

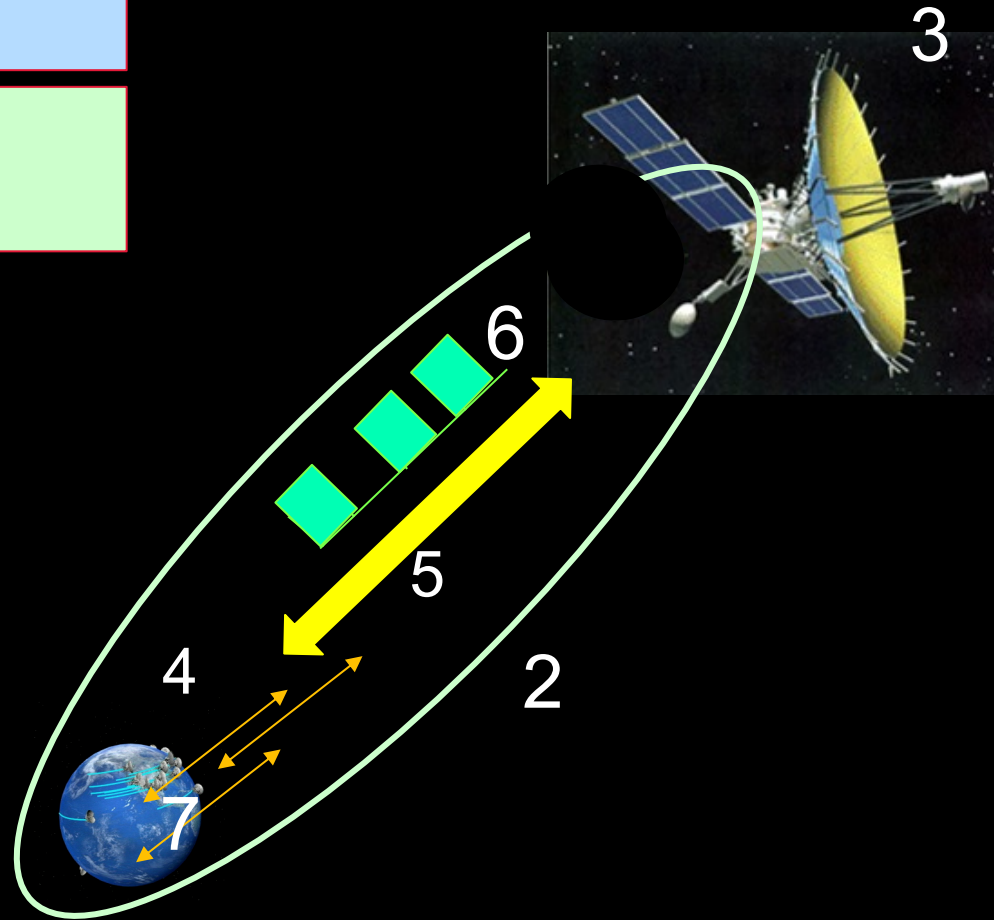
PHYS 3250

Introduction to space communications

Professor N Bartel

Sketch of the 7 chapters

- 2 Orbital aspects
- 3 Spacecraft
- 4 Earth station
- 5 Communications link
- 6 Modulation and multiplexing techniques
- 7 Multiple access to a satellite

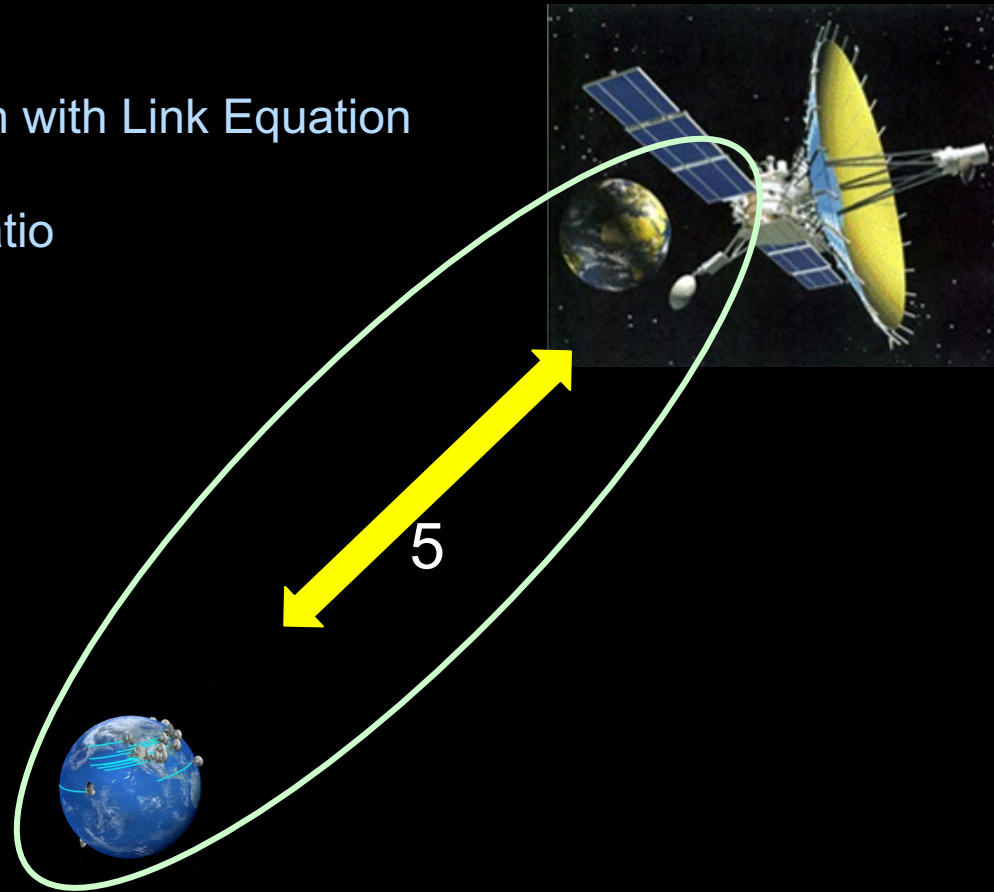


5. Communications Link

5.1 Transmission path with Link Equation

5.2 System noise

5.3 Carrier to noise ratio



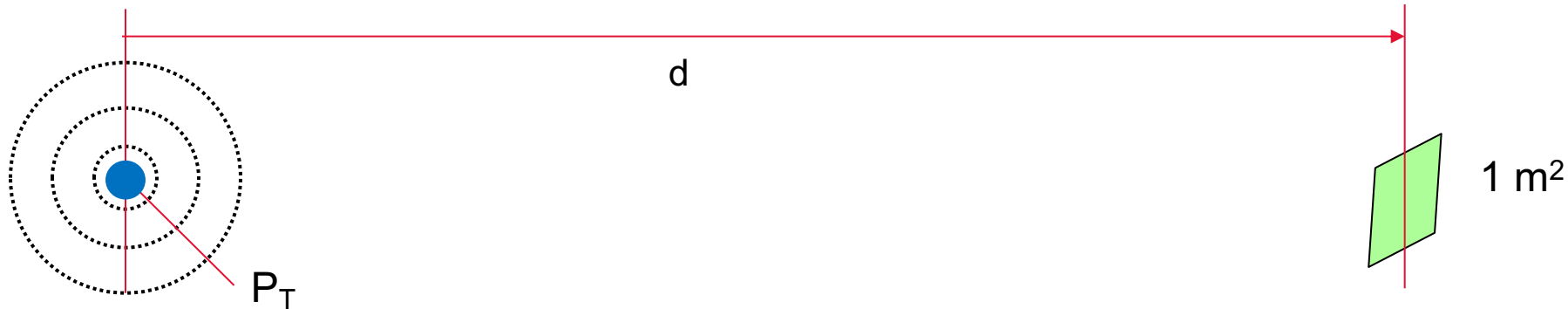
5.1 Transmission Path and Link Equation

Here we first want to introduce the relevant parameters important for this section.

- P_T : transmitted power [W]
- P_R : received power [W]
- G_T : gain of transmit antenna
- G_R : gain of receive antenna
- d : distance between receive and transmit antenna
- $A_{\text{eff}, T}$: effective aperture of transmit antenna
- $A_{\text{eff}, R}$: effective aperture of receive antenna
- D_T : diameter of transmit antenna
- D_R : diameter of receive antenna
- η_T : efficiency of transmit antenna
- η_R : efficiency of receive antenna

Assumption: No power loss other than that resulting from spreading of signal in space.

1. Assume that power is isotropically radiated.



Example 5-1

$$P_T = 1\text{W}$$

$$D = 37,000\text{ km}$$

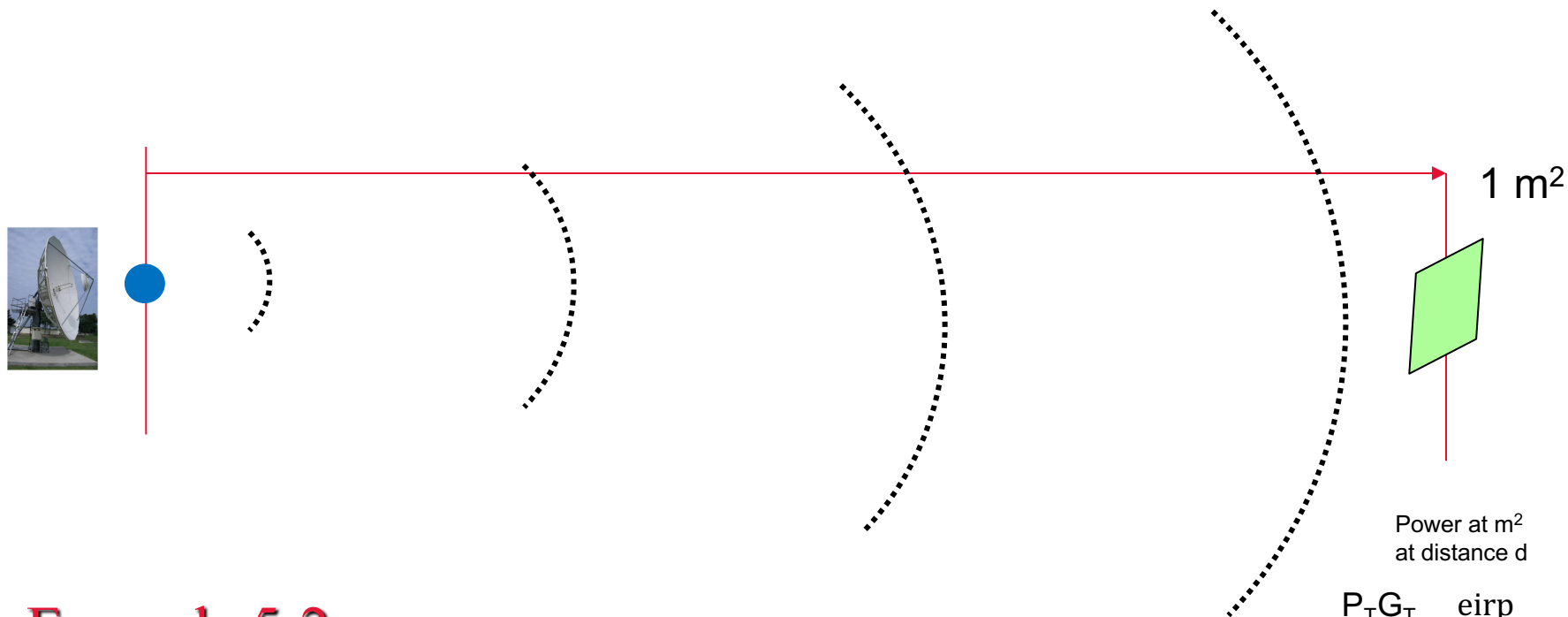
What is the power per m^2 at distance d ?

$$\frac{P_T}{4\pi d^2} = 5.8 \cdot 10^{-17} \left[\frac{\text{W}}{\text{m}^2} \right]$$

$$\left[\frac{P_r}{4\pi d^2} \right] = [P_r] - [4\pi d^2]$$

$$-162.4 \frac{\text{dBW}}{\text{m}^2} = (0 - 162.4) \frac{\text{dBW}}{\text{m}^2}$$

2. Now assume that power is radiated in certain direction (transmit antenna has $G_T = I_{\max}/ I_0$)



Example 5-2

$P_T = 1\text{W}$
 $D = 37,000\text{ km}$
 $D_T = 5\text{m}$
 $\eta_T = 0.6$
 $\lambda_T = 5\text{ cm}$

What is the power per m^2 at distance d ?

$$G = \eta \left(\frac{\pi D}{\lambda} \right)^2$$

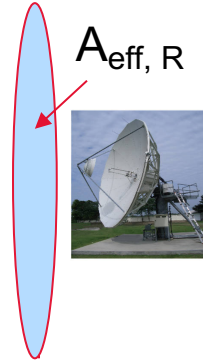
$$G_T = 59,200 = 47.7\text{ dB}$$

eirp: equivalent isotropic radiated power

$$\text{eirp} = P_T G_T$$

$$\begin{aligned}
 \frac{P_T G_T}{4\pi d^2} &= [P_T] + [G_T] - [4\pi d^2] \\
 &= 0 + 47.7 - 162.4. \text{ dBW/m}^2 \\
 &= -114.7 \text{ dBW/m}^2 \\
 &= 3.39 \text{ pW/m}^2
 \end{aligned}$$

3. Now assume receive antenna with $A_{\text{eff}, R}$



Example 5-3

$P_T = 1\text{W}$
 $D = 37,000\text{ km}$
 $D_T = 5\text{m}$
 $\eta_T = 0.6$
 $\lambda_T = 5\text{ cm}$
 $D_R = 1\text{m}$
 $\eta_R = 0.7$

$G_T = 59,200 = 47.7\text{ dB}$

$G_R = 2,760 = 34.4\text{ dB}$

What is P_R ?

$$\begin{aligned}
 [P_R] &= 47.4 + 34.4 - (32.4 + 91.4 + 75.6)\text{ dBW} \\
 &= 47.4 + 34.4 - 199.4 \\
 [FSL] &= 10^{19.94} \quad \text{!!!!}
 \end{aligned}$$

$$\begin{aligned}
 &= -117.3\text{ dBW} \\
 &= 1.9 \cdot 10^{-12}\text{ W} \\
 &= 1.9\text{ pW}
 \end{aligned}$$

$A_{\text{eff},R}$ is usually given in terms of G_R

$$A_{\text{eff},R} = \eta \frac{\pi D^2}{4}$$

$$G_R = \eta \left(\frac{\pi D}{\lambda} \right)^2$$

$$A_{\text{eff},R} = G_R \frac{\lambda^2}{4\pi}$$

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d} \right)^2$$

$$P_R = \text{eirp} \cdot G_R \left(\frac{\lambda}{4\pi d} \right)^2$$

$$[P_R] = [\text{eirp}] + [G_R] - \left[\left(\frac{4\pi d}{\lambda} \right)^2 \right]$$

$$[P_R] = [\text{eirp}] + [G_R] - [FSL]$$

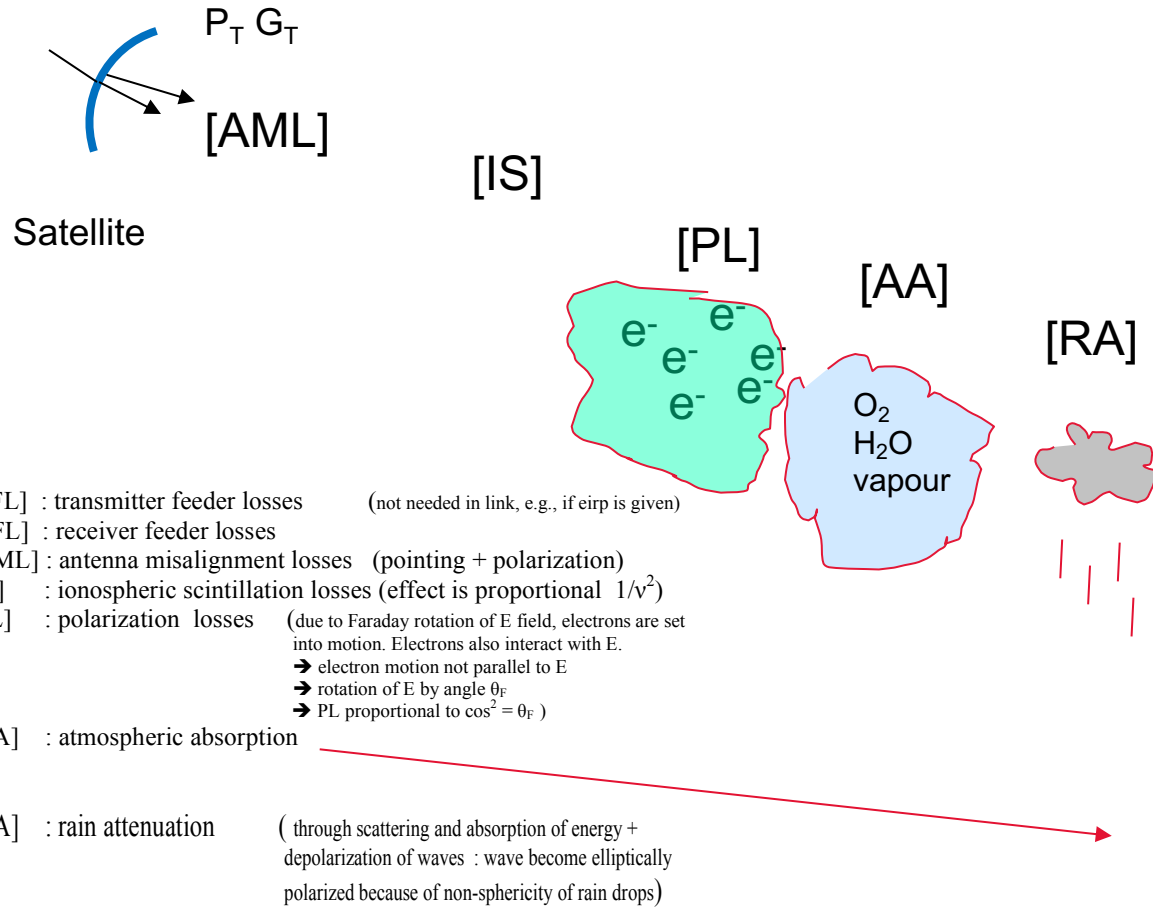
$$FSL = \left(\frac{4\pi d}{\lambda} \right)^2$$

FSL: free space loss
[FSL] = 32.44 + 20 log d(km) + 20 log v(MHz) dB

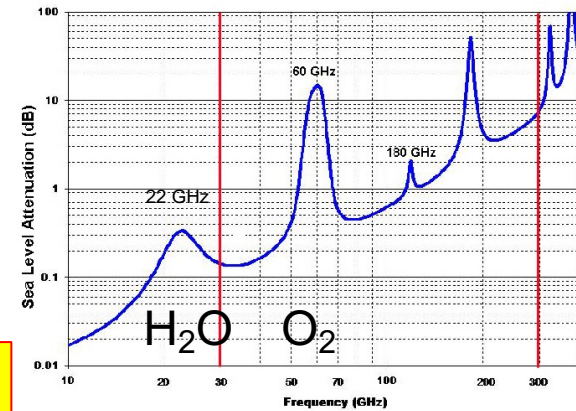
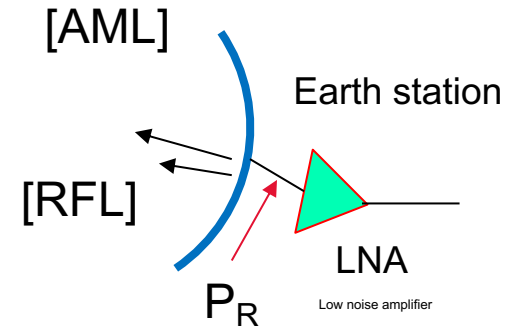
$$\frac{P_T G_T A_{\text{eff},R}}{4\pi d^2} = \frac{\text{eirp} \cdot A_{\text{eff},R}}{4\pi d^2}$$

Power at m^2 at distance d

Apart from FSL there are other losses.



- [TFL] : transmitter feeder losses (not needed in link, e.g., if eirp is given)
- [RFL] : receiver feeder losses
- [AML] : antenna misalignment losses (pointing + polarization)
- [IS] : ionospheric scintillation losses (effect is proportional $1/v^2$)
- [PL] : polarization losses (due to Faraday rotation of E field, electrons are set into motion. Electrons also interact with E.
 - electron motion not parallel to E
 - rotation of E by angle θ_F
 - PL proportional to $\cos^2 = \theta_F$)
- [AA] : atmospheric absorption
- [RA] : rain attenuation (through scattering and absorption of energy + depolarization of waves : wave become elliptically polarized because of non-sphericity of rain drops)



The Link Equation

$$[P_R] = [eirp] + [G_R] - [loss]$$

$$[loss] = [FSL] + [RFL] + [AML] + [IS] + [PL] + [AA] + [RA]$$

Wikipedia, research gate

5.2. System noise

P_R is the power at the input of the LNA

P_N is the noise power of thermal noise sources

LNA amplifies P_R and P_N , and power of other noise sources.

Measurement of satellite link performance is:

$$\left[\frac{C}{N}\right] = [P_R] - [P_N]$$

$$N_0 = \frac{h\nu}{\exp\left(\frac{h\nu}{kT_N}\right) - 1} \quad [\text{W/Hz}] \text{ or } [\text{J}]$$

N_0 : noise power spectral density

h : Planck's constant: $6.62 \cdot 10^{-34}$ [Js]

k : Boltzmann's constant : $1.38 \cdot 10^{-23}$ [JK⁻¹]

T_N : absolute temperature of device or noise temperature [K]

For $h\nu \ll kT_N$: ($\nu = 10$ GHz, $T_N = 1000$ K)

$$6.62 \cdot 10^{-34} \cdot 10^{10} \text{ J} \ll 1.38 \cdot 10^{-23} \cdot 10^3 \text{ [J]}$$

$$N_0 = \frac{h\nu}{1 + \left(\frac{h\nu}{kT_N}\right) - 1}$$

$$N_0 = kT_N \quad [\text{J}]$$

$$P_N = kT_N B_N \quad [\text{W}]$$

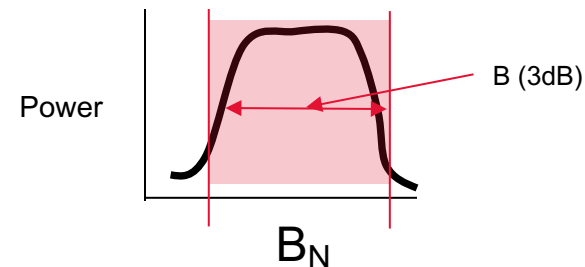
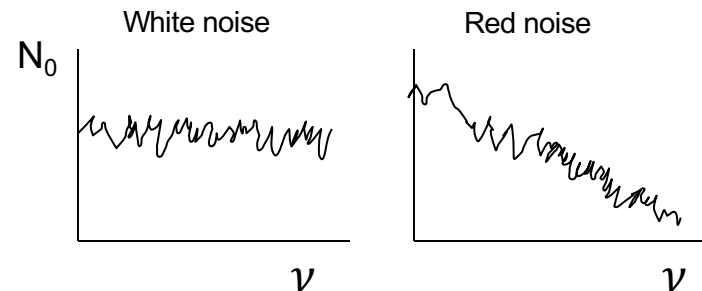
B_N : noise power bandwidth = 1.12 B (B: 3dB bandwidth of filter)

Example 5-4

$$T_N = 100 \text{ K}$$

$$B_N = 36 \text{ MHz}$$

$$\begin{aligned} \rightarrow P_N &= 5 \cdot 10^{-14} \text{ W} \\ &= 5 \cdot 10^{-2} \text{ pW} \end{aligned}$$

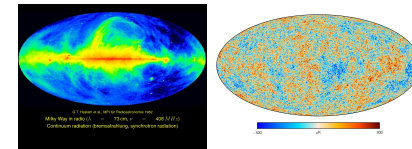


Types of thermal noise

Antenna noise -- i) sky noise, ii) noise due to antenna losses
(T_{ant})

Amplifier noise -- from thermal motion of electrons in amplifiers
 T_{e1}, T_{e2}, \dots (equivalent input temperatures for amplifiers 1, 2, ...)

Absorptive network noise – from resistive elements or devices in the network, that is attenuators, transmission lines, waveguides. They introduce losses by absorbing energy from signal and converting it to heat
→ thermal noise

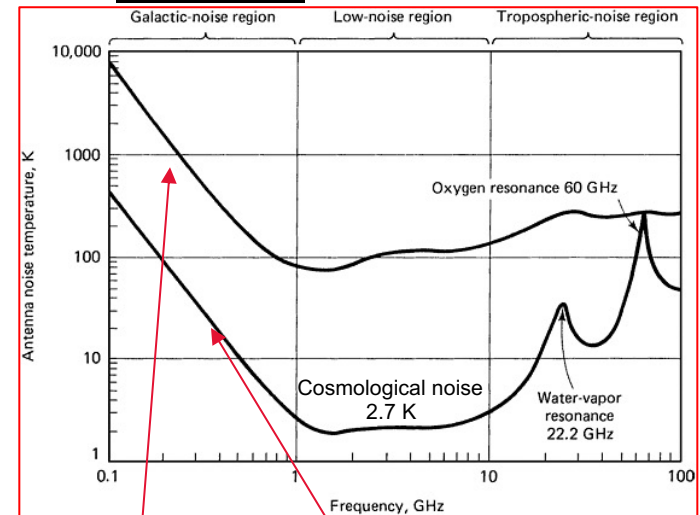


Antenna noise

i) Sky noise

Here is a typical diagram of the sky noise. The sky noise is expressed as a temperature, called antenna temperature, T_{ant} . The diagram corresponds to T_{ant} for zenith pointing (EL=90 deg.) and for pointing away from the galactic disk, earth, sun, moon and planets.

Note: T_{ant} increases for pointing towards lower elevations
 “ galactic plane
 “ galactic center
 “ earth, sun, moon, planets.
 T_{ant} increases at lower frequencies



Just above horizon pointing

Zenith pointing

D. Roddy, ch.: 12.5.1

ii) Noise due to antenna losses

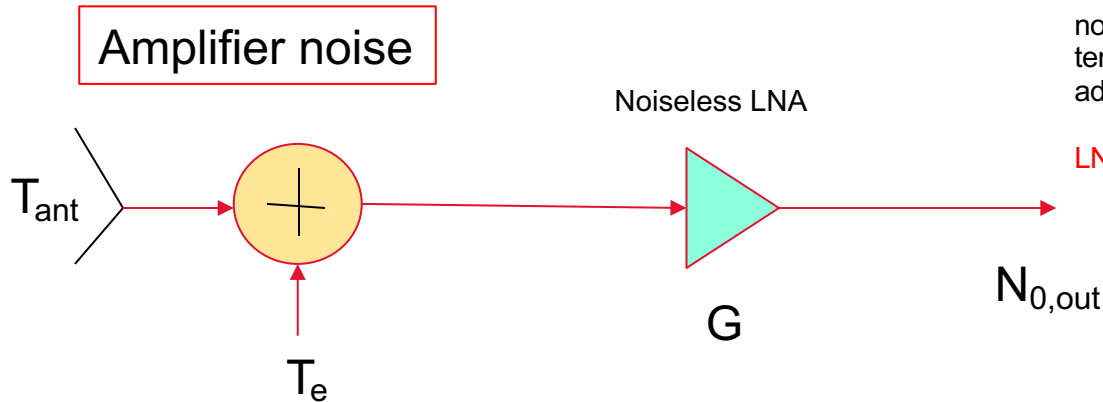
- Ohmic losses of antenna itself
- Feed

Example 5-4

What is T_{ant} for satellite with antenna pointing towards earth?



$T_{\text{ant}} = 290 \text{ K}$, if the 3 dB beamwidth of the antenna is smaller than the angular diameter of the earth.



We consider this network of one amplifier as a noiseless amplifier with gain G where the noise temperature of the amplifier is considered to be added at the input of the amplifier.

LNA: Low noise amplifier

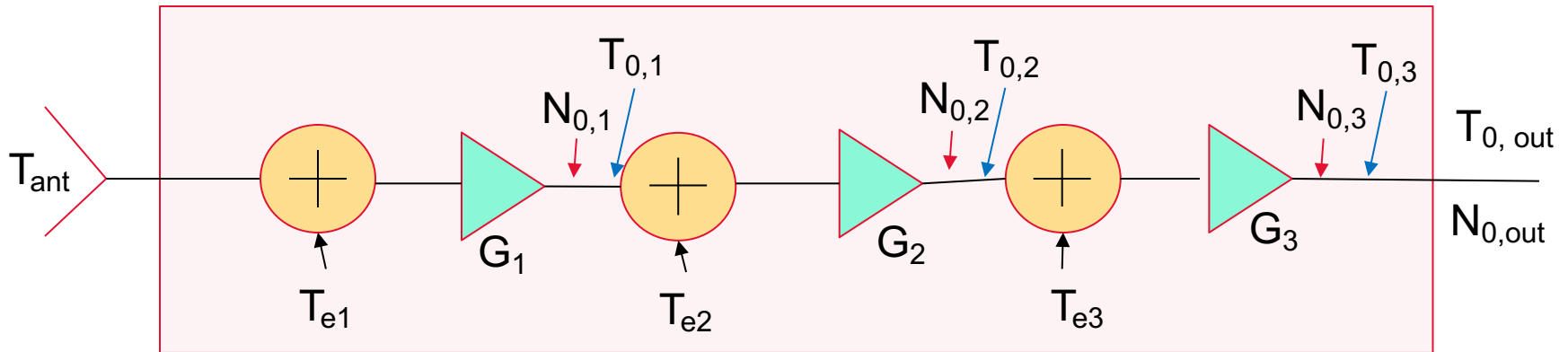
T_e : equivalent input temperature of amplifier [K]

$N_{0,\text{out}}$: noise power spectral density of amplifier output [W/Hz]

G : gain of amplifier

$$N_{0,\text{out}} = Gk(T_{\text{ant}} + T_e)$$

Several amplifiers in cascade



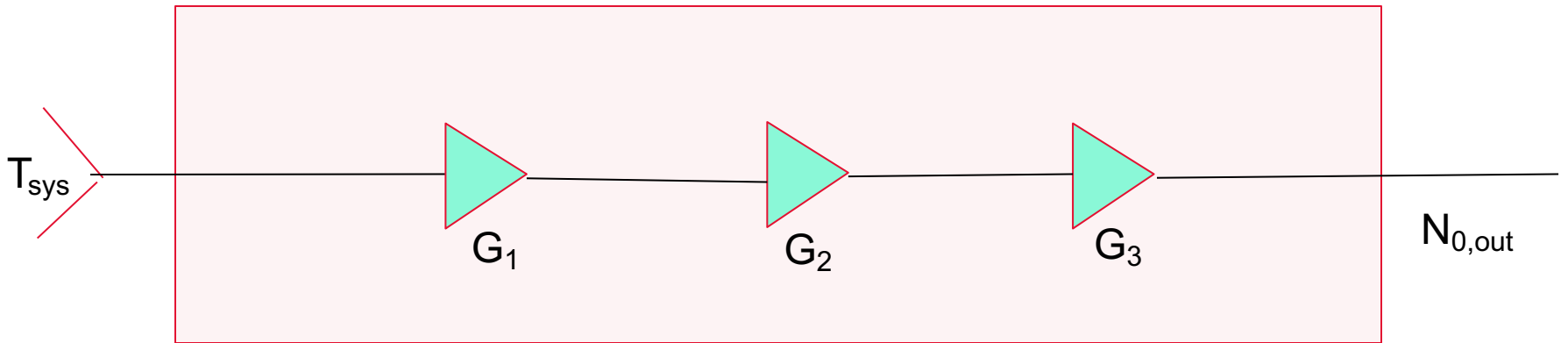
What is $T_{0, out}$?

$$\begin{aligned}
 T_{0,1} &= G_1 (T_{ant} + T_{e1}) \\
 T_{0,2} &= G_2 [G_1 (T_{ant} + T_{e1}) + T_{e2}] \\
 T_{0,3} &= G_3 [G_2 [G_1 (T_{ant} + T_{e1}) + T_{e2}] + T_{e3}] \\
 &= T_{0, out}
 \end{aligned}$$

What is $N_{0, out}$?

$$\begin{aligned}
 N_{0,1} &= G_1 k (T_{ant} + T_{e1}) \\
 N_{0,2} &= G_2 [G_1 k (T_{ant} + T_{e1}) + kT_{e2}] \\
 N_{0,3} &= G_3 [G_2 [G_1 k (T_{ant} + T_{e1}) + kT_{e2}] + kT_{e3}] \\
 &= N_{0, out}
 \end{aligned}$$

Instead of having all kinds of T's (T_{ant} , T_{e1} , T_{e2} ...), we want to have one representative temperature referred to the input. This temperature is called, system temperature, T_{sys} .



$$N_{0, \text{out}} = G_1 G_2 G_3 k T_{\text{sys}}$$

$$\rightarrow T_{\text{sys}} = T_{\text{ant}} + T_{e1} + T_{e2}/G_1 + T_{e3}/(G_1 G_2)$$

Generalization leads to:

$$T_{\text{sys}} = T_{\text{ant}} + T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \dots + \frac{T_{en}}{G_1 G_2 \dots G_n}$$

T_{rec}

$$T_{\text{sys}} = T_{\text{ant}} + T_{\text{rec}}$$

T_{rec} : effective receiver input noise temperature

To keep T_{sys} as low as possible, first stage should have High G and low noise temperature.

That means:

LNA should be positioned right after feed

LNA should be cooled

Cables should be as short as possible

Example 5-5

$$T_{\text{ant}} = 10\text{K}$$

$$T_{e1} = 50\text{ K} \quad G_1 = 23\text{ dB}$$

$$T_{e2} = 500\text{ K} \quad G_2 = 13\text{ dB}$$

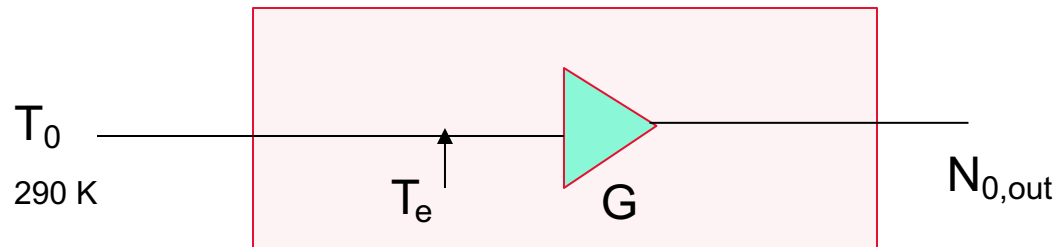
$$T_{e3} = 1000\text{K} \quad G_3 = 10\text{ dB}$$

$$T_{\text{sys}} = [10 + 50 + 500/200 + 1000/4000] = 62.5\text{ K}$$

$$N_{0, \text{out}} = 200 \cdot 20 \cdot 10 \cdot 1.38 \cdot 10^{-23} \cdot 62.5 = 3.5 \cdot 10^{-17}\text{ W/Hz}$$

An alternative way of representing amplifier noise is through the *noise factor* F.

It is introduced because in many practical cases the noise temperature of the input source is at room temperature, $T_0 = 290\text{ K}$.



$$N_{0,out} = Gk(T_0 + T_e) = GkFT_0$$

$$FT_0 = T_0 + T_e$$

$$F = 1 + T_e/T_0$$

$$T_e = (F - 1) T_0$$

← definition of noise factor

← equivalent input temperature of device with noise factor F

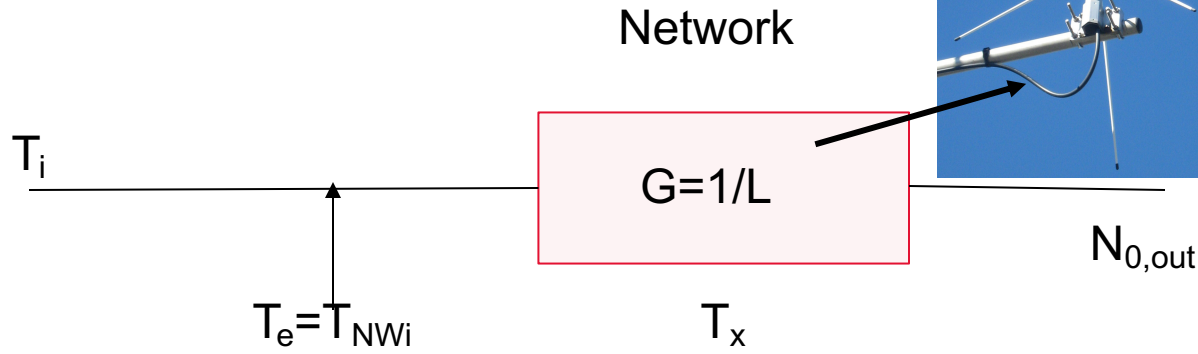
$$[F] = 10 \log F$$

← definition of noise figure

Noise temperatures are used for LNA's and converters

Noise figures and factors are used for the main receiver unit.

Absorptive network noise



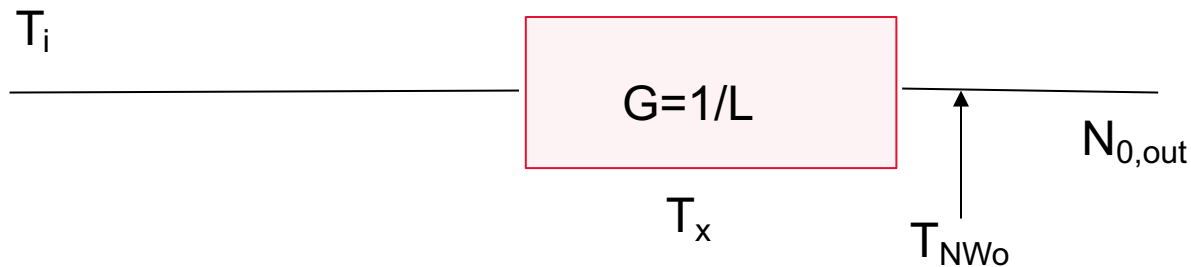
$$N_{0,out} = Gk(T_i + T_{NW_i})$$

T_{NW_i} = network temperature referred to the input (function of T_x)

T_x = physical temperature of network

L = loss

We can also look at the network by using T_{NW_o} (network temperature referred to the output).



T_{NW_o} = network temperature referred to the output

Our goal is to find T_{NW_i} and T_{NW_o} so that attenuators can just be treated like amplifiers only with a different equivalent noise temperature and a gain < 1 .

For passive devices, boundary conditions exist.

If $T_i = T_x$, then source and attenuator are “one unit” and

$$\begin{aligned} N_{0, out} &= kT_x = Gk(T_x + T_{NW_i}) \\ T_x &= G(T_x + T_{NW_i}) \\ T_{NW_i} &= T_x(1/G - 1) \\ T_{NW_i} &= T_x(L - 1) \end{aligned}$$



Special case

$$\begin{aligned} \text{If } T_x &= T_0 = 290 \text{ K} \\ T_x(L-1) &= T_0(L-1) \\ &= T_0(F-1) \\ L &= F \end{aligned}$$

But also:

$$\begin{aligned} N_{0, out} &= kT_x = GkT_x + kT_{NW_o} \\ T_x &= GT_x + T_{NW_o} \\ T_{NW_o} &= T_x(1 - G) \\ T_{NW_o} &= T_x(1 - 1/L) \end{aligned}$$

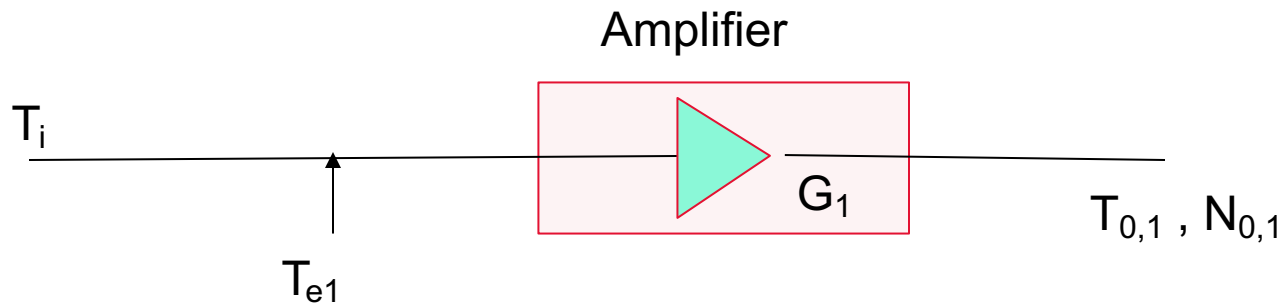
In general: $T_i \neq T_x$, and

$$N_{0, out} = Gk [T_i + T_x(L - 1)]$$



Equivalent noise temperature referred to the input

Difference between amplifier and passive attenuator

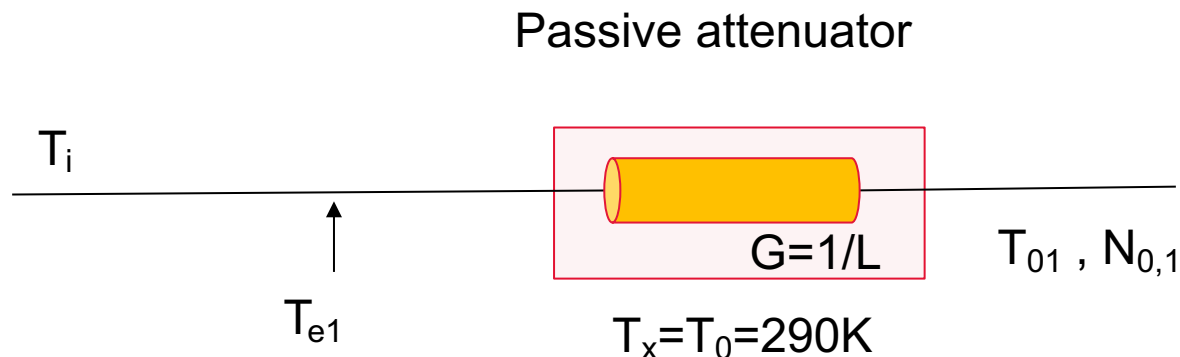


$$T_{0,1} = G_1 (T_i + T_{e1})$$

$$N_{0,1} = G_1 k(T_i + T_{e1})$$

Equivalent temperature at the output

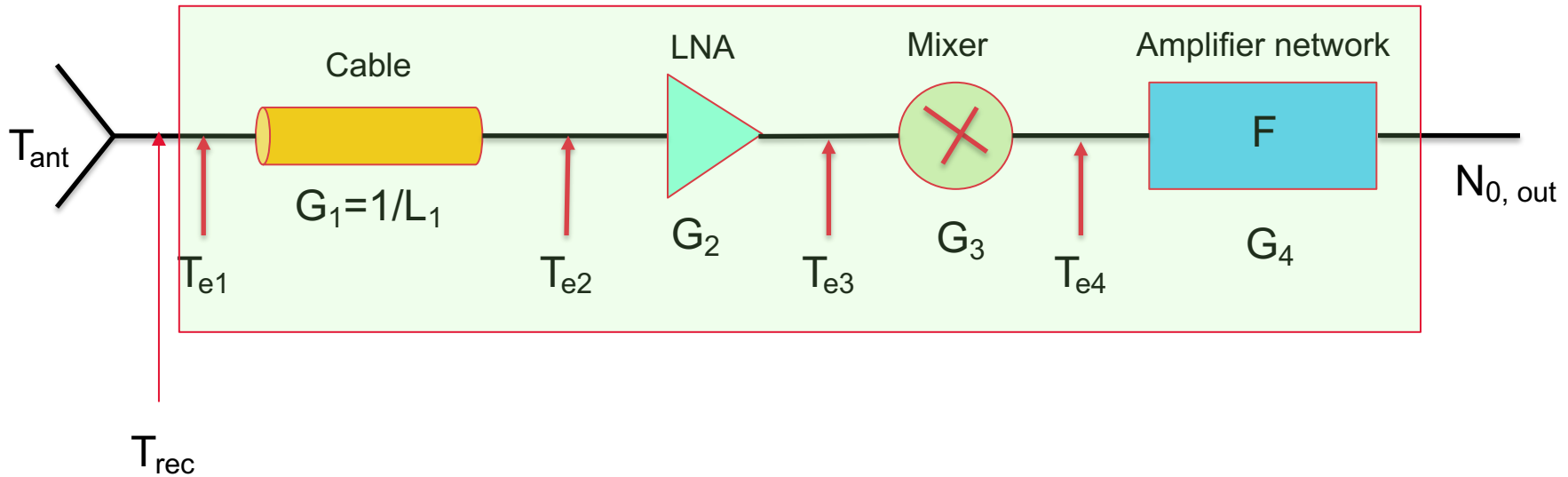
Noise power spectral density at the output



$$T_{0,1} = G_1 [T_i + (L_1 - 1)T_0] = G_1 T_i + (1 - 1/L_1)T_0$$

$$N_{0,1} = kG_1 [T_i + (L_1 - 1)T_0] = k[G_1 T_i + (1 - 1/L_1)T_0]$$

Example 5-6



Cable, Low noise amplifier, mixer and amplifier network are each sketched as consisting of two components:

- 1) A noiseless unit
- 2) An effective input temperature at the input terminals of the noiseless unit

T_{rec} is the effective input temperature for the whole receiver unit within the green box.

$$\begin{aligned}
 G_1 &= -2\text{dB} = 0.63 & T_{\text{ant}} &= 10 \text{ K} \\
 G_2 &= 23\text{dB} = 200 & T_1 &= (L_1 - 1)T_x = 168 \text{ K} \quad , T_x = 290 \text{ K}, \quad L = 1.58 \\
 G_3 &= 0\text{dB} = 1 & T_2 &= 50 \text{ K} \\
 G_4 &= 30\text{dB} = 1000 & T_3 &= 500 \text{ K} \\
 & & F &= 12 \text{ dB} \\
 & & \rightarrow T_4 &= (F-1)T_0 = (15.85 - 1) 290 \text{ K}
 \end{aligned}$$

$$T_{\text{sys}} = T_{\text{ant}} + T_{e1} + T_{e2}/G_1 + T_{e3}/(G_1 G_2) + T_{e4}/(G_1 G_2 G_3)$$

$$N_{0, \text{out}} = G_1 G_2 G_3 G_4 k T_{\text{sys}}$$

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 G_1 &= -2\text{dB} = 0.63 & T_{\text{ant}} &= 10 \text{ K} \\
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 T_{\text{sys}} &= T_{\text{ant}} + T_{e1} + T_{e2}/G_1 + T_{e3}/(G_1 G_2) + T_{e4}/(G_1 G_2 G_3) \\
 &= 10 + 168 + \frac{50}{0.63} + \frac{500}{0.63 \cdot 200} + \frac{4307}{0.63 \cdot 200 \cdot 1} = 295.5 \text{ K}
 \end{aligned}$$

$$\begin{aligned}
 N_{0, \text{out}} &= G_1 G_2 G_3 G_4 k T_{\text{sys}} \\
 &= 0.63 \cdot 200 \cdot 1 \cdot 1000 \cdot 1.38 \cdot 10^{-23} \cdot 295.5 = 5.14 \cdot 10^{-16} \text{ W Hz}^{-1}
 \end{aligned}$$

$$T_{\text{rec}} =$$

$$\begin{aligned}
 G_1 &= -2\text{dB} = 0.63 & T_{\text{ant}} &= 10 \text{ K} \\
 G_2 &= 23\text{dB} = 200 & T_1 &= (L_1 - 1)T_x = 168 \text{ K} \quad , T_x = 290 \text{ K}, \quad L = 1.58 \\
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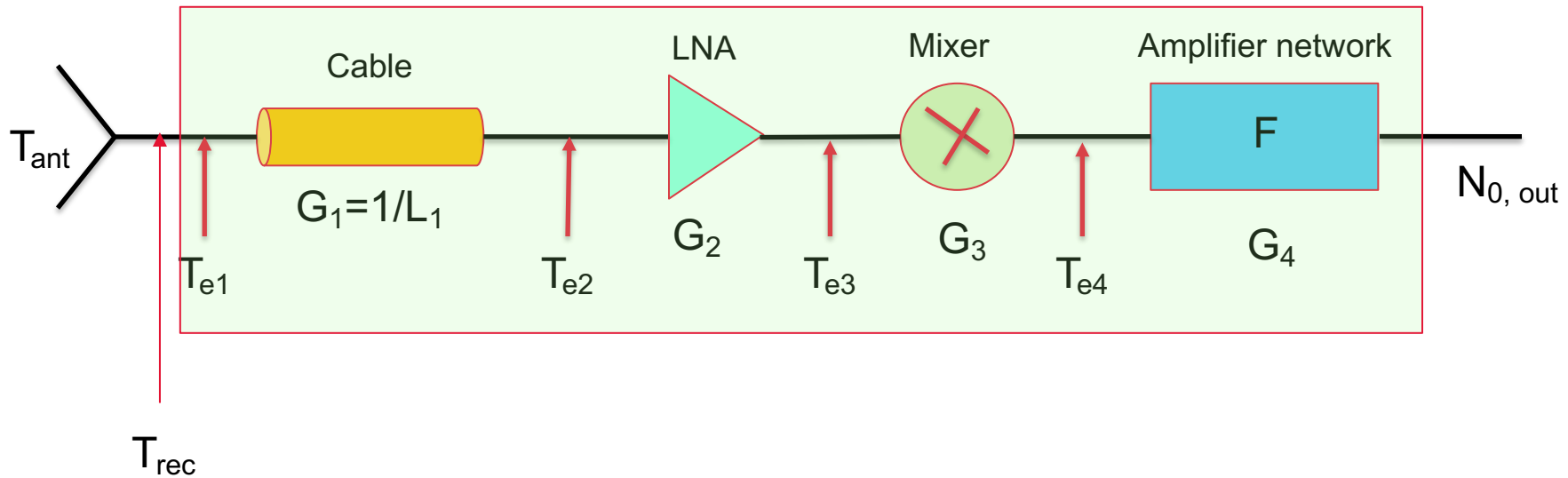
$$\begin{aligned}
 T_{\text{sys}} &= T_{\text{ant}} + T_{e1} + T_{e2}/G_1 + T_{e3}/(G_1 G_2) + T_{e4}/(G_1 G_2 G_3) \\
 &= 10 + 168 + \frac{50}{0.63} + \frac{500}{0.63 \cdot 200} + \frac{4307}{0.63 \cdot 200 \cdot 1} = 295.5 \text{ K}
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$$\begin{aligned}
 N_{0, \text{out}} &= G_1 G_2 G_3 G_4 k T_{\text{sys}} \\
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 \end{aligned}$$

$$T_{\text{rec}} = T_{e1} + T_{e2}/G_1 + T_{e3}/(G_1 G_2) + T_{e4}/(G_1 G_2 G_3)$$

$$T_{\text{sys}} = T_{\text{ant}} + T_{\text{rec}} \quad \text{System temperature} = \text{Antenna temperature} + \text{receiver temperature}$$

Example 5-7

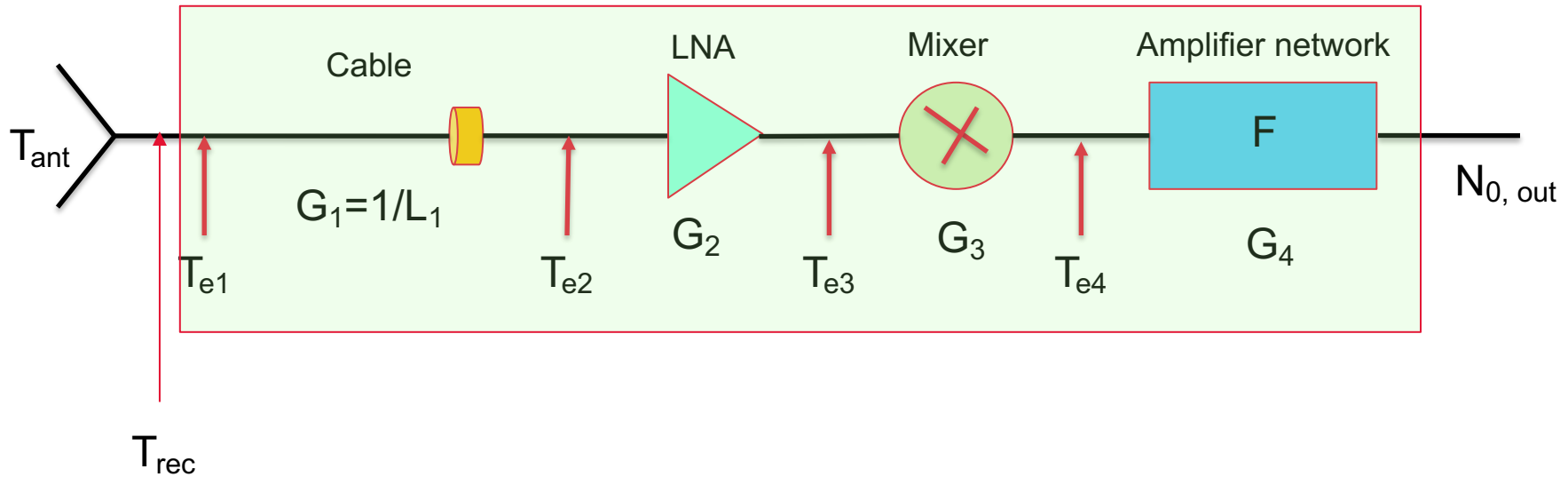


Cable, Low noise amplifier, mixer and amplifier network are each sketched as consisting of two components:

- 1) A noiseless unit
- 2) An effective input temperature at the input terminals of the noiseless unit

T_{rec} is the effective input temperature for the whole receiver unit within the green box.

Example 5-8



Cable, Low noise amplifier, mixer and amplifier network are each sketched as consisting of two components:

- 1) A noiseless unit
- 2) An effective input temperature at the input terminals of the noiseless unit

T_{rec} is the effective input temperature for the whole receiver unit within the green box.

$$\begin{aligned}
G_1 &= -0.2\text{dB} = 0.95 & T_{\text{ant}} &= 10 \text{ K} \\
G_2 &= 23\text{dB} = 200 & T_1 &= (L_1 - 1)T_x = 13.67 \text{ K} \quad , T_x = 290 \text{ K}, \quad L_1 = 1.047 \\
G_3 &= 0\text{dB} = 1 & T_2 &= 50 \text{ K} \\
G_4 &= 30\text{dB} = 1000 & T_3 &= 500 \text{ K} \\
& & F &= 12 \text{ dB} \\
& & \rightarrow T_4 &= (F-1)T_0 = (15.85 - 1) 290 \text{ K}
\end{aligned}$$

$$\begin{aligned}
T_{\text{sys}} &= T_{\text{ant}} + T_{e1} + T_{e2}/G_1 + T_{e3}/(G_1 G_2) + T_{e4}/(G_1 G_2 G_3), & T_{e1}, T_{e2}, \dots &= T_1, T_2 \\
&= 10 + 13.67 + \frac{50}{0.955} + \frac{500}{0.955 \cdot 200} + \frac{4307}{0.955 \cdot 200 \cdot 1} & &= 101.2 \text{ K}
\end{aligned}$$

$$\begin{aligned}
N_{0, \text{out}} &= G_1 G_2 G_3 G_4 k T_{\text{sys}} \\
&= 0.95 \cdot 200 \cdot 1 \cdot 1000 \cdot 1.38 \cdot 10^{-23} \cdot 101.2 = 2.65 \cdot 10^{-16} \text{ W Hz}^{-1}
\end{aligned}$$

$$T_{\text{rec}} = T_{e1} + T_{e2}/G_1 + T_{e3}/(G_1 G_2) + T_{e4}/(G_1 G_2 G_3)$$

$$T_{\text{sys}} = T_{\text{ant}} + T_{\text{rec}} \quad \text{System temperature} = \text{Antenna temperature} + \text{receiver temperature}$$

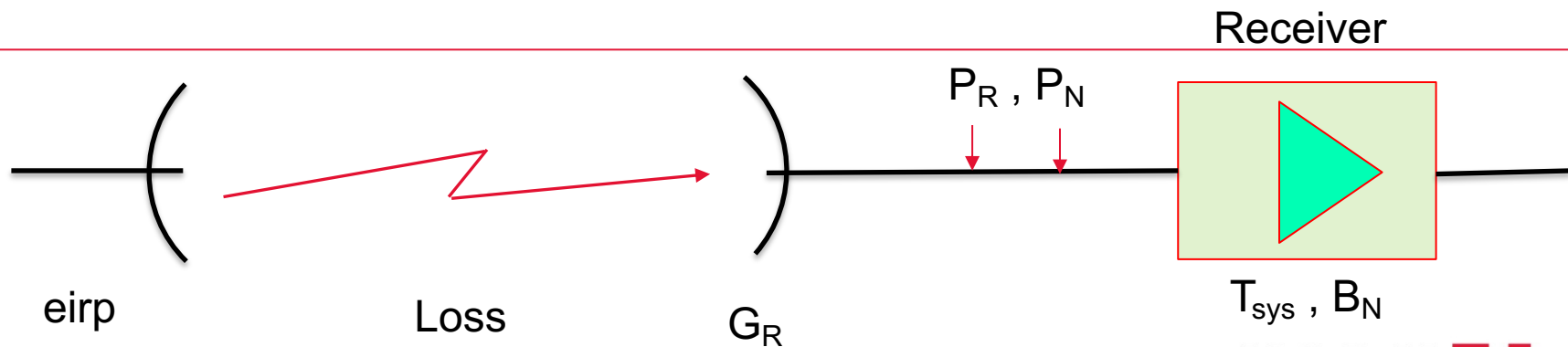
LNA (Low noise amplifier)

- *Parametric amplifiers* (not used anymore)
They operate on the principle of a pump oscillator to vary the capacitance of a varactor diode. They are expensive and refrigeration is necessary.
- *Tunnel diodes*
They permit electron tunneling through energy barrier. Amplification occurs in the negative resistance region.
- *Field-effect transistors (FETs)*
Solid state amplifiers (GaAs Fets, MosFETs). They operate on the basis of enhancement of carriers (electrons or holes) in the channel generated by a gate and by the applied gate voltage. The gain is moderate. FETS are used in cascades.
- *High-electron mobility transistors (HEMTs)*
Solid state amplifiers. Operation is similar to that of FETs.

5.3. Carrier to noise ratio

So far we have covered the power issues and the noise issues and introduced the carrier to noise ratio already. Now we want to look at these issues in more detail.

C/N : Carrier to noise ratio. (carrier power to noise power ratio)
 C/N_0 : Carrier to noise spectral density ratio. (...to noise power spectral density ratio)
 G/T : G/T ratio, Figure of merit (Characteristic of receiving antenna)
 A_0 : Effective aperture of isotropic antenna
 Φ_s : Saturation flux density
 BO_i : Input back-off
 BO_o : Output back-off



$$C/N = P_R / P_N \quad P_N = k T_N B_N , \quad T_N = T_{sys} \text{ (at receiver input)}$$

$$\begin{aligned} [C/N] &= [P_R] - [P_N] \\ &= [\text{eirp}] + [G_R] - [\text{loss}] - [k] - [T_{sys}] - [B_N] \\ &= [\text{eirp}] + [G/T] - [\text{loss}] - [k] - [B_N] \end{aligned}$$

$$[G/T] = [G_R] - [T_{sys}]$$

G/T defines the quality of an earth station (receiving station). It is a fundamental parameter.

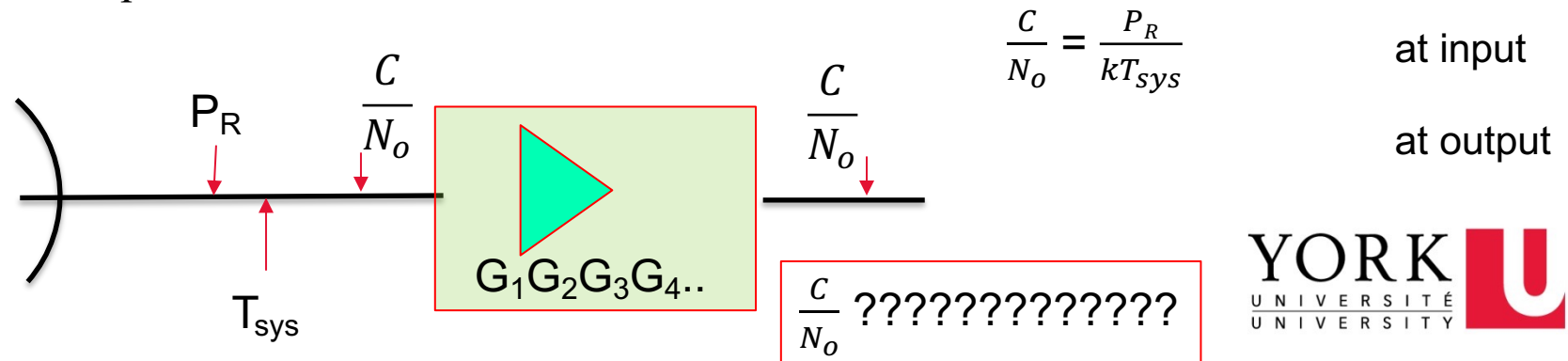
$C/N \propto G/T$ for given satellite system.

The “standard A” earth station in an Intelsat network is required to have $[G/T] \geq 40.7 \text{ dBK}^{-1}$ at 4 GHz and for $EL \geq 5^\circ$. G/T can also be given for satellite receive-antenna system.

$$[C/N_0] = [\text{eirp}] + [G/T] - [\text{loss}] - [k]$$

Example 5-9

In order to calculate C/N and C/N₀ we used P_R and P_N at LNA input. What is C/N₀ at receiver output?



$$C/N = P_R / P_N \quad P_N = k T_N B_N , \quad T_N = T_{sys} \text{ (at receiver input)}$$

$$\begin{aligned} [C/N] &= [P_R] - [P_N] \\ &= [\text{eirp}] + [G_R] - [\text{loss}] - [k] - [T_{sys}] - [B_N] \\ &= [\text{eirp}] + [G/T] - [\text{loss}] - [k] - [B_N] \end{aligned}$$

$$[G/T] = [G_R] - [T_{sys}]$$

G/T defines the quality of an earth station (receiving station). It is a fundamental parameter.

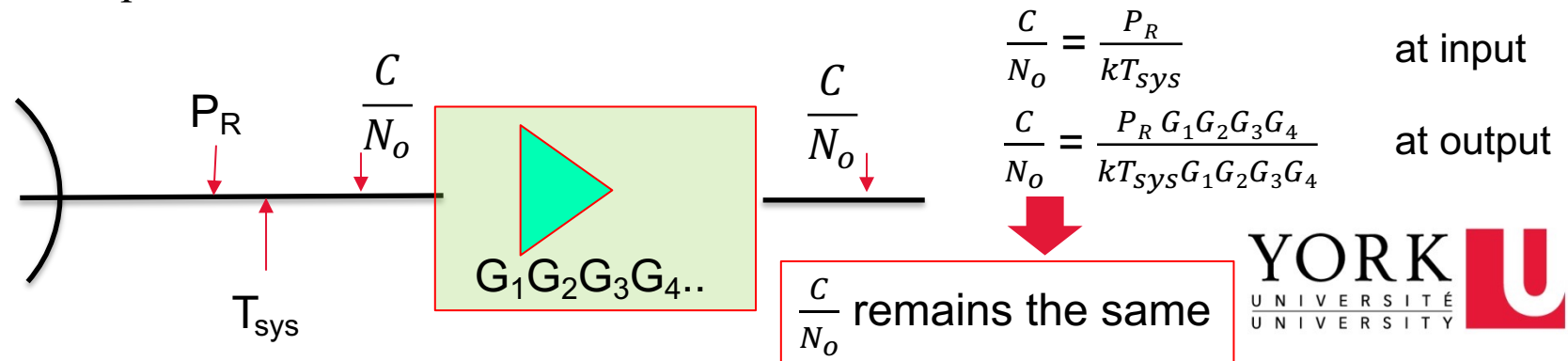
$C/N \propto G/T$ for given satellite system.

The “standard A” earth station in an Intelsat network is required to have $[G/T] \geq 40.7 \text{ dBK}^{-1}$ at 4 GHz and for $EL \geq 5^\circ$. G/T can also be given for satellite receive-antenna system.

$$[C/N_0] = [\text{eirp}] + [G/T] - [\text{loss}] - [k]$$

Example 5-6

In order to calculate C/N and C/N₀ we used P_R and P_N at LNA input. What is C/N₀ at receiver output?



Example 5-10

$$[\text{eirp}] = 48 \text{ dBW}$$

$$[\text{FSL}] = 206 \text{ dB}$$

$$[\text{AML}] = 1 \text{ dB}$$

$$[\text{AA}] = 2 \text{ dB}$$

$$[\text{RFL}] = 1 \text{ dB}$$

$$\rightarrow [\text{loss}] = 210 \text{ dB}$$

$$[\text{G/T}] = 19.5 \text{ dBK}^{-1}$$

$$[\text{B}_N] = 75.6 \text{ dBHz (36 MHz)}$$

$$\rightarrow [\text{C/N}_0] = 48 + 19.5 - 210 + 228.6 \text{ dBHz}$$
$$= 86.1 \text{ dBHz}$$

$$\rightarrow [\text{C/N}] = 86.1 - 75.6 \text{ dB}$$
$$\text{C/N} = 11.2$$

Saturation flux density

Φ is the flux density which is given sometimes instead of eirp.

$$\Phi = \frac{P_R}{A_{eff}} \quad \left[\frac{W}{m^2} \right]$$

$$G_R = \frac{4\pi A_{eff}}{\lambda^2} = \frac{\eta\pi\frac{D^2}{4}}{\lambda^2} = \eta \left(\frac{\pi D}{\lambda} \right)^2$$

$$A_{eff} = \frac{G_R \lambda^2}{4\pi} = G_R A_0$$

$$A_0 = \frac{\lambda^2}{4\pi}$$

$$\begin{aligned}
[P_R] &= [\text{eirp}] + [G_R] - [\text{loss}] \\
&= [\Phi] + [A_{\text{eff}}] \\
&= [\Phi] + [G_R] + [A_0] \\
[\text{eirp}] &= [\Phi] + [A_0] + [\text{loss}] \\
[\text{eirp}_s] &= [\Phi_s] + [A_0] + [\text{loss}]
\end{aligned}$$

eirp_s is the minimum eirp of an earth station that is required to saturate the satellite transponder.

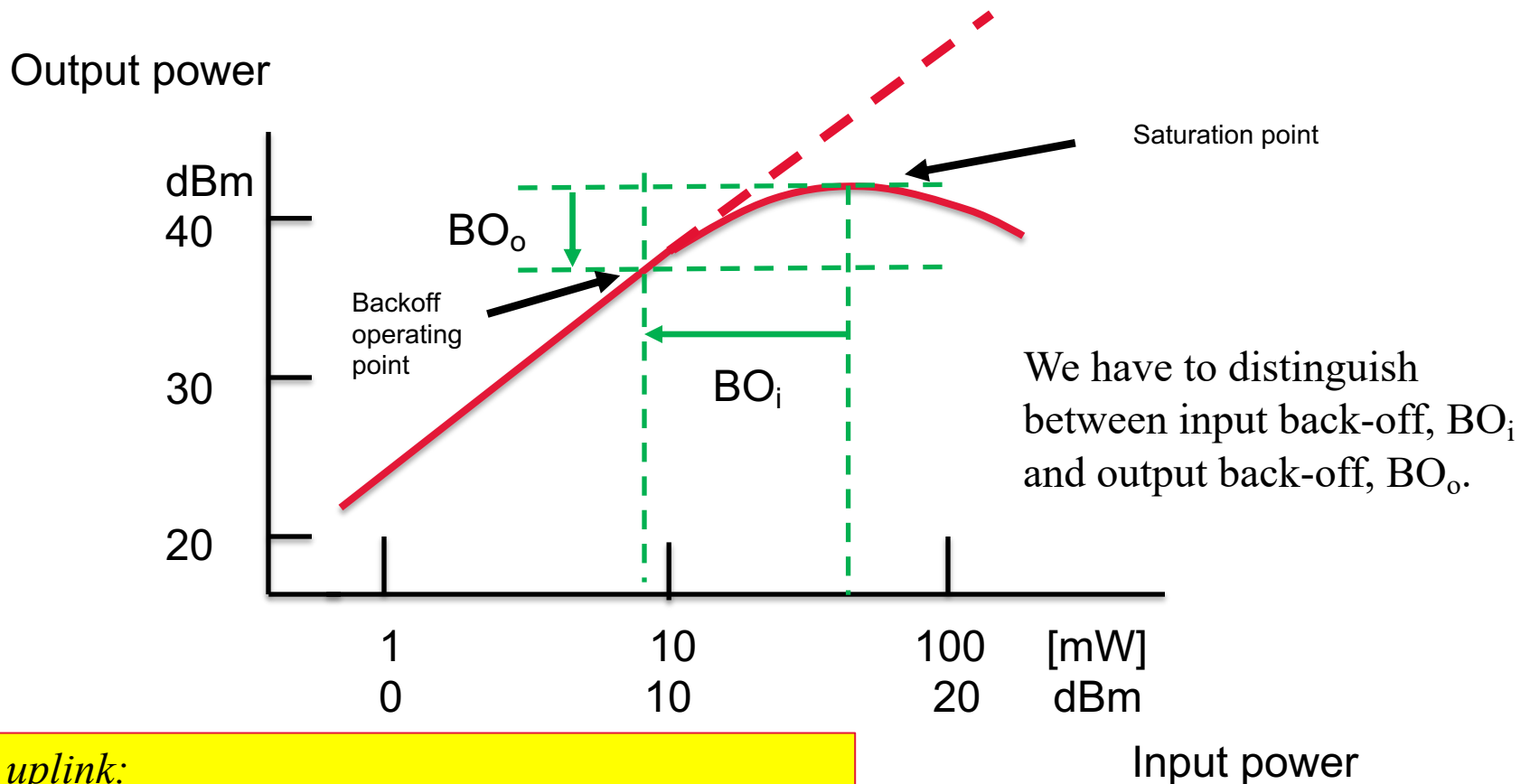
Back-off BO

With multicarrier operation we do not want to saturate the transponder. Why?

→ eirp_s has to be reduced, namely by BO

$$[\text{eirp}] = [\text{eirp}_s] - [\text{BO}]$$

Transponder amplification curve



For uplink:

$$[C/N_0]_U = [\text{eirp}]_u + [G/T]_U - [\text{loss}]_U - [k]$$

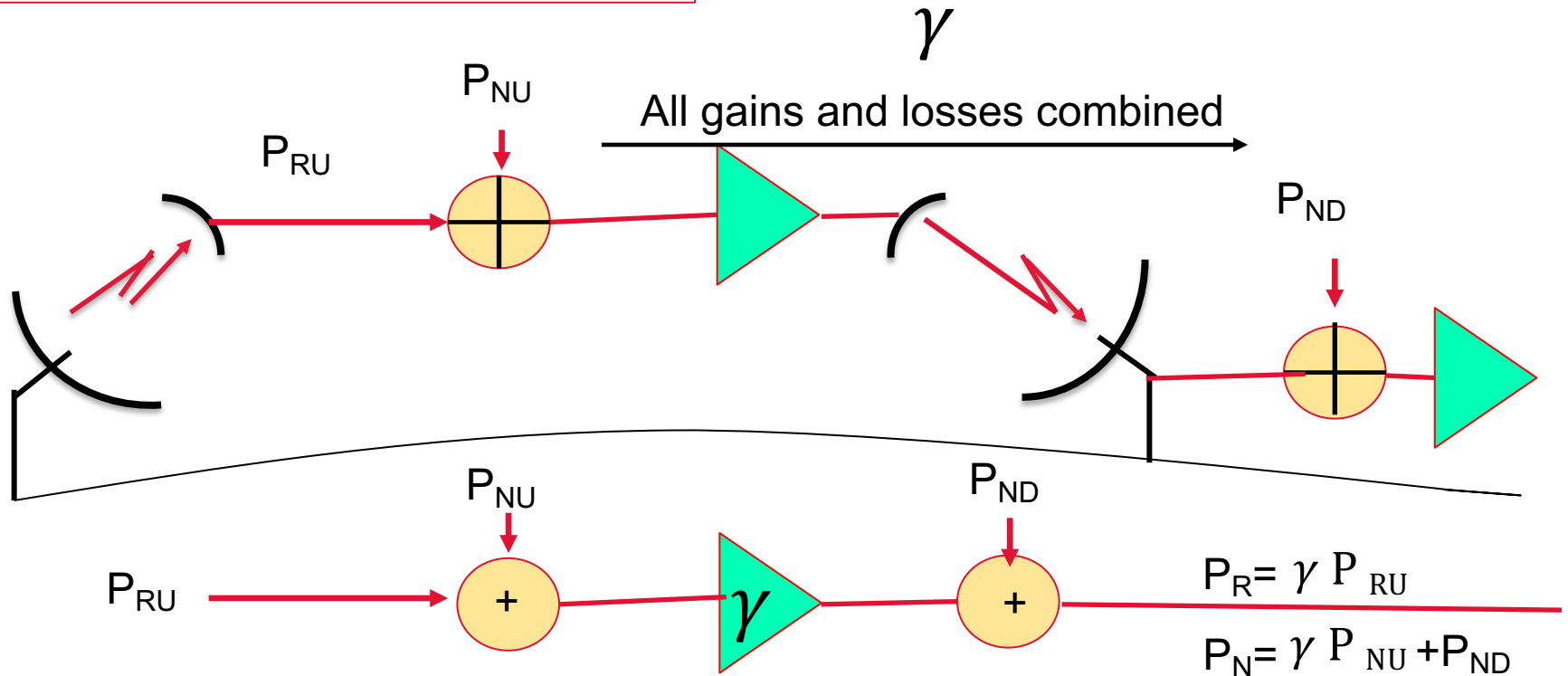
$$= [\Phi_s] + [A_0]_U - [BO]_i + [G/T] - [k]$$

For downlink:

$$[C/N_0]_D = [\text{eirp}]_D + [G/T]_D - [\text{loss}]_D - [k]$$

$$= [\text{eirp}_s]_D - [BO]_o + [G/T]_D - [\text{loss}]_D - [k]$$

Combined uplink and downlink C/N



$$(C/N)_U = P_{RU} / P_{NU}$$

$$(C/N)_D = P_R / P_{ND}$$

$$C/N = P_R / P_N$$

$$N/C = P_N / P_R = (\gamma P_{NU} + P_{ND}) / P_R$$

$$= (\gamma P_{NU} / \gamma P_{RU}) + (P_{ND} / P_R)$$

$$N/C = (N/C)_U + (N/C)_D$$

$$N/C = \sum_{i=1}^n (N/C)_i$$



In general

Types of C/N ratios and of noise

- **Thermal noise:** (From thermal motion of electrons in an electronic device)

$$(C/N)_U, (C/N)_D$$

- **Intermodulation noise:** (From the passage of multiple carriers through devices with non-linear amplification characteristics)

$$(C/N)_{IM}$$

- **Intrasystem interference noise:** (From imperfect isolations between different
 - a) bandpasses,
 - b) polarization channels,
 - c) antenna beams,
 - d) satellite systems due to sidelobe radiation from different earth stations to our satellite, or sidelobe reception of signal from other satellites into our earth station.)

Example 5-11

$$(C/N)_U = 23 \text{ dB}$$

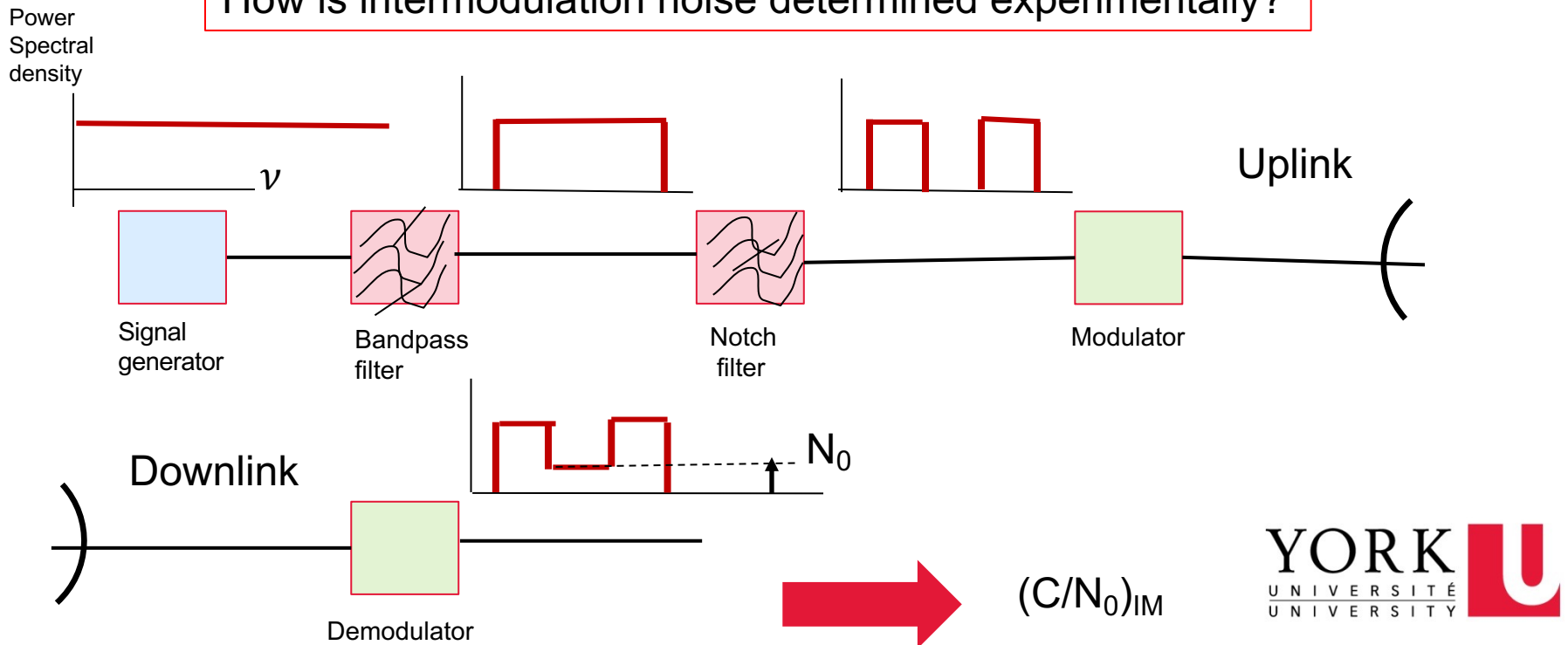
$$(C/N)_D = 16 \text{ dB}$$

$$(C/N)_{IM} = 26 \text{ dB}$$

$$(C/N)_{int} = 30 \text{ dB}$$

$$C/N = (1/200 + 1/40 + 1/400 + 1/1000)^{-1} = 29.85 \rightarrow 14.75 \text{ dB}$$

How is intermodulation noise determined experimentally?



Example 5-12

Multicarrier satellite circuit at C-band,

Uplink. $\nu = 6\text{GHz}$, $\lambda = 5\text{ cm}$

$$[\Phi_s] = -67.5\text{ dBW m}^{-2}$$

$$[\text{BO}]_i = 11\text{ dB}$$

$$D = 0.4\text{ m}$$

$$\eta = 60\%$$

$$T_{\text{sys}} = 1000\text{ K}$$

$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [k]$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [k]$$

Example 5-12

Multicarrier satellite circuit at C-band,

Uplink. $\nu = 6\text{GHz}$, $\lambda = 5\text{ cm}$

$$[\Phi_s] = -67.5\text{ dBW m}^{-2}$$

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$$D = 0.4\text{ m}$$

$$\eta = 60\%$$

$$T_{\text{sys}} = 1000\text{ K}$$

$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [k]$$

$$\begin{aligned}\frac{G}{T} &= \frac{\eta \left(\frac{\pi D}{\lambda}\right)^2}{T_{\text{sys}}} = \frac{0.6 \left(\frac{\pi 0.4}{0.05}\right)^2}{1000} = 0.38\text{ K}^{-1} \\ &= -4.2\text{ dB K}^{-1}\end{aligned}$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [k]$$

Example 5-12

Multicarrier satellite circuit at C-band,

Uplink. $\nu = 6\text{GHz}$, $\lambda = 5\text{ cm}$

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$$[\text{BO}]_i = 11\text{ dB}$$

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$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [\text{k}]$$

$$\frac{G}{T} = \frac{\eta \left(\frac{\pi D}{\lambda}\right)^2}{T_{\text{sys}}} = \frac{0.6 \left(\frac{\pi 0.4}{0.05}\right)^2}{1000} = 0.38\text{ K}^{-1}$$
$$= -4.2\text{ dB K}^{-1}$$

$$A_0 = \frac{\lambda^2}{4\pi} = \frac{0.05^2}{4\pi} = -37\text{ dB m}^2$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [\text{k}]$$

Example 5-12

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$$[\text{BO}]_i = 11\text{ dB}$$

$$D = 0.4\text{ m}$$

$$\eta = 60\%$$

$$T_{\text{sys}} = 1000\text{ K}$$

$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [\text{k}]$$

$$= -67.5 - 37.0 - 11 - 4.2 + 228.6 = 108.9\text{ dB Hz}$$

$$\frac{G}{T} = \frac{\eta \left(\frac{\pi D}{\lambda}\right)^2}{T_{\text{sys}}} = \frac{0.6 \left(\frac{\pi 0.4}{0.05}\right)^2}{1000} = 0.38\text{ K}^{-1}$$
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$$A_0 = \frac{\lambda^2}{4\pi} = \frac{0.05^2}{4\pi} = -37\text{ dB m}^2$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [\text{k}]$$

Example 5-12

Multicarrier satellite circuit at C-band,

Uplink. $\nu = 6\text{GHz}$, $\lambda = 5\text{ cm}$

$$[\Phi_s] = -67.5\text{ dBW m}^{-2}$$

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$$D = 0.4\text{ m}$$

$$\eta = 60\%$$

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$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [k]$$

$$= -67.5 - 37.0 - 11 - 4.2 + 228.6 = 108.9\text{ dB Hz}$$

$$\frac{G}{T} = \frac{\eta \left(\frac{\pi D}{\lambda}\right)^2}{T_{\text{sys}}} = \frac{0.6 \left(\frac{\pi 0.4}{0.05}\right)^2}{1000} = 0.38\text{ K}^{-1}$$
$$= -4.2\text{ dB K}^{-1}$$

$$A_0 = \frac{\lambda^2}{4\pi} = \frac{0.05^2}{4\pi} = -37\text{ dB m}^2$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [k]$$

$$= 26.6 + 41 - 200 + 228.6 = 96.2\text{ dBHz}$$

Example 5-12

Multicarrier satellite circuit at C-band,

Uplink. $\nu = 6\text{GHz}$, $\lambda = 5\text{ cm}$

$$[\Phi_s] = -67.5\text{ dBW m}^{-2}$$

$$[\text{BO}]_i = 11\text{ dB}$$

$$D = 0.4\text{ m}$$

$$\eta = 60\%$$

$$T_{\text{sys}} = 1000\text{ K}$$

$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [k]$$

$$= -67.5 - 37.0 - 11 - 4.2 + 228.6 = 108.9\text{ dB Hz}$$

$$\frac{G}{T} = \frac{\eta \left(\frac{\pi D}{\lambda}\right)^2}{T_{\text{sys}}} = \frac{0.6 \left(\frac{\pi 0.4}{0.05}\right)^2}{1000} = 0.38\text{ K}^{-1}$$
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$$A_0 = \frac{\lambda^2}{4\pi} = \frac{0.05^2}{4\pi} = -37\text{ dB m}^2$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [k]$$

$$= 26.6 + 41 - 200 + 228.6 = 96.2\text{ dBHz}$$

$$\frac{C}{N_0} = \left\{ \left(\frac{C}{N_0}\right)_U^{-1} + \left(\frac{C}{N_0}\right)_D^{-1} \right\}^{-1}$$

$$1.288 \times 10^{-11}$$

$$2.399 \times 10^{-10}$$

Example 5-12

Multicarrier satellite circuit at C-band,

Uplink. $\nu = 6\text{GHz}$, $\lambda = 5\text{ cm}$

$$[\Phi_s] = -67.5\text{ dBW m}^{-2}$$

$$[\text{BO}]_i = 11\text{ dB}$$

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$$T_{\text{sys}} = 1000\text{ K}$$

$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [\text{k}]$$

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$$= -4.2\text{ dB K}^{-1}$$

$$A_0 = \frac{\lambda^2}{4\pi} = \frac{0.05^2}{4\pi} = -37\text{ dB m}^2$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [\text{k}]$$

$$= 26.6 + 41 - 200 + 228.6 = 96.2\text{ dBHz}$$

$$\frac{C}{N_0} = \left\{ \left(\frac{C}{N_0}\right)_U + \left(\frac{C}{N_0}\right)_D \right\}^{-1}$$

$$\left[\frac{C}{N_0}\right] = 96.0\text{ dBHz}$$

Example 5-12

Multicarrier satellite circuit at C-band,

Uplink. $\nu = 6\text{GHz}$, $\lambda = 5\text{ cm}$

$$[\Phi_s] = -67.5\text{ dBW m}^{-2}$$

$$[\text{BO}]_i = 11\text{ dB}$$

$$D = 0.4\text{ m}$$

$$\eta = 60\%$$

$$T_{\text{sys}} = 1000\text{ K}$$

$$\left[\frac{C}{N_0}\right]_U = [\Phi_s] + [A_0]_U - [\text{BO}]_i + \left[\frac{G}{T}\right]_U - [\text{k}]$$

$$= -67.5 - 37.0 - 11 - 4.2 + 228.6 = 108.9\text{ dB Hz}$$

$$\frac{G}{T} = \frac{\eta \left(\frac{\pi D}{\lambda}\right)^2}{T_{\text{sys}}} = \frac{0.6 \left(\frac{\pi \cdot 0.4}{0.05}\right)^2}{1000} = 0.38\text{ K}^{-1}$$
$$= -4.2\text{ dB K}^{-1}$$

$$A_0 = \frac{\lambda^2}{4\pi} = \frac{0.05^2}{4\pi} = -37\text{ dB m}^2$$

Downlink

$$[\text{eirp}] = 26.6\text{ dBW}$$

$$[\text{loss}] = 200\text{ dB}$$

$$\left[\frac{G}{T}\right] = 41\text{ dB K}^{-1}$$

$$\left[\frac{C}{N_0}\right]_D = [\text{eirp}]_D + \left[\frac{G}{T}\right]_D - [\text{loss}]_D - [\text{k}]$$

$$= 26.6 + 41 - 200 + 228.6 = 96.2\text{ dBHz}$$

$$\frac{C}{N_0} = \left\{ \left(\frac{C}{N_0}\right)_U^{-1} + \left(\frac{C}{N_0}\right)_D^{-1} \right\}^{-1}$$

$$= 96.0\text{ dBHz}$$

For a 36 MHz = 75.6 dBHz
transponder channel

$$\left[\frac{C}{N}\right] = 96.0 - 75.6 = 20.4\text{ dB}$$

Could we communicate with hypothetical aliens in our Galaxy?

Example 5-13

Assume that we have the 300m Arecibo antenna

Assume that the aliens also have a reflector antenna with a diameter of 300 m

Assume an efficiency of the antennas of 60 %

Assume for the frequency 10 GHz

Assume a minimum $[C/N_0] = 0\text{dB}$ for the aliens to be able to pick up your carrier.

How far can the aliens be located away from Earth to detect your carrier?

$$[C/N_0] = [eirp] + [G/T] - [\text{loss}] - [k]$$

$$[\text{loss}] = [eirp] + [G/T] - [C/N_0] - [k]$$

$$[C/N_0] = [eirp] + [G/T] - [\text{loss}] - [k]$$

$$[\text{loss}] = [eirp] + [G/T] - [C/N_0] - [k]$$

$$[eirp] = [P_T] + [G_T]$$

$$\begin{aligned} G_T = G_R &= \eta \left(\frac{\pi D}{\lambda} \right)^2 = 0.6 \left(\frac{\pi 300}{0.03} \right)^2 = 5.92 \cdot 10^8 = 87.72 \text{ dB} \\ G/T &= G_R / T_{\text{sys}} = 5.92 \cdot 10^8 / 40 = 1.48 \cdot 10^7 \text{ K}^{-1} = 71.70 \text{ dBK}^{-1} \\ k &= 1.38 \cdot 10^{-23} \text{ JK}^{-1} = -228.60 \text{ dB JK}^{-1} \end{aligned}$$

$$[C/N_0] = [eirp] + [G/T] - [\text{loss}] - [k]$$

$$[\text{loss}] = [eirp] + [G/T] - [C/N_0] - [k]$$

$$[eirp] = [P_T] + [G_T]$$

$$G_T = G_R = \eta \left(\frac{\pi D}{\lambda} \right)^2 = 0.6 \left(\frac{\pi 300}{0.03} \right)^2 = 5.92 \cdot 10^8 = 87.72 \text{ dB}$$

$$G/T = G_R / T_{\text{sys}} = 5.92 \cdot 10^8 / 40 = 1.48 \cdot 10^7 \text{ K}^{-1} = 71.70 \text{ dBK}^{-1}$$

$$k = 1.38 \cdot 10^{-23} \text{ JK}^{-1} = -228.60 \text{ dB JK}^{-1}$$

$$[\text{loss}] = 60 + 87.72 + 71.70 - 0 + 228.60$$

$$= 448.02 \text{ dB}$$

$$\text{loss} = 10^{44.802}$$

$$\text{FSL} = \left(\frac{4\pi d}{\lambda} \right)^2$$

$$[\text{FSL}] = 32.44 + 20 \log d(\text{km}) + 20 \log \nu(\text{MHz}) \text{ dB}$$

$$20 \log d(\text{km}) = [\text{FSL}] - 32.44 - 20 \log \nu(\text{MHz})$$

$$= 448.02 - 32.44 - 80$$

$$= 335.58$$

$$d(\text{km}) = 10^{335.58/20} = 10^{16.78} \text{ km}$$

$$= 6.03 \times 10^{16} \text{ km}$$

Or:

$$d = (\text{FSL})^{1/2} \frac{\lambda}{4\pi} = 10^{22.401} \times 0.03 / 12.56$$

$$= 6.01 \times 10^{19} \text{ m}$$

$$1 \text{ pc} = 3.086 \cdot 10^{13} \text{ km}$$

$$1 \text{ pc} = 3.26 \text{ ly}$$

$$[C/N_0] = [\text{eirp}] + [G/T] - [\text{loss}] - [k]$$

$$[\text{loss}] = [\text{eirp}] + [G/T] - [C/N_0] - [k]$$

$$[\text{eirp}] = [P_T] + [G_T]$$

$$G_T = G_R = \eta \left(\frac{\pi D}{\lambda} \right)^2 = 0.6 \left(\frac{\pi 300}{0.03} \right)^2 = 5.92 \cdot 10^8 = 87.72 \text{ dB}$$

$$G/T = G_R / T_{\text{sys}} = 5.92 \cdot 10^8 / 40 = 1.48 \cdot 10^7 \text{ K}^{-1} = 71.70 \text{ dBK}^{-1}$$

$$k = 1.38 \cdot 10^{-23} \text{ JK}^{-1} = -228.60 \text{ dB JK}^{-1}$$

$$[\text{loss}] = 60 + 87.72 + 71.70 - 0 + 228.60$$

$$= 448.02 \text{ dB}$$

$$d = 1952.6 \text{ pc}$$

$$d = 6350 \text{ ly}$$

$$\text{FSL} = \left(\frac{4\pi d}{\lambda} \right)^2$$

$$[\text{FSL}] = 32.44 + 20 \log d(\text{km}) + 20 \log \nu(\text{MHz}) \text{ dB}$$

$$20 \log d(\text{km}) = [\text{FSL}] - 32.44 - 20 \log \nu(\text{MHz})$$

$$= 448.02 - 32.44 - 80$$

$$= 335.58$$

$$d(\text{km}) = 10^{335.58/20} = 10^{16.78} \text{ km}$$

$$1 \text{ pc} = 3.086 \cdot 10^{13} \text{ m}$$

$$1 \text{ pc} = 3.26 \text{ ly}$$

$$\rightarrow d = 1952.6 \text{ pc}$$

$$\rightarrow d = 6350 \text{ ly}$$