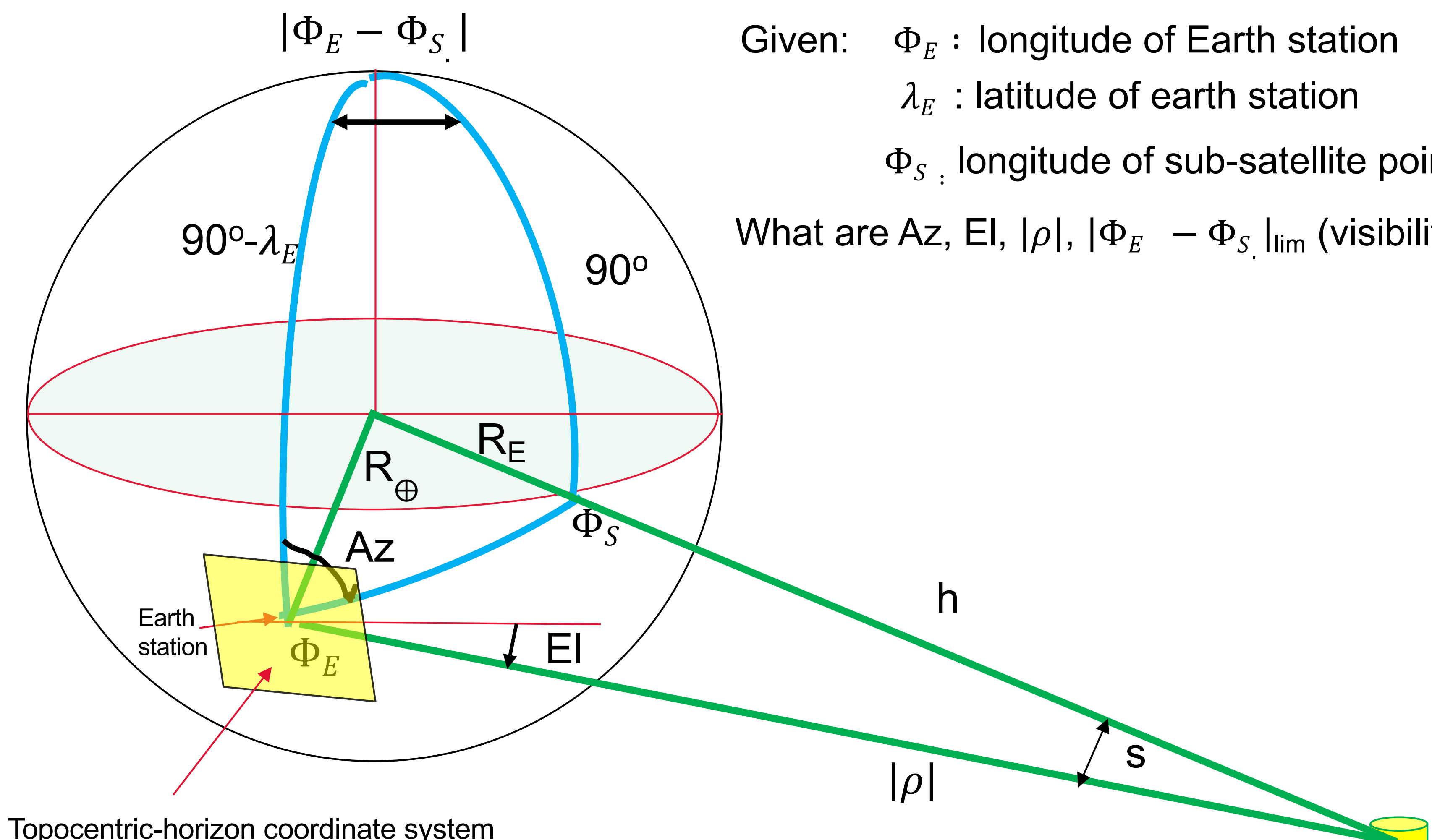
Special case:

Antenna look angles (Az, El) and range ($|\rho|$) for a geostationary satellite



Given: Φ_E : longitude of Earth station

λ_E : latitude of earth station

Φ_S : longitude of sub-satellite point

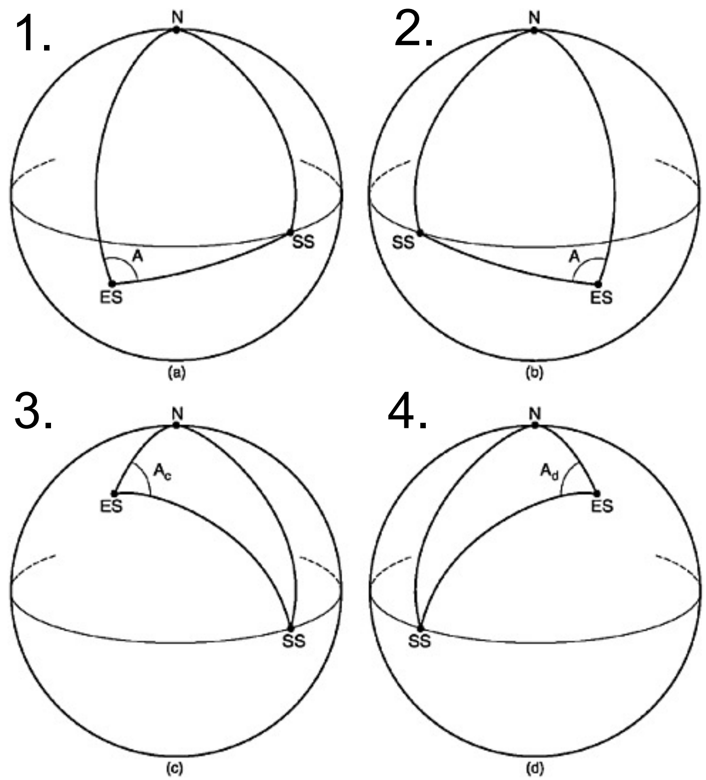
What are Az, El, $|\rho|$, $|\Phi_E - \Phi_S|_{\lim}$ (visibility limits)?

Az (Azimuth)

$$\tan A = \frac{-\tan(|\Phi_E - \Phi_S|)}{\sin \lambda_E}$$

1. With station in southern hemisphere (λ_E <0): $\Phi_E - \Phi_S < 0$: $\mathrm{Az} = \mathrm{A}$
2. With station in southern hemisphere (λ_E <0): $\Phi_E - \Phi_S > 0$: $\mathrm{Az} = 360^{o} - \mathrm{A}$

3. With station in northern hemisphere (λ_E >0): $\Phi_E - \Phi_S < 0$: $\mathrm{Az} = 180^{o} + \mathrm{A}$
4. With station in northern hemisphere (λ_E >0): $\Phi_E - \Phi_S > 0$: $\mathrm{Az} = 180^{o} - \mathrm{A}$



D. Roddy (2006)

Az (Azimuth)

$$\tan A = \frac{-\tan(|\Phi_E - \Phi_S|)}{\sin \lambda_E}$$

1. With station in southern hemisphere (λ_E <0): $\Phi_E - \Phi_S < 0$: $\mathrm{Az} = \mathrm{A}$
2. With station in southern hemisphere (λ_E <0): $\Phi_E - \Phi_S > 0$: $\mathrm{Az} = 360^o - \mathrm{A}$

3. With station in northern hemisphere (λ_E >0): $\Phi_E - \Phi_S < 0$: $\mathrm{Az} = 180^o + \mathrm{A}$
4. With station in northern hemisphere (λ_E >0): $\Phi_E - \Phi_S > 0$: $\mathrm{Az} = 180^o - \mathrm{A}$

Example 2-8

1. $|\Phi_E - \Phi_S|$=60^o λ_E = -15^o

$$\tan A = \frac{-\tan(60)}{\sin(-15)} = \frac{-1.732}{-0.259} = 6.687$$

A=81.5^o

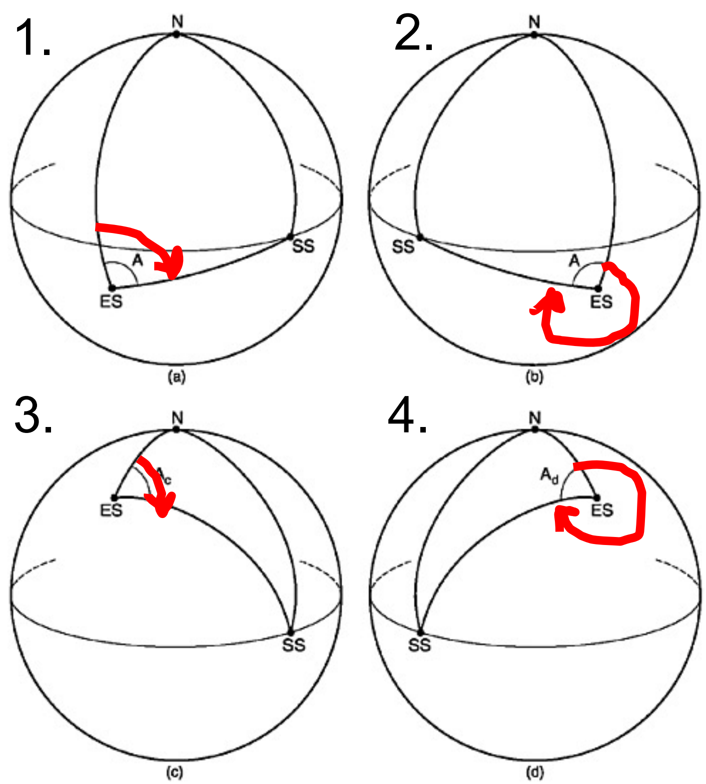
Az=81.5^o

4. $|\Phi_E - \Phi_S|$=60^o λ_E = 50^o

$$\tan A = \frac{-\tan(60)}{\sin(50)} = \frac{-1.732}{0.766} = -2.261$$

A= -66.1^o

Az =246.1^o



D. Roddy (2006)

El (Elevation)

$$\cos El = \frac{R_E+h}{\rho}\sin c \qquad \text{with} \qquad \cos c = \cos\lambda_E \cos(\Phi_E - \Phi_S)$$

$|\rho|$ (Range)

$$|\rho| = \left[R_\oplus^2 + (R_E+h)^2 - 2R_\oplus(R_E+h)\cos c\right]^{1/2}$$

Note: $R_\oplus = Re\,(1 - \frac{\sin^2\lambda_e}{298.257})$

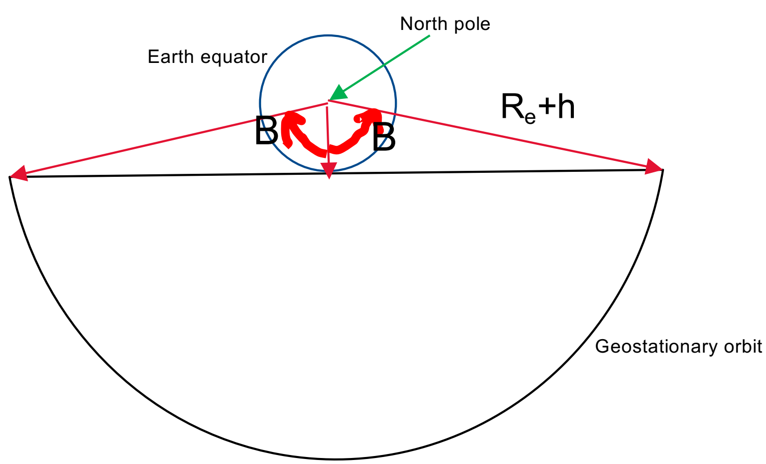
B (Limits of visibility)

$$B = \cos^{-1}\left\{\frac{\sin\left[El_{min} + \sin^{-1}\left(\frac{R_\oplus \cos El_{min}}{R_E+h}\right)\right]}{\cos\lambda_E}\right\}$$

El_{min} = minimum pointing elevation for antenna

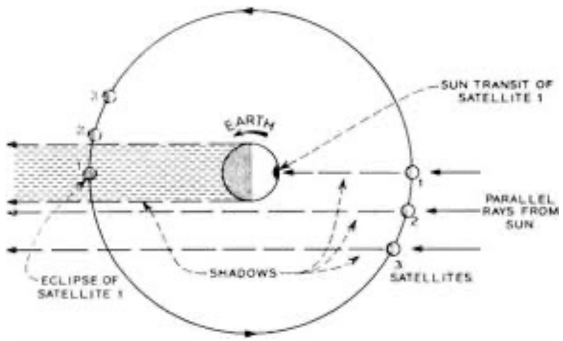
Satellites can be seen with sub-satellite longitudes +B and –B from the earth station longitude.

For earth station on equator and El_{min} =0° $\quad B=\cos^{-1}(\frac{6378.14}{42164.17})$ = 81.3°

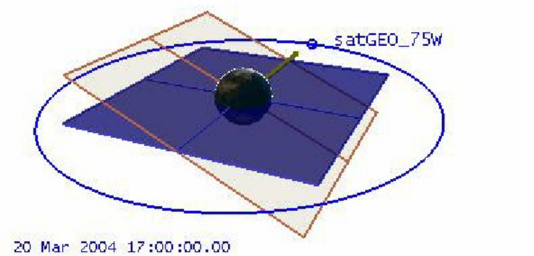


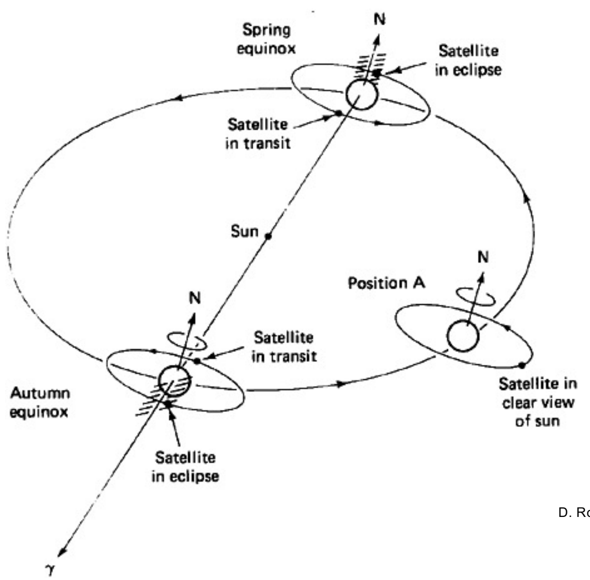
R_e+h: Geostationary orbit radius ➔ a

Earth eclipse of satellite



https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=6769530

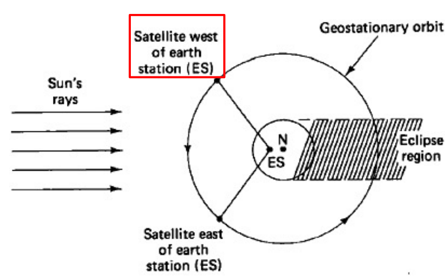




D. Roddy (2006)

Earth eclipse of satellite: Satellite gets into the shadow of Earth. That happens for LEO satellites frequently and would happen once each day also for geostationary satellites if the Earth equator were not tilted with respect to the Earth orbit.

Because of the obliquity of 23.4° geostationary satellites are in full view of the Sun throughout the year except around the equinoxes. For ± 23 days around the equinoxes a geostationary satellite is in the Earth shadow for 10 to 72 min/day. During these times batteries need to be used on the satellite.



Preferred positions for geostationary satellites: Satellites east of ES enter shadow early in the evening during busy times. A satellite west of ES enters shadow in the early morning hours when usage is low.

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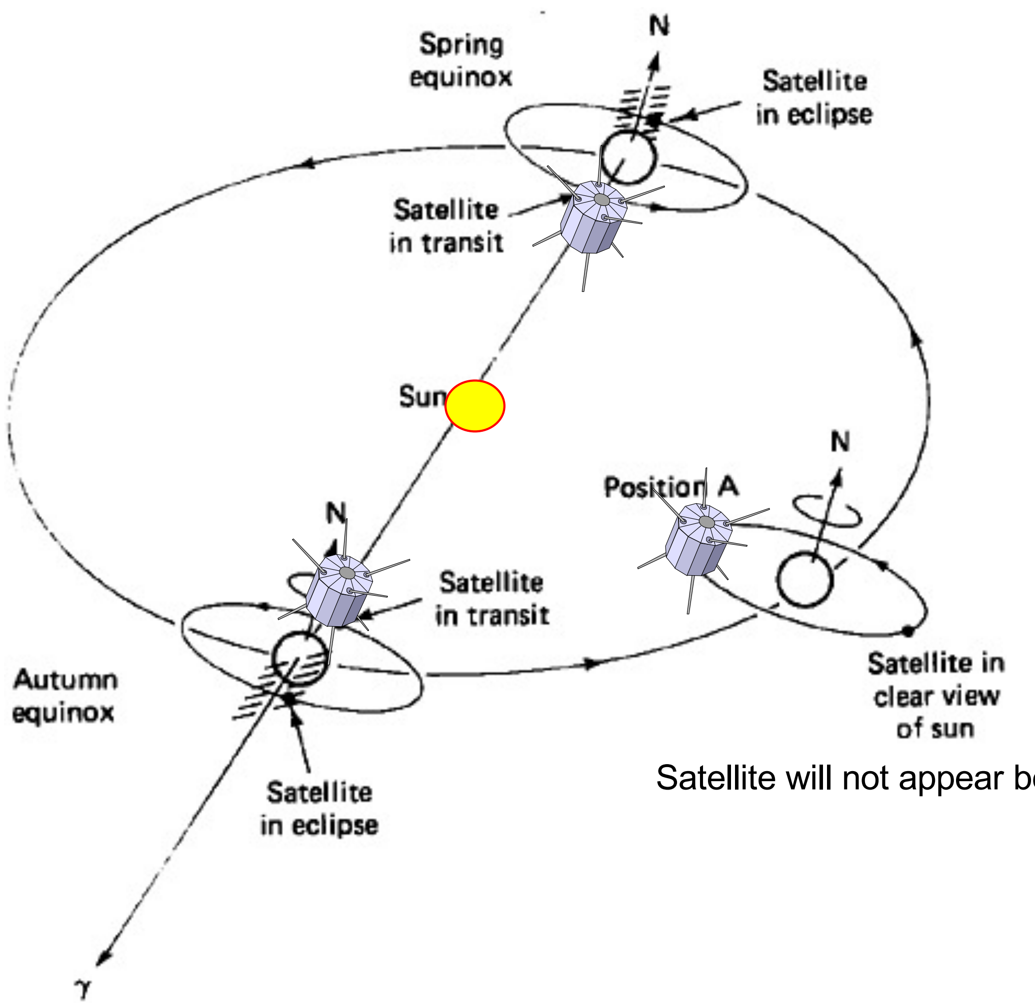
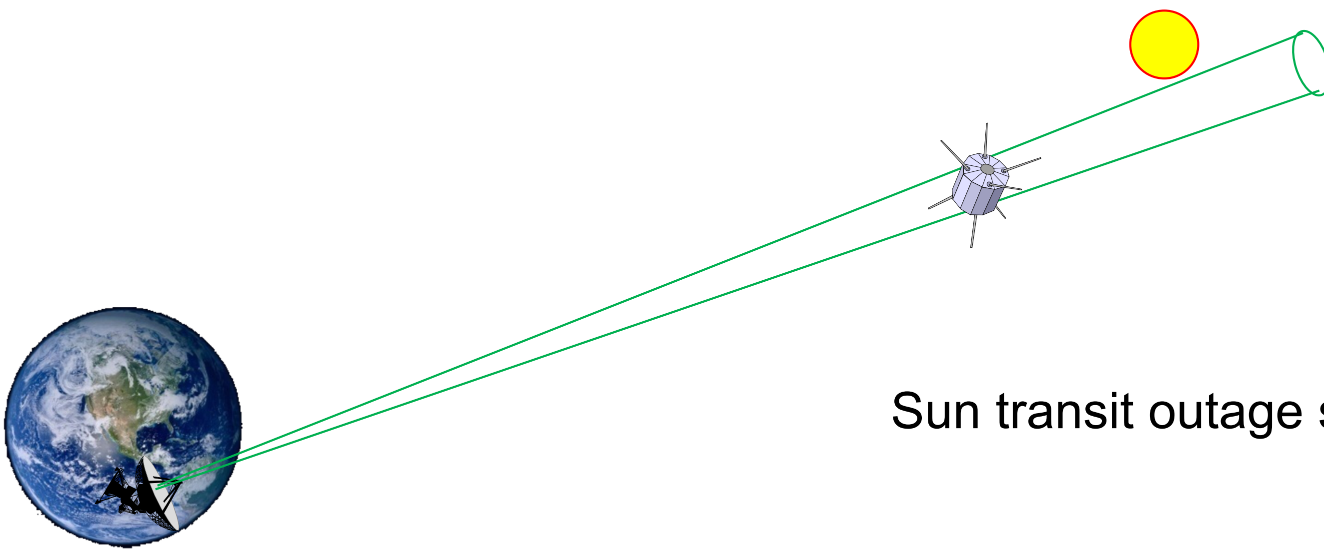
Sun transit outage

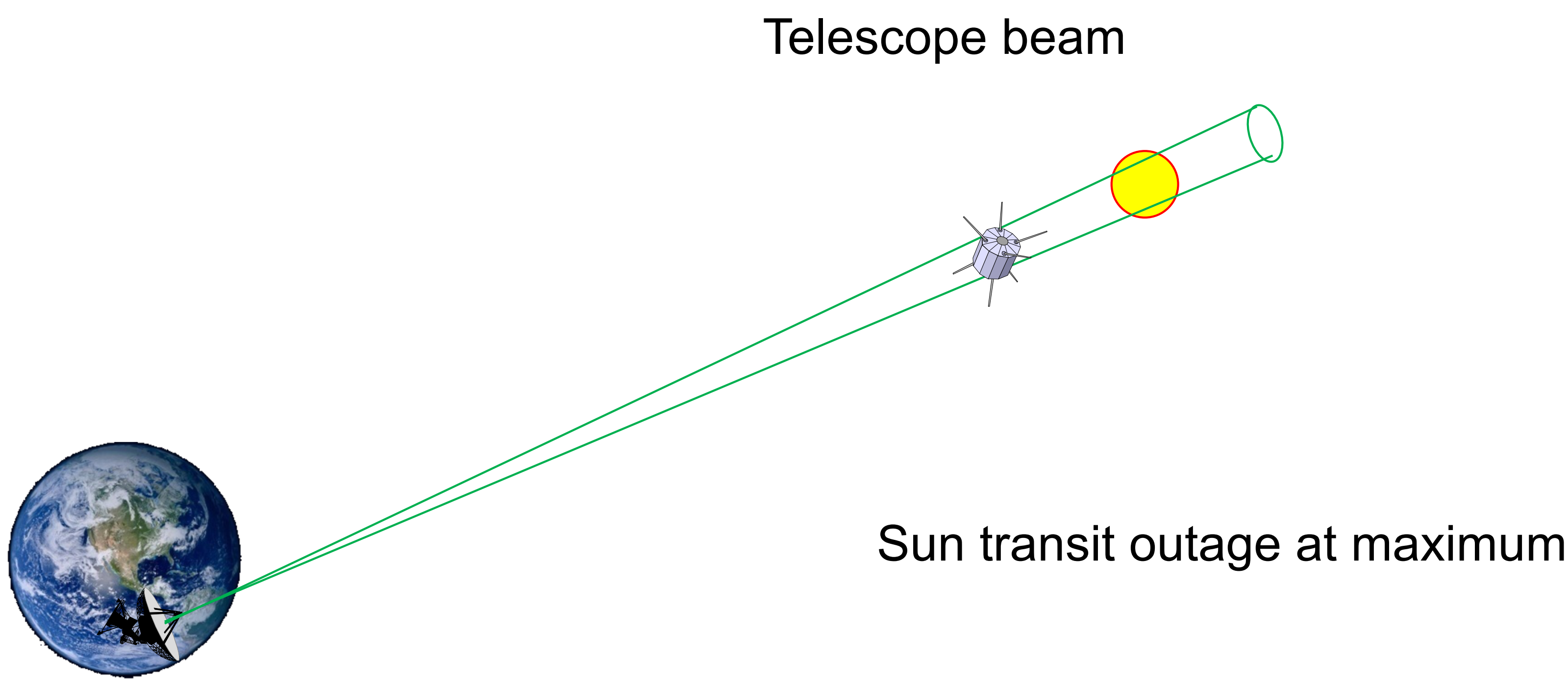
Sun transit outage: When a satellite comes close to the line of sight to the Sun, the earth station picks up a lot of noise from the Sun that blanks out the signal from the satellite. For geostationary satellites that happens for $\pm$6 days around the equinoxes for 10 min/day.



Satellite will not appear before the Sun



Sun transit outage starts



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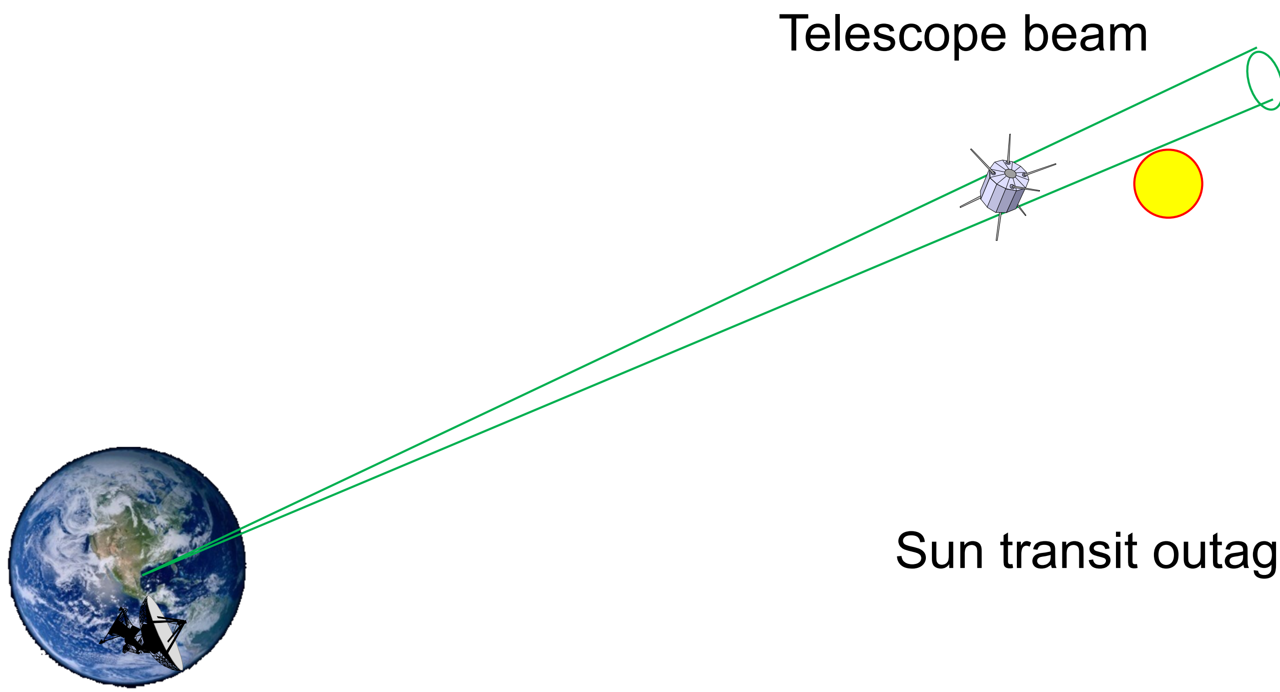
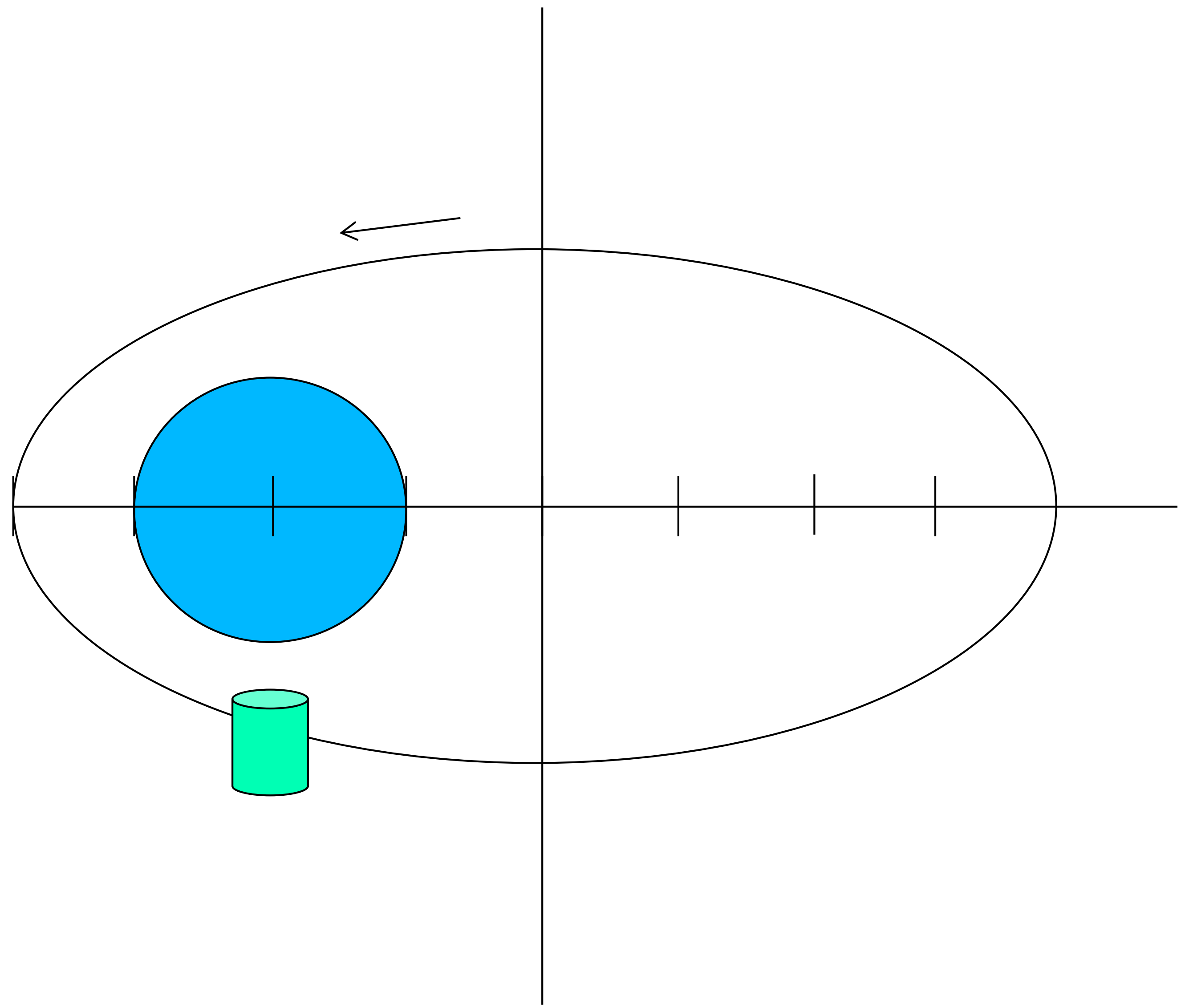


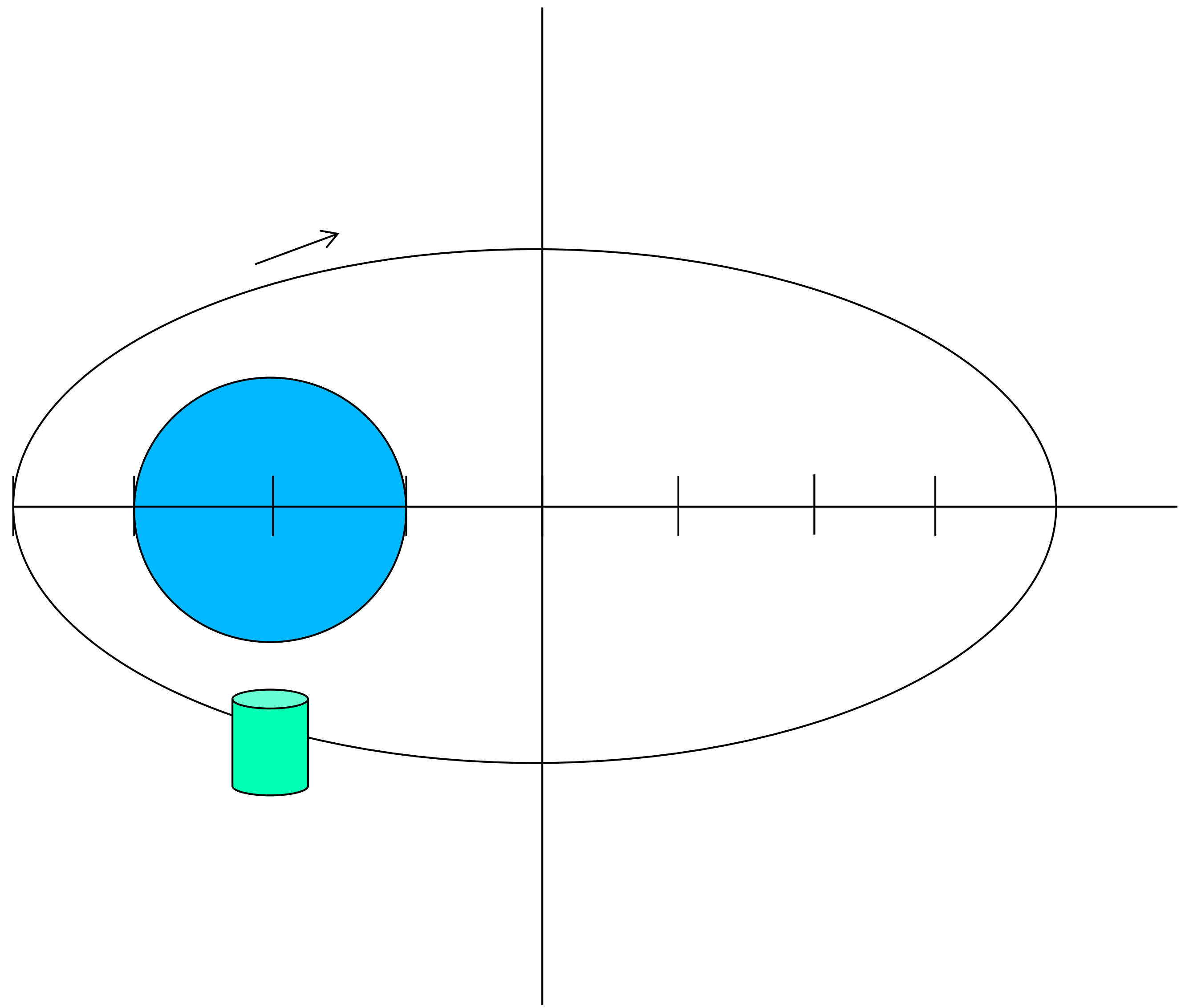
Sun transit outage ends

Practice questions

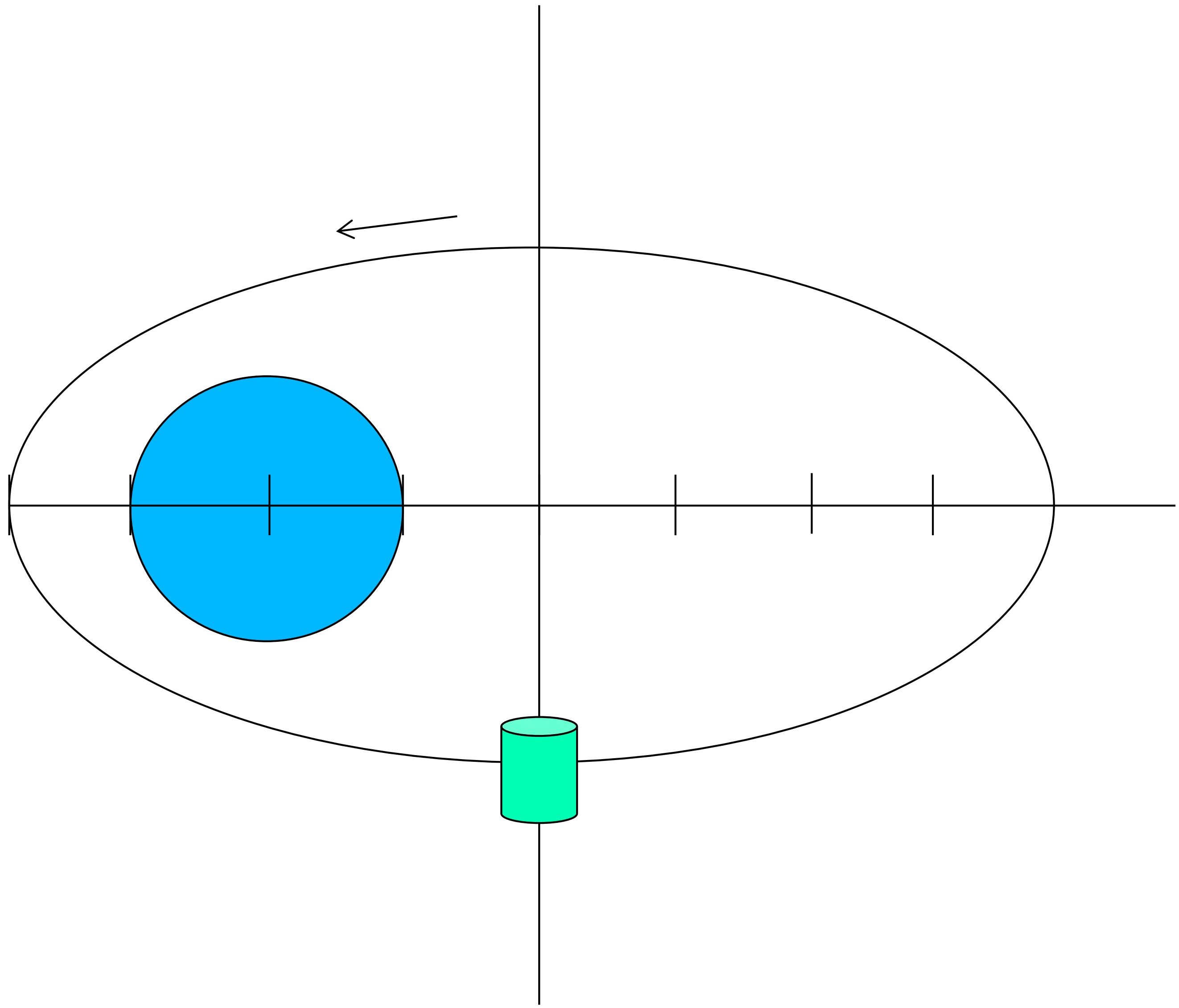
1) Give the values of the orbital parameters and show how they are defined



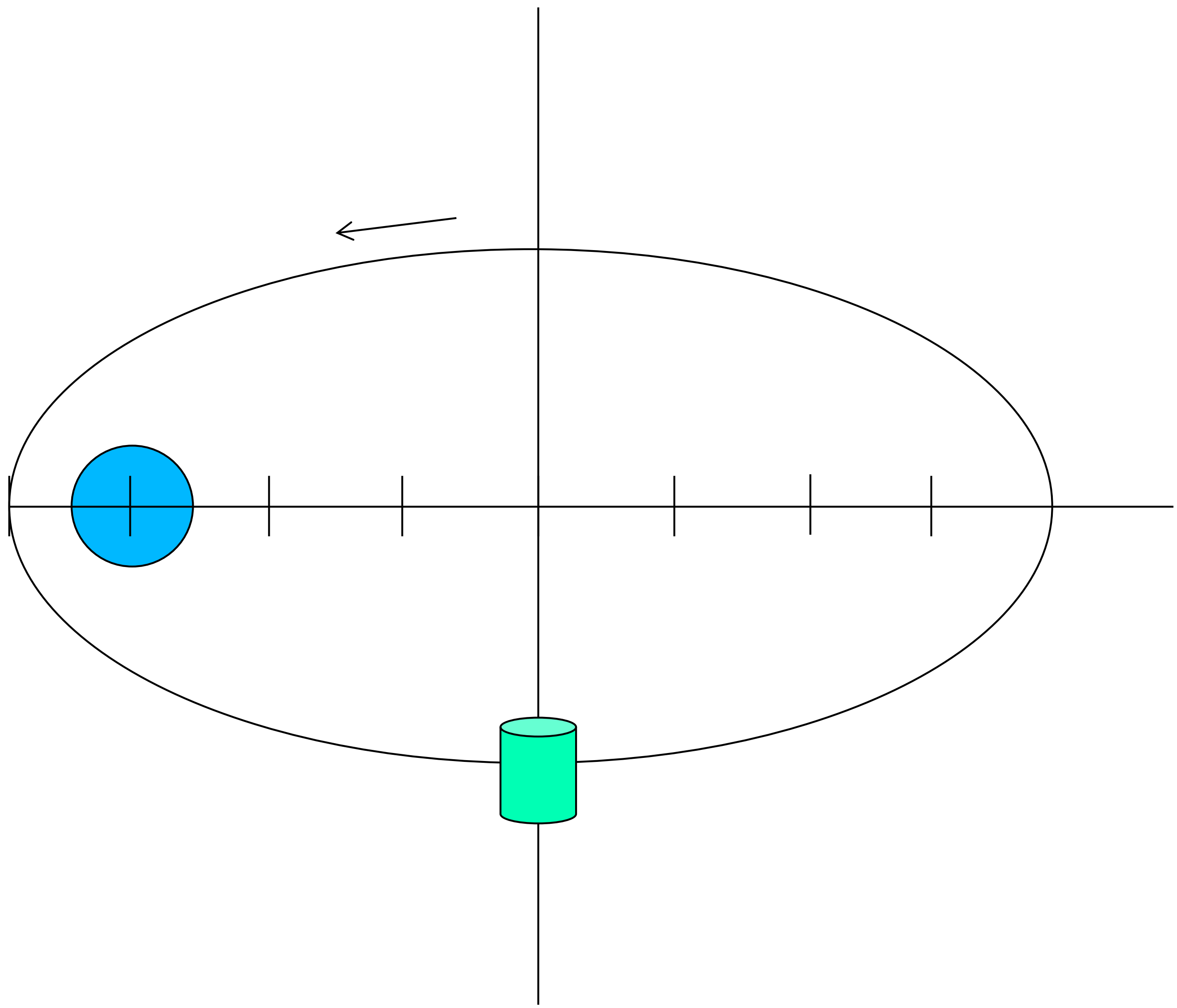
1) Give the values of the orbital parameters and show how they are defined



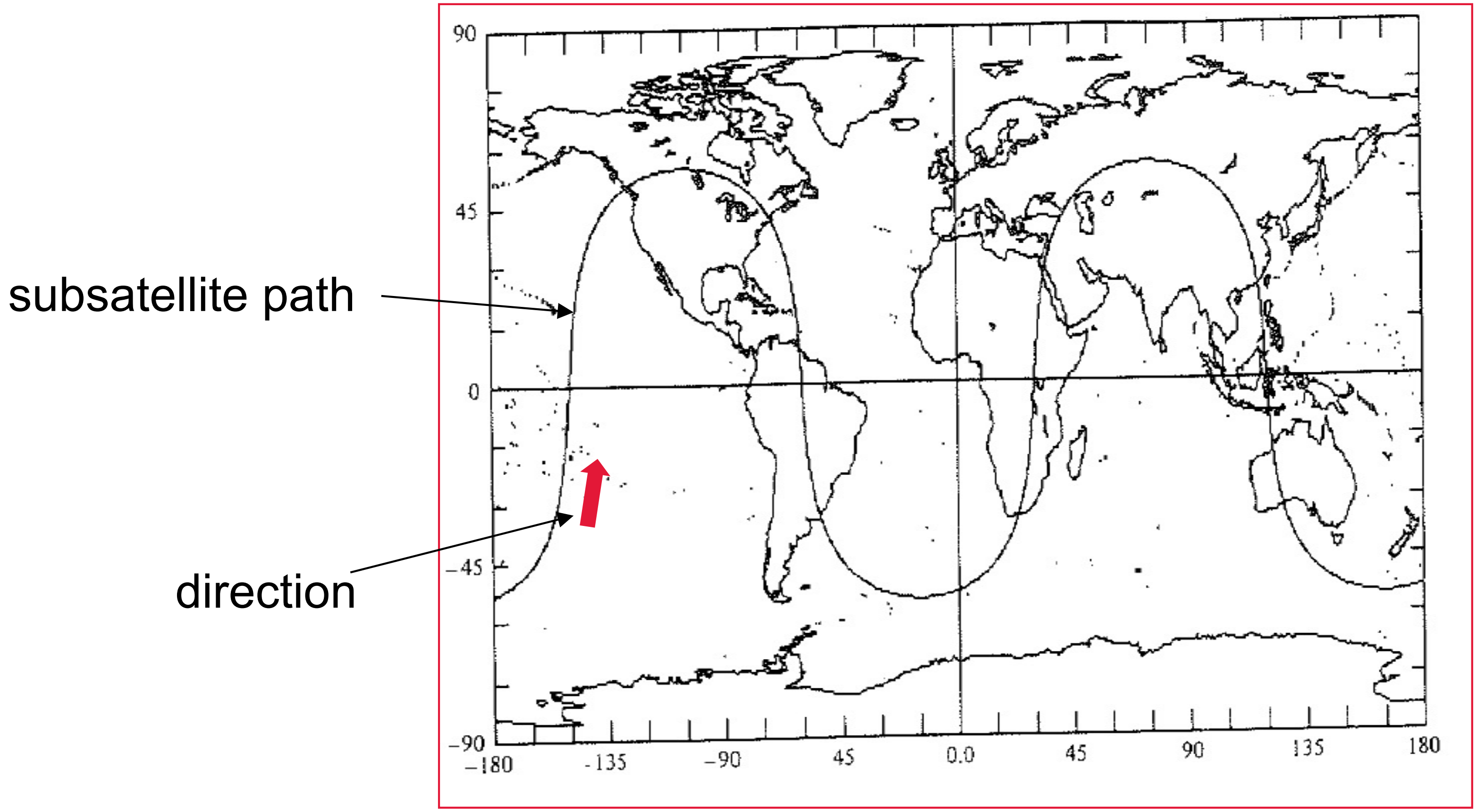
1) Give the values of the orbital parameters and show how they are defined



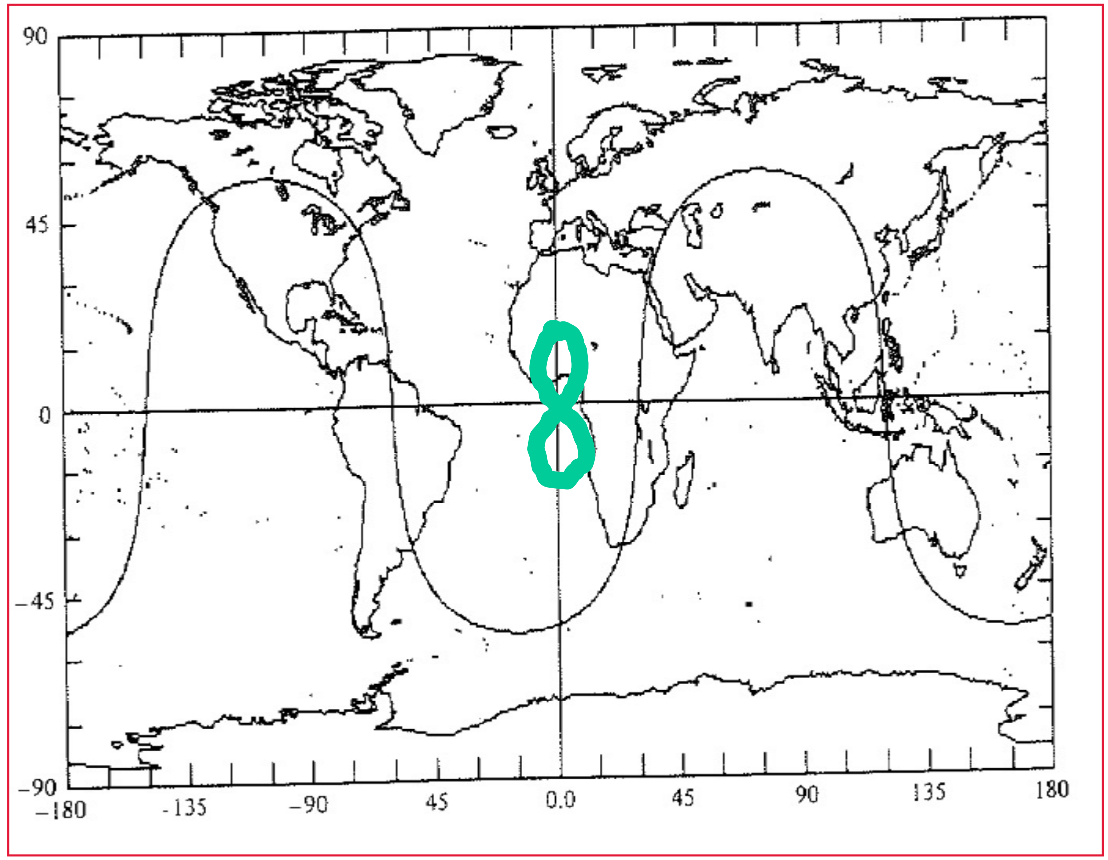
1) Give the values of the orbital parameters and show how they are defined



3) Whic Keplerian elements can be inferred from the subsatellite path on the figure below?



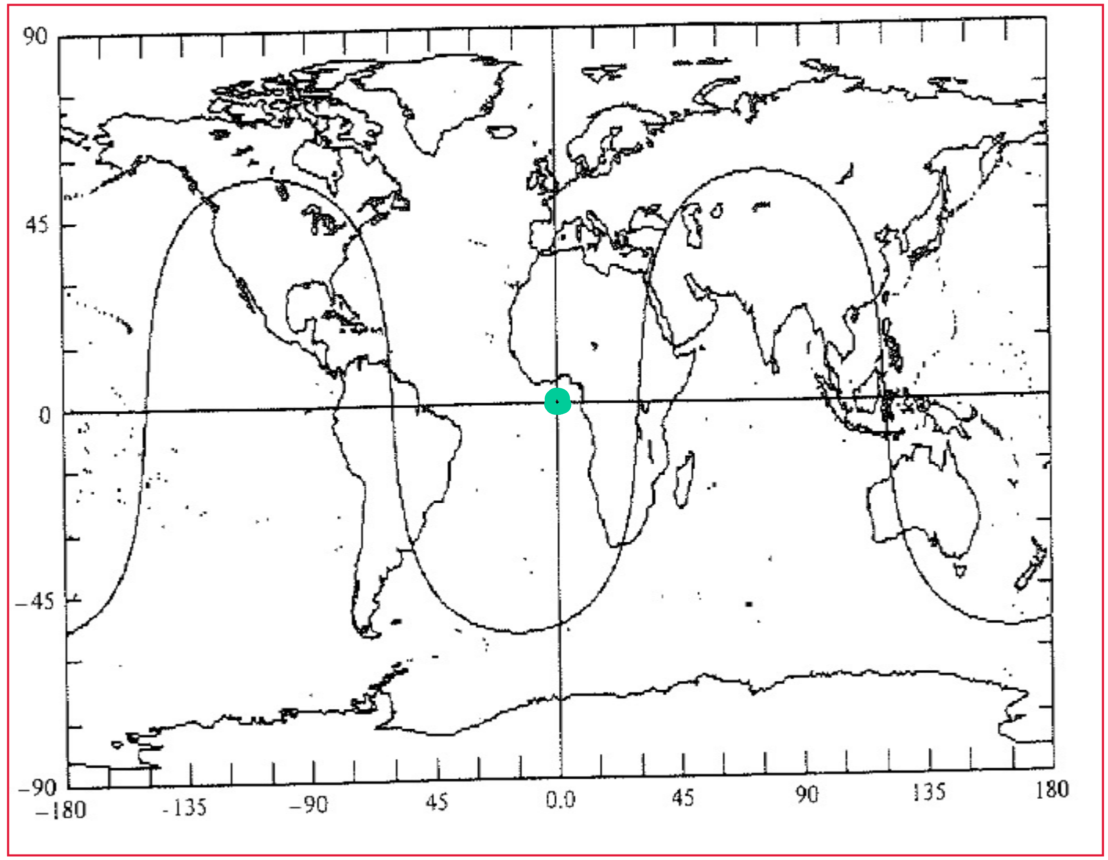
3) Which of the Keplerian elements of the satellite with the green path on the figure below is correct?



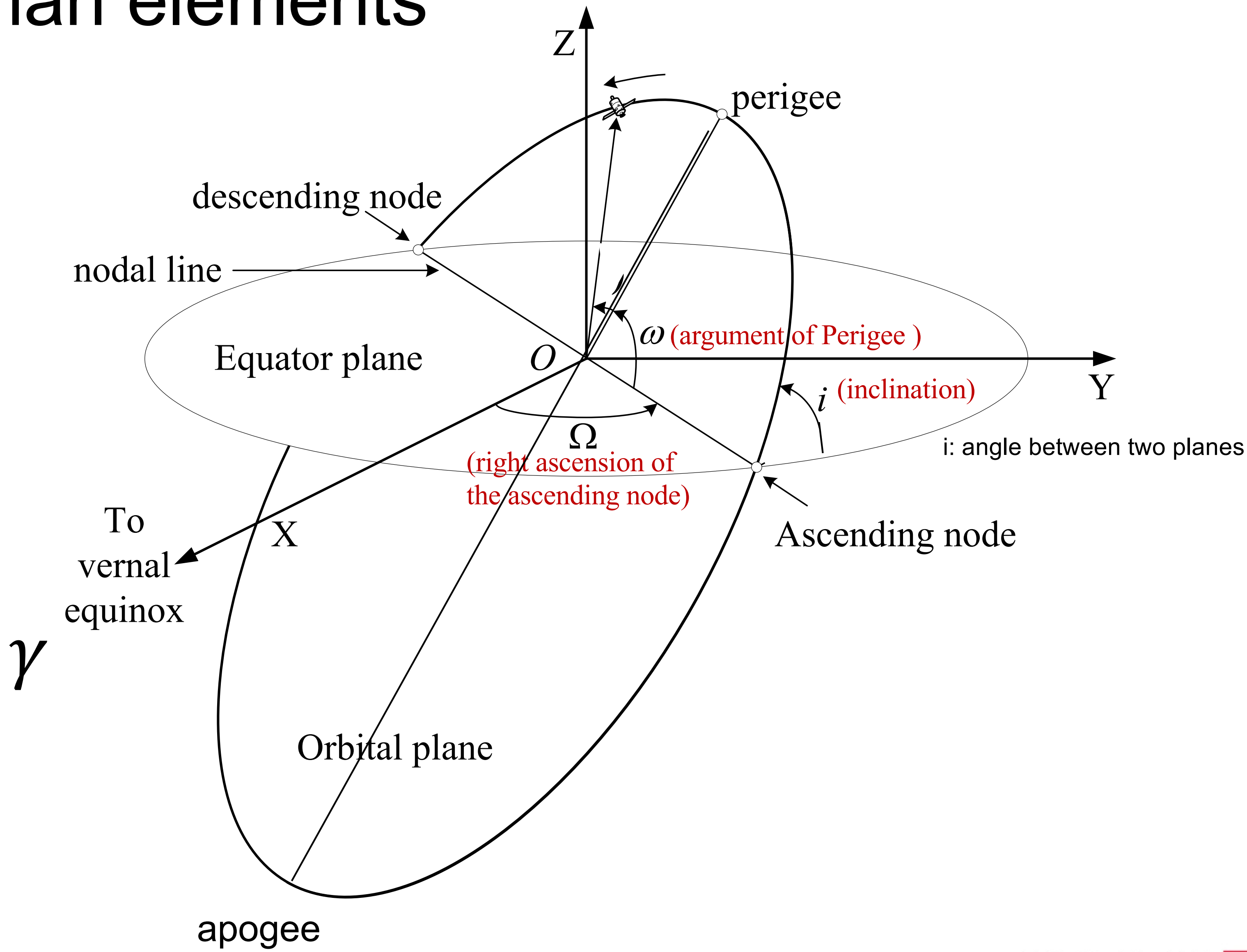
A. a= 21,164 km,
B. a=42,164 km
C. e=0.2
D. e=-0.2
E. $i=40^0$

3) Which of the Keplerian elements of the satellite with the green path on the figure below is correct?

A. a= 21,164 km,
B. a=42,164 km
C. e=0.2
D. e=-0.2
E. $i=40^0$



Keplerian elements



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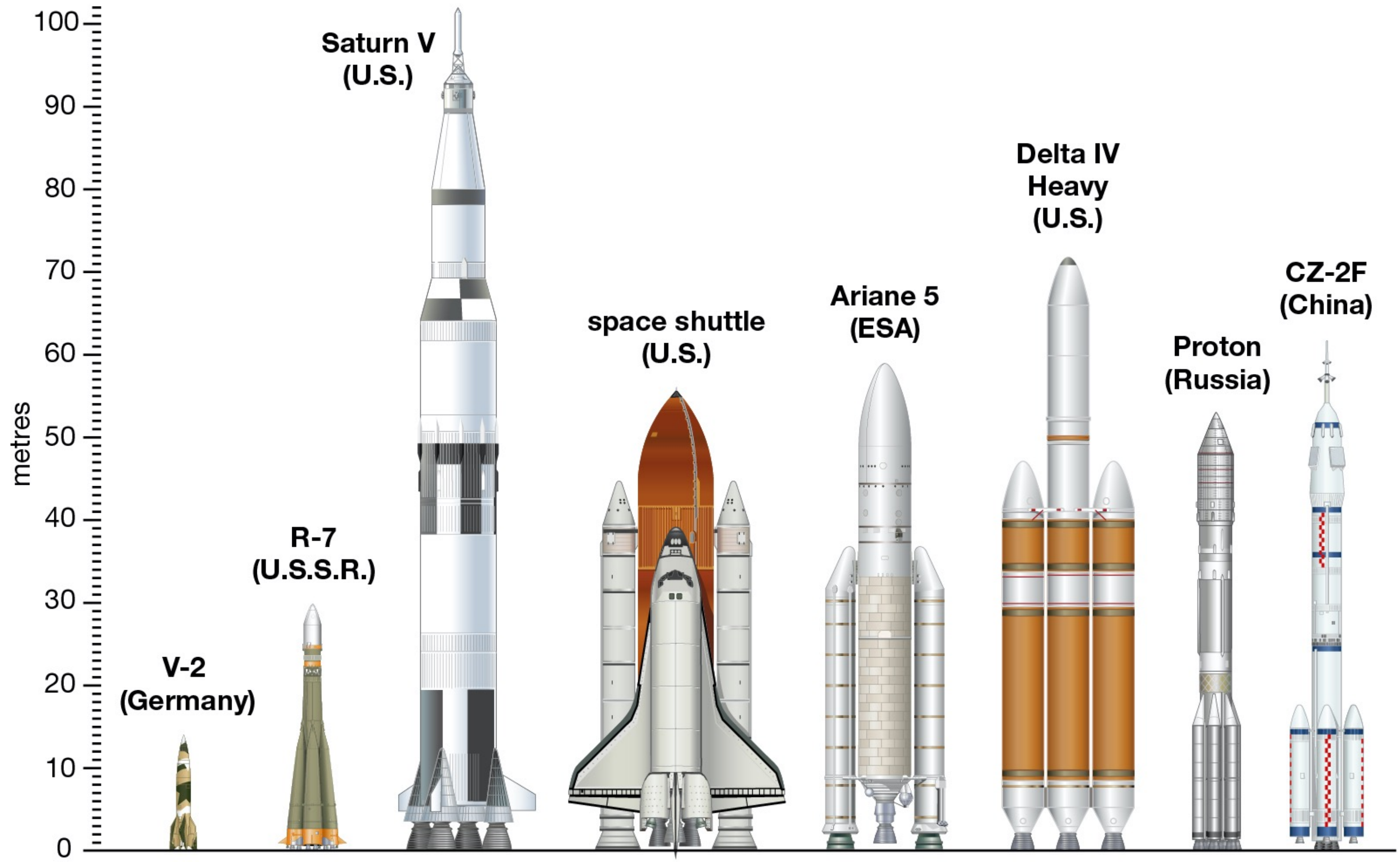
2.4 Launch



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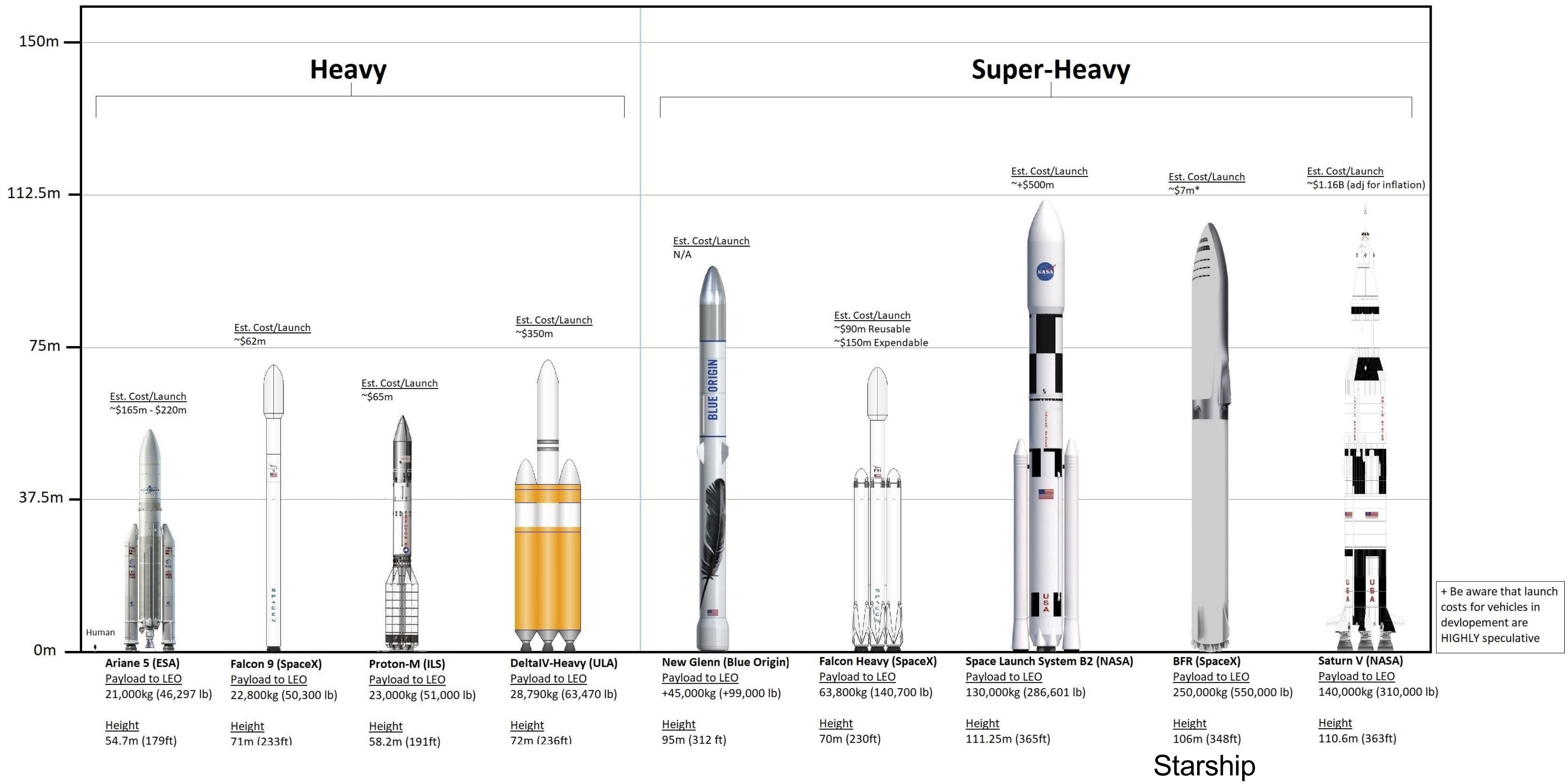

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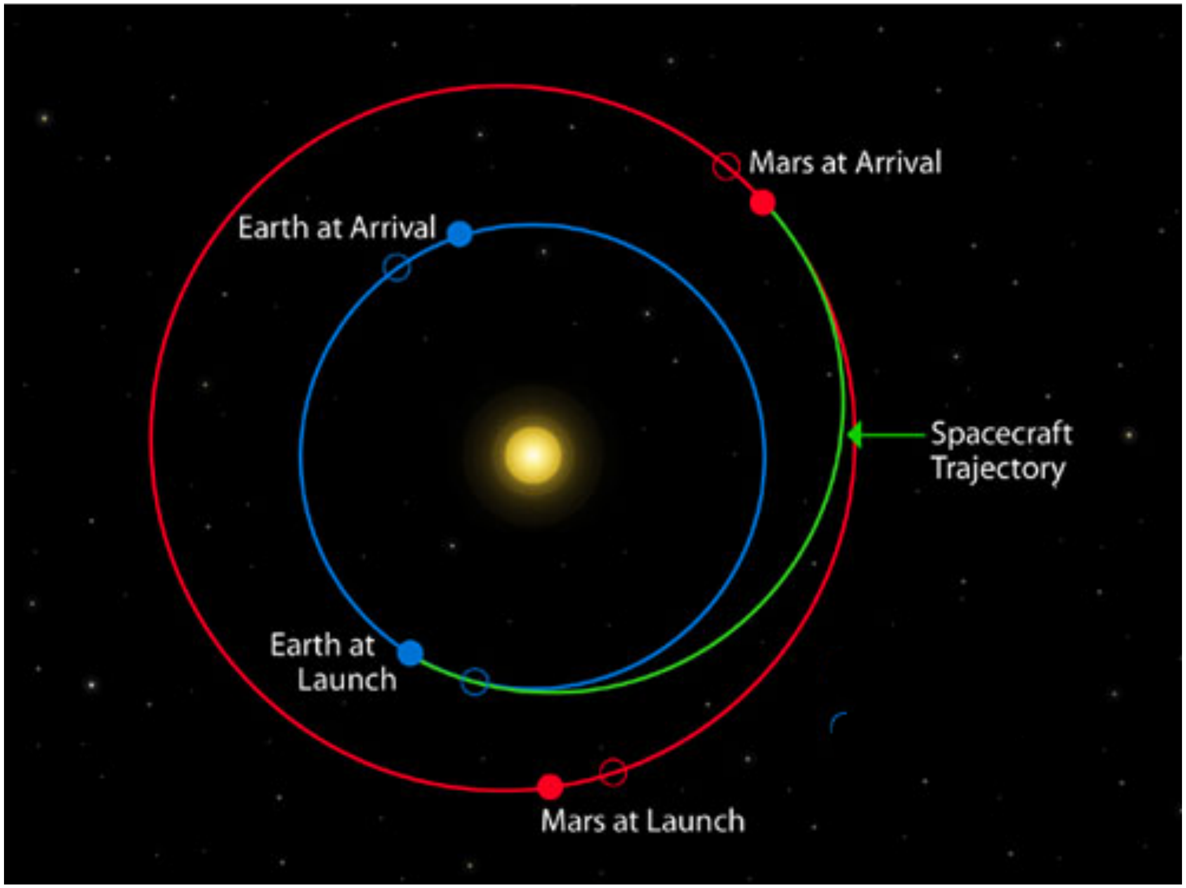
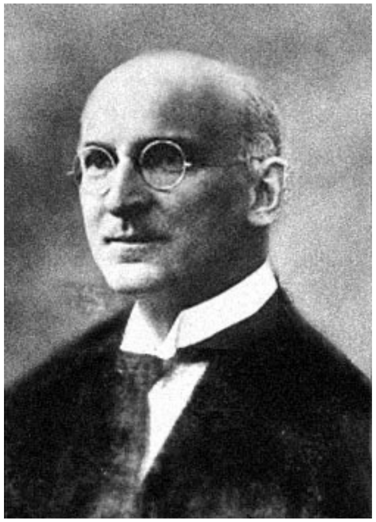
Launch of Haruka on board of a M-V rocket
VLBI space observatory program

Launch of Gravity Probe B

Launch of RadioAstron

Hohmann transfer orbit





Walter Hohmann
1880-1945

Image credit: Smithsonian Institution

Hohmann transfer orbit is an elliptical orbit between two circular orbits in the same plane around the same central body using considered to be the lowest possible amount of propellant.

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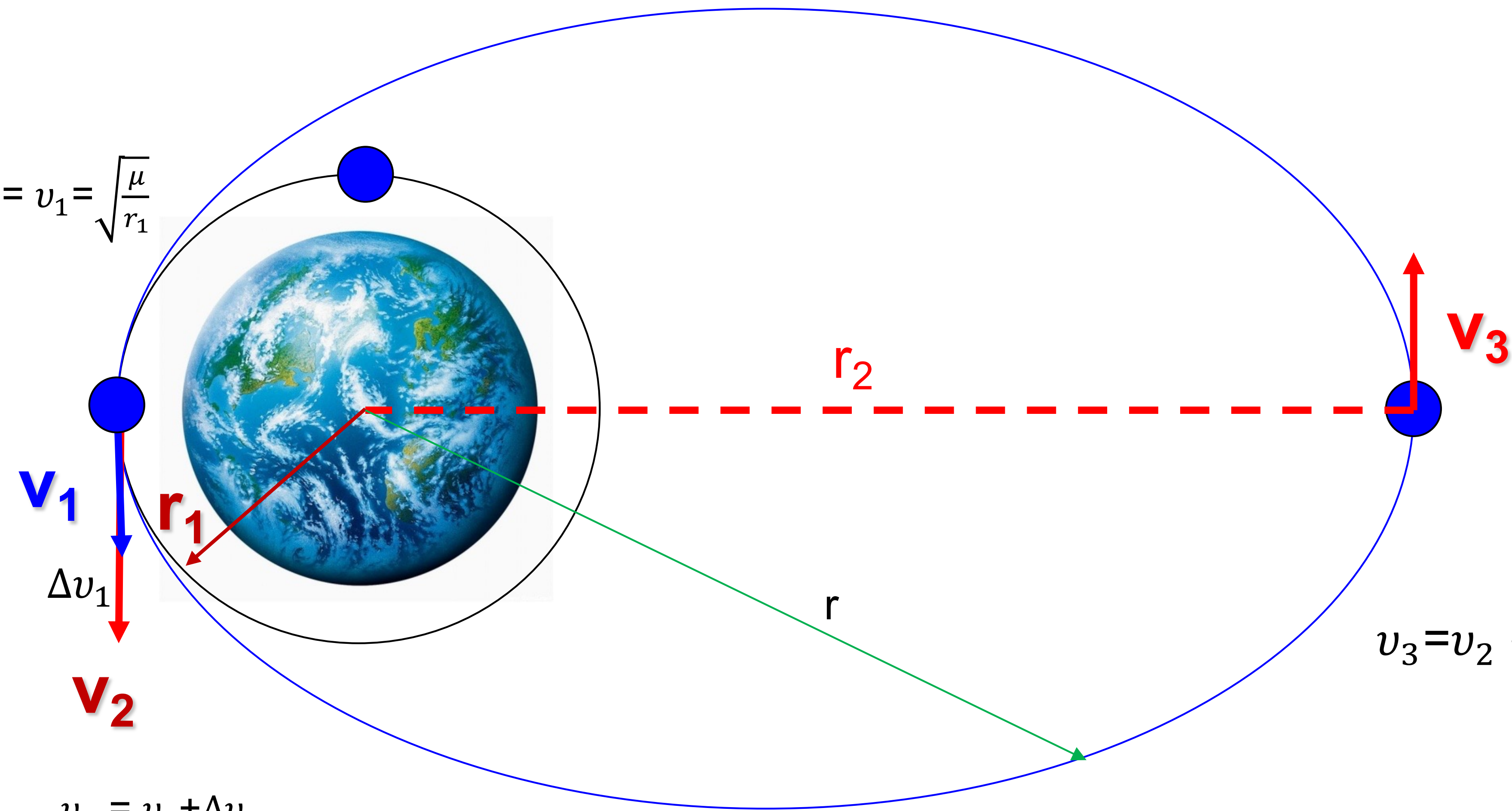
Stages of placing a satellite into geostationary orbit

$$E_{tot} = \frac{mv^2}{2} - \frac{GMm}{r}$$

$$\Rightarrow \quad v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$v_{circ} = v_1 = \sqrt{\frac{\mu}{r_1}}$$



$$v_3 = v_2 \cdot \frac{1-e}{1+e}$$

$$v_2 = v_1 + \Delta v_1$$

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right)$$

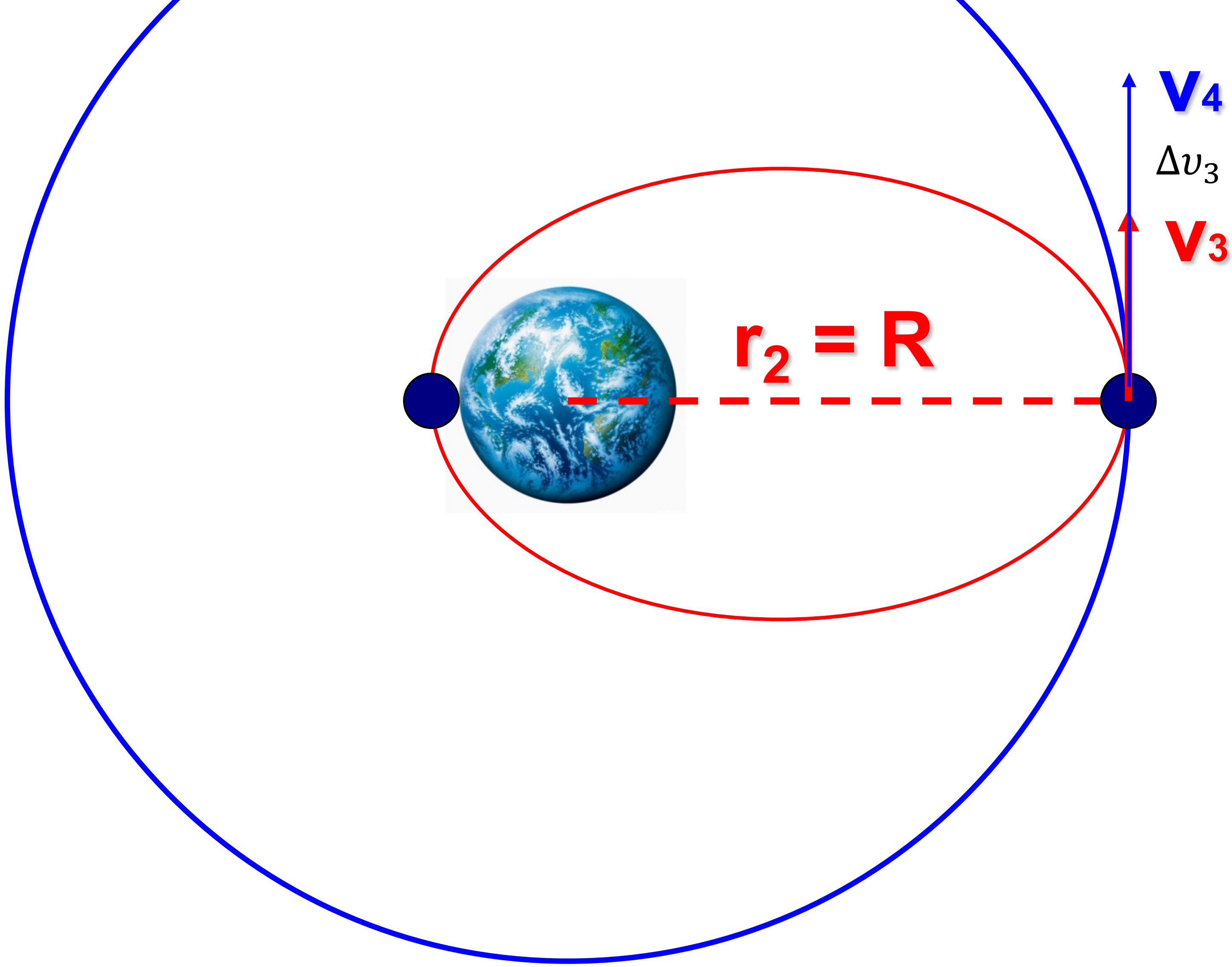
Stages of placing a satellite into geostationary orbit

1. Satellite is sent into circular orbit (or transfer orbit directly)
2. Acquisition is obtained via omnidirectional satellite antenna
3. Keplerian elements of transfer orbit are determined
4. Apogee kick motor is fired when satellite is at apogee
5. Satellite velocity and orientation are adjusted, system is checked out

$$\upsilon_4 = \upsilon_3 + \Delta\upsilon_3$$

$$\Delta\upsilon_3 = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right)$$

$$\upsilon_4 = \sqrt{\frac{\mu}{R}}$$



Derivation for pundits

Conservation of momentum: $m v_p r_1 = m v_a r_2$ ➔ $v_a = \frac{r_1}{r_2} v_p$

Conservation of energy: $\frac{1}{2} m v_p^2 - \frac{\mu m}{r_1} = \frac{1}{2} m v_a^2 - \frac{\mu m}{r_2}$

$$\frac{1}{2} v_p^2 - \frac{1}{2}\left(\frac{r_1}{r_2} v_p\right)^2 = \frac{\mu}{r_1} - \frac{\mu}{r_2}$$

$$v_p^2 \left(1 - \left(\frac{r_1}{r_2}\right)^2\right) = 2\,\mu \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$v_p^2 \left(\frac{r_2^2 - r_1^2}{r_2^2}\right) = 2\,\mu \left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

with $v_2 = v_p$

$$v_2^2 = 2\,\mu \frac{r_2}{r_1(r_1 + r_2)}$$

with $v_2 = v_1 + \Delta v_1$ ➔ $\Delta v_1 = v_2 - v_1$

$$\Delta v_1 = \sqrt{2\mu \frac{r_2}{r_1(r_1+r_2)}} - \sqrt{\frac{\mu}{r_1}}$$

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2 r_2}{r_1 + r_2}} - 1\right)$$

Similarly, with : $v_4 = v_3 + \Delta v_3$ ➔ $\Delta v_1 = v_2 - v_1$

$$\Delta v_3 = \sqrt{\frac{\mu}{r_2}} \left(1 - \sqrt{\frac{2 r_1}{r_1 + r_2}}\right)$$