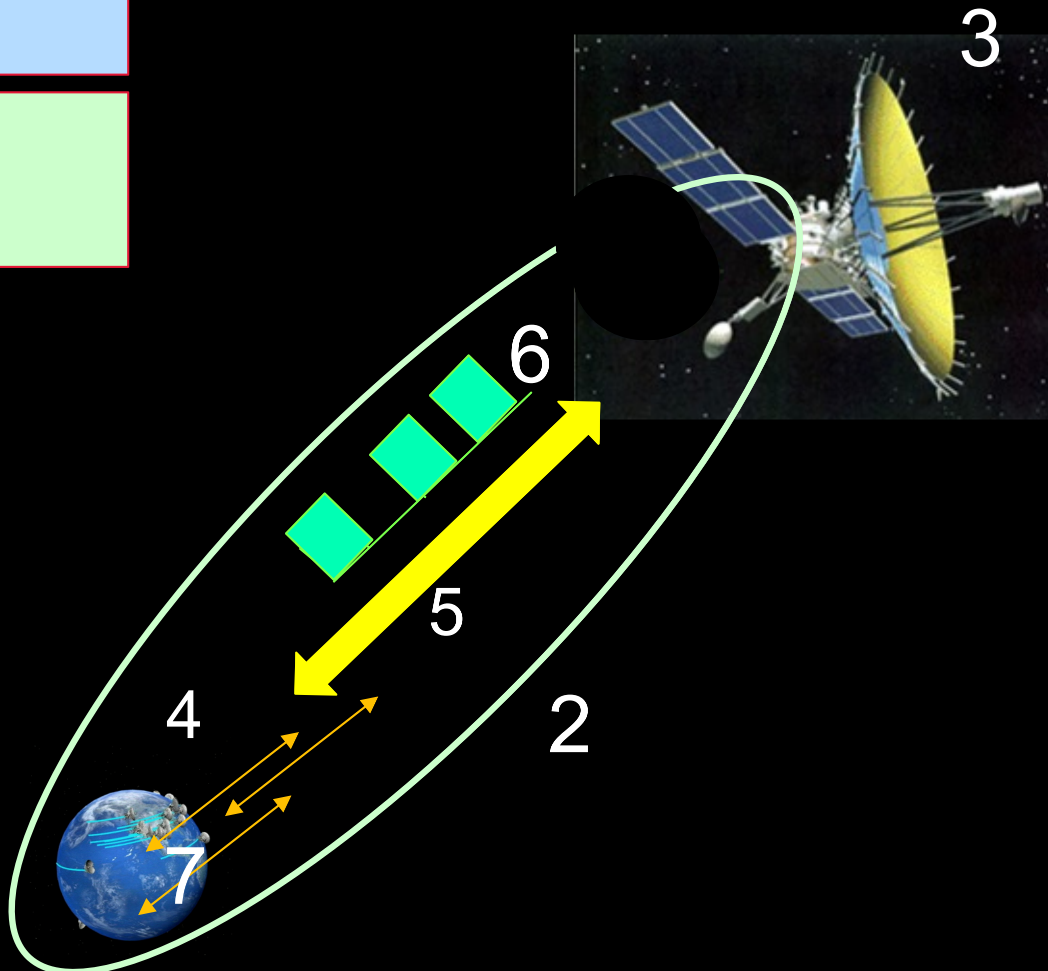
PHYS 3250
Introduction to space communications

Professor N Bartel

Sketch of the 7 chapters

2 Orbital aspects
3 Spacecraft
4 Earth station
5 Communications link
6 Modulation and multiplexing techniques
7 Multiple access to a satellite



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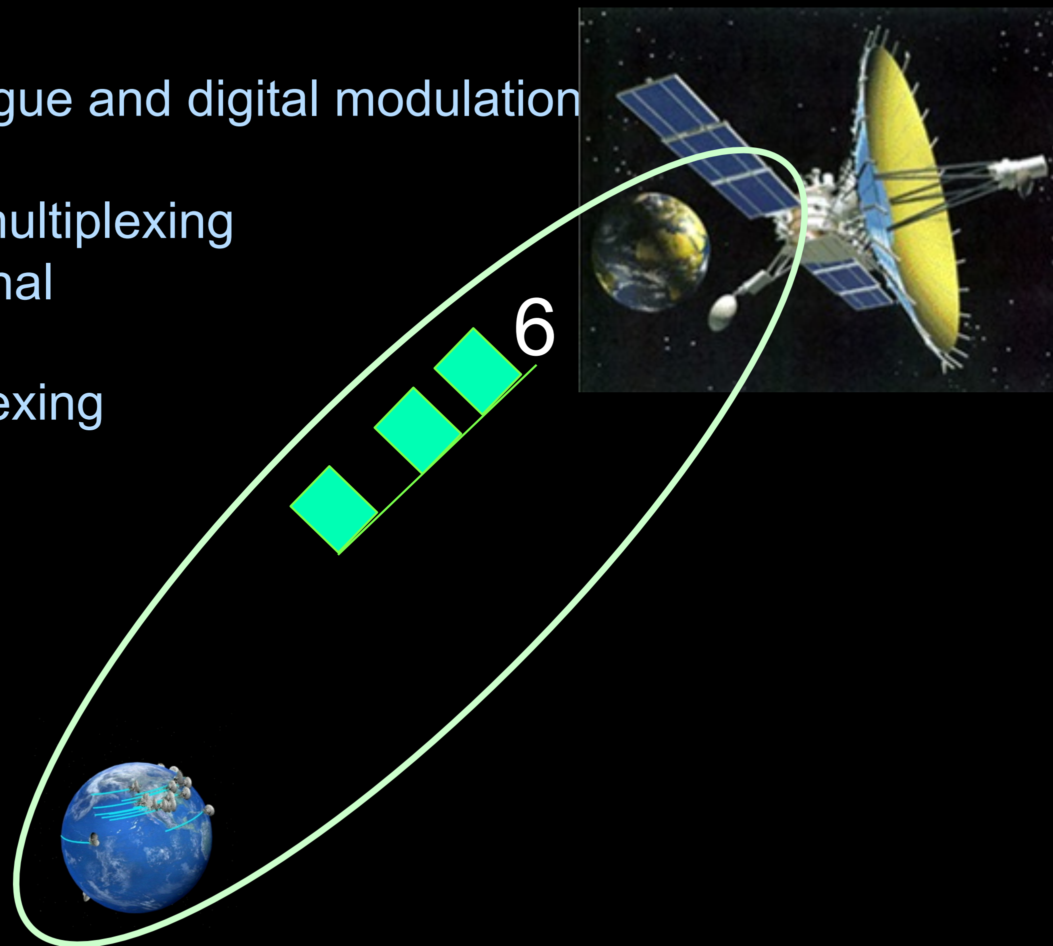
6. Modulation and multiplexing techniques

6.1 Introduction to analogue and digital modulation
6.2 Analogue modulation
6.3 Frequency division multiplexing
6.4 Digital baseband signal
6.5 Digital modulation
6.6 Time division multiplexing



6.1 Introduction to analogue and digital modulation

First we want to compare the basics of the transmission with analogue and digital modulation

The quality of transmission with analogue modulation is S/N (signal to noise ratio).

$$S/N = \frac{\text{signal power}}{\text{baseband noise power}}$$

S/N can be greater than C/N. For instance for FM (frequency modulation) TV transmission: [S/N] can be up to 35 dB larger than [C/N].

The quality of transmission with digital modulation is BER (bit error rate).

$$BER = \frac{\text{number of improperly detected bits during time t}}{\text{total number of bits detected during time t}}$$

e.g. BER = 10^{-5} ➔ on average 1 bit in 100,000 is wrong.

S/N and BER depend on C/N and on the modulation technique.

Analogue modulation

- Amplitude modulation (AM)
- Frequency modulation (FM)
- Phase modulation (PM)

Digital modulation

- Amplitude shift keying (ASK)
- Frequency shift keying (FSK)
- Phase shift keying (PSK)
    - Binary PSK (BPSK)
    - Quadrature PSK (QPSK)

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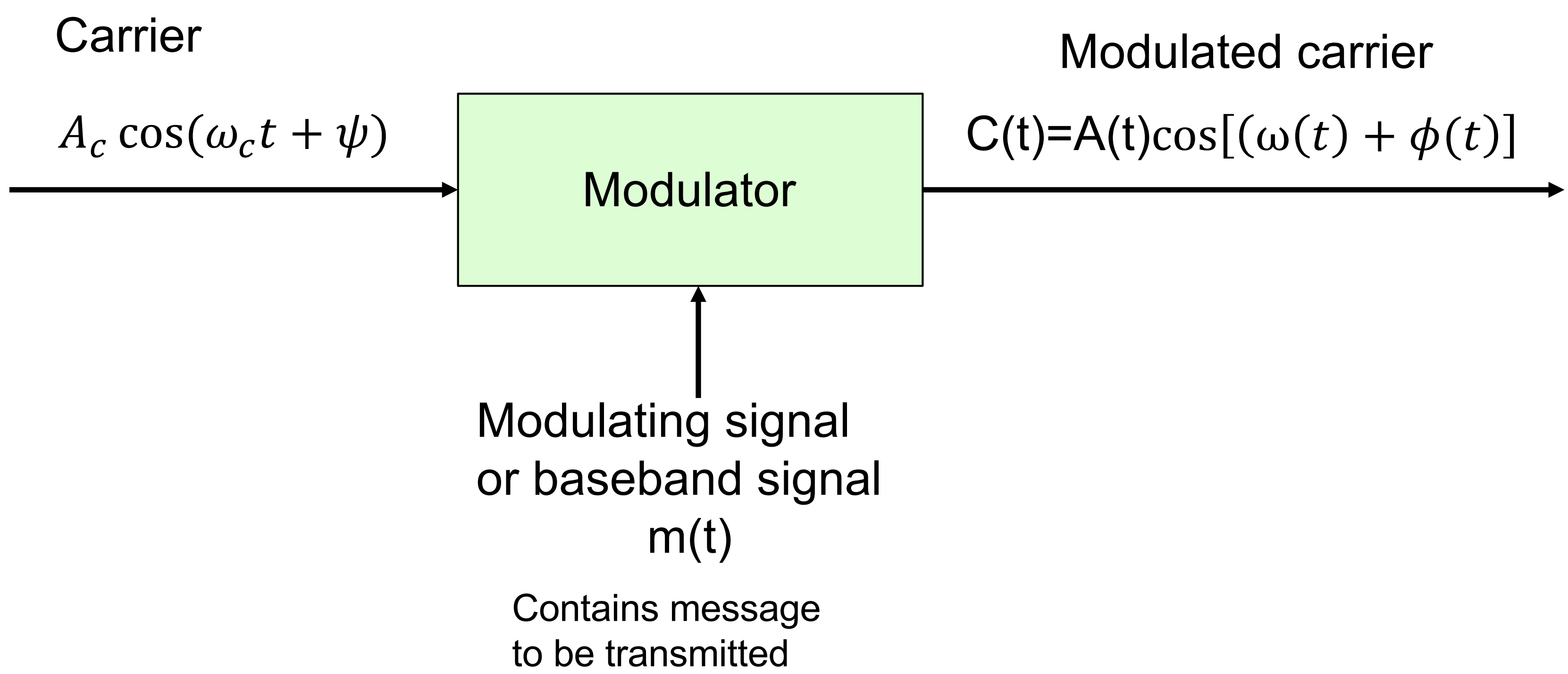
6.2. Analogue modulation

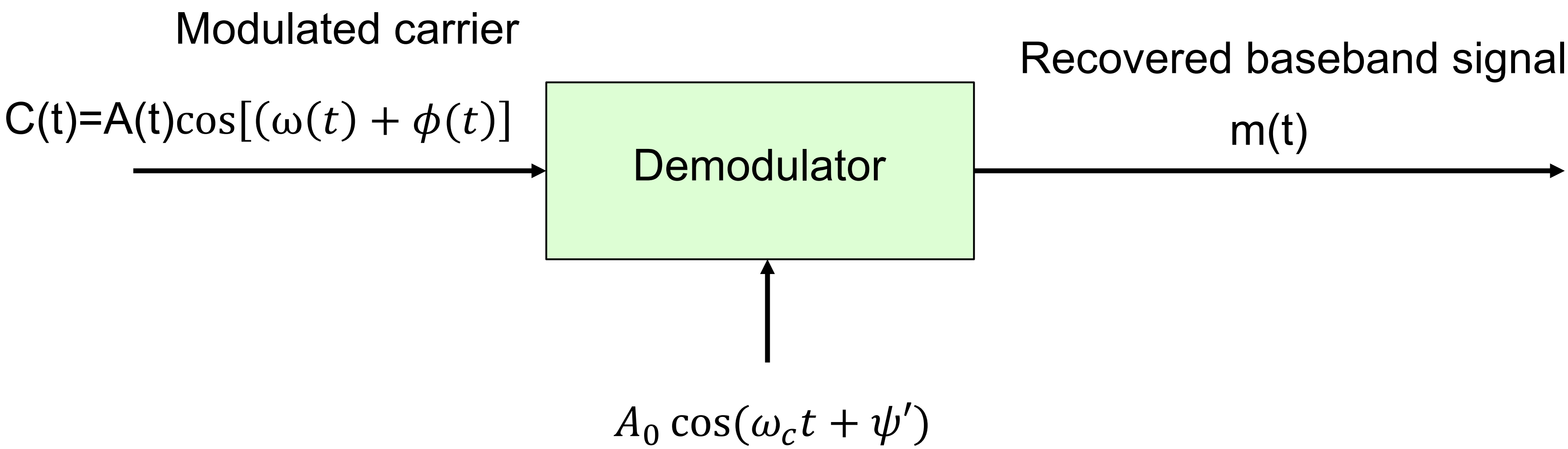


AM: Message is carried in A(t): C(t)=A(t) $\cos[\omega_c t + \phi_0]$

FM: Message is carried in $\omega(t)$: C(t)=$A_c \cos[\omega(t) + \phi_0]$

PM: Message is carried in ϕ(t): C(t)=$A_c \cos[\omega_c t + \phi(t)]$

Analogue demodulation



6.2.1 Amplitude modulation (AM)

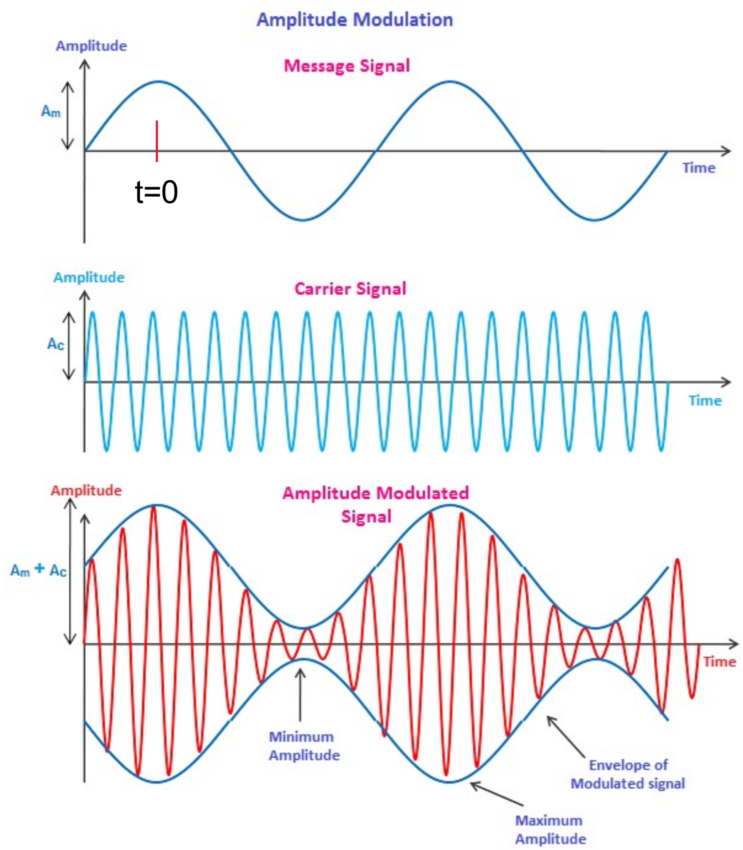
C(t) = A(t) cos (ω_ct + ϕ_0) ω_c: carrier angular frequency

= A_c [1+ $\frac{A_m}{A_c}$ m(t)] cos (ω_ct + ϕ_0)

$\frac{A_m}{A_c}$: $0 \le \frac{A_m}{A_c} \le 1$: amplitude modulation index

m(t) : $|m(t)| \le 1$: baseband signal

Graphical example for a special case: m(t) = cos ω_mt



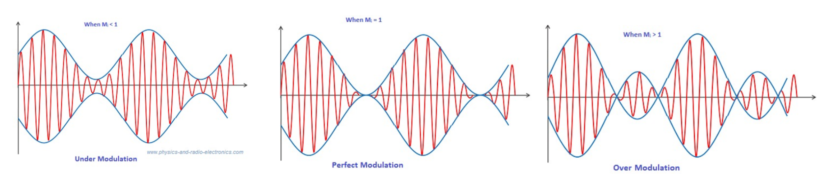
$$m(t) = A_m \cos(\omega_m t)$$ Baseband signal

$$A_c \cos(\omega_c t)$$ Carrier

Modulated carrier

$$C(t) = A_c \left[1 + \frac{A_m}{A_c}\cos(\omega_m t)\right] \cdot \cos(\omega_c t)$$

$\frac{A_m}{A_c}$: Modulation index



$\frac{A_m}{A_c}$ =0.8 1.0 >1.0

https://www.physics-and-radio-electronics.com/blog/amplitude-modulation/

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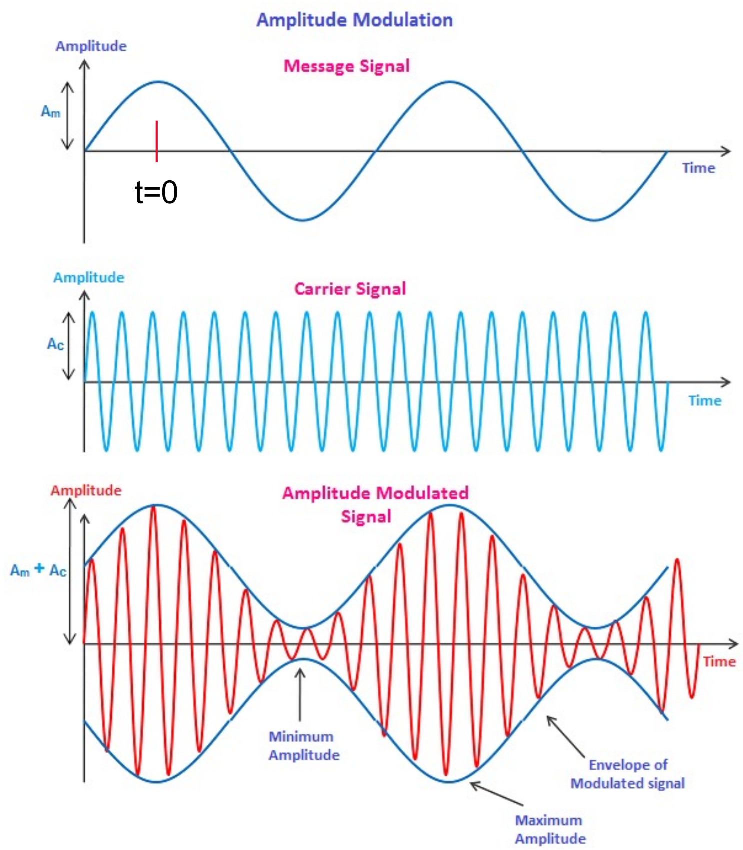
Graphical example for a special case: m(t) = cos $\omega_m t$

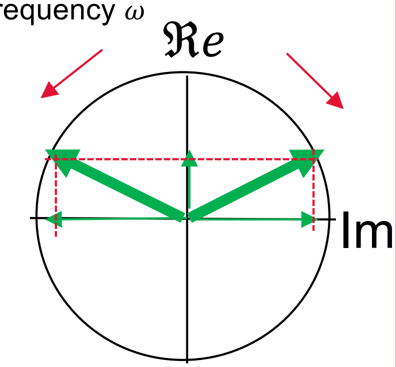


$$m(t) = A_m \cos(\omega_m t)$$

$$A_c \cos(\omega_c t)$$

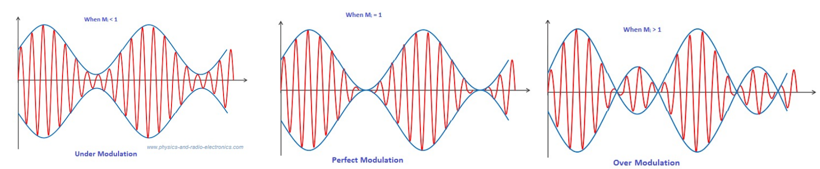
$$\cos \omega t = \frac{1}{2}\left(e^{+i\omega t} + e^{-i\omega t}\right)$$

Vectors rotate clockwise and counter-clockwise with frequency ω



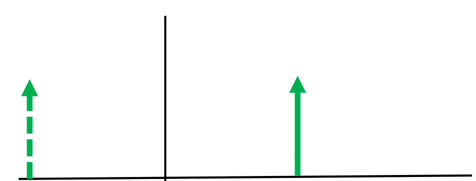
$$C(t) = A_c\left[1 + \frac{A_m}{A_c}\cos(\omega_m t)\right] \cdot \cos(\omega_c t)$$

$\frac{A_m}{A_c}$: Modulation index



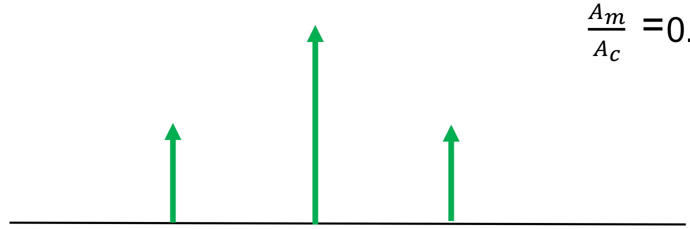
$\frac{A_m}{A_c}$ =0.8 1.0 >1.0

AM spectrum



$-f_m$ 0 f_m

Baseband



$f_c - f_m$ f_c $f_c + f_m$

Modulated carrier

https://www.physics-and-radio-electronics.com/blog/amplitude-modulation/

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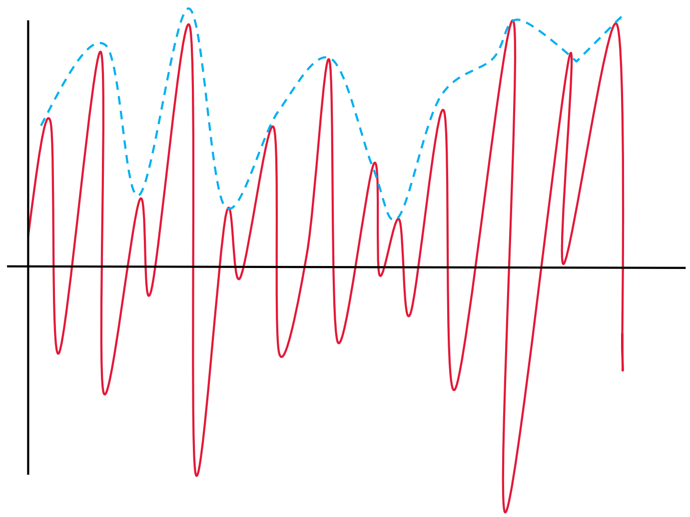
General case:
baseband signal m(t) consists of cosine waves with a continuous range of frequencies.

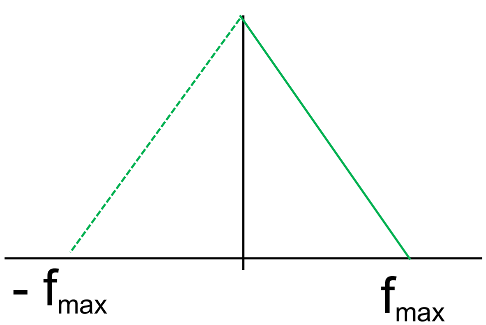


$S/N \approx C/N$ at detector output

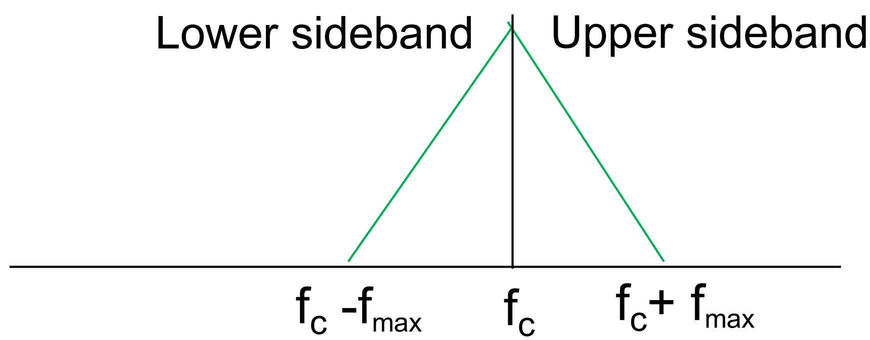
1. AM does not provide improvement of S/N over C/N.
2. AM is very susceptible to non-linear distortions as produced in satellite transponders, and is easily disturbed by link noise and interference.

➔ AM is essentially never used in satellite communications.

AM spectrum for general case



Triangular baseband spectrum



6.2.2 Frequency modulation (FM)

Modulated carrier

$C(t) = A_0 \cos(\omega(t)\, t + \phi_0)$

$= A_0 \cos(\int_0^t (\omega_c + k_\omega m(t))dt + \phi_0)$

$= A_0 \cos(\omega_c t + k_\omega \int_0^t m(t)dt + \phi_0)$

ω_c: carrier angular frequency

$\omega_i(t) = \omega_c + k_\omega m(t)$: instantaneous frequency,

k_ω : frequency modulator=constant

Example 6-1

If $\omega_i(t)$ = constant, then $C(t) = A_0 \cos(\int_0^t \omega_i dt + \phi_0) = A_0 \cos(\omega_i t + \phi_0)$

Integration of frequency over time gives the phase

$\omega_i(t) = \frac{d}{dt}\theta(t)$

Graphical example for a special case: $m(t) = \cos \omega_m t$

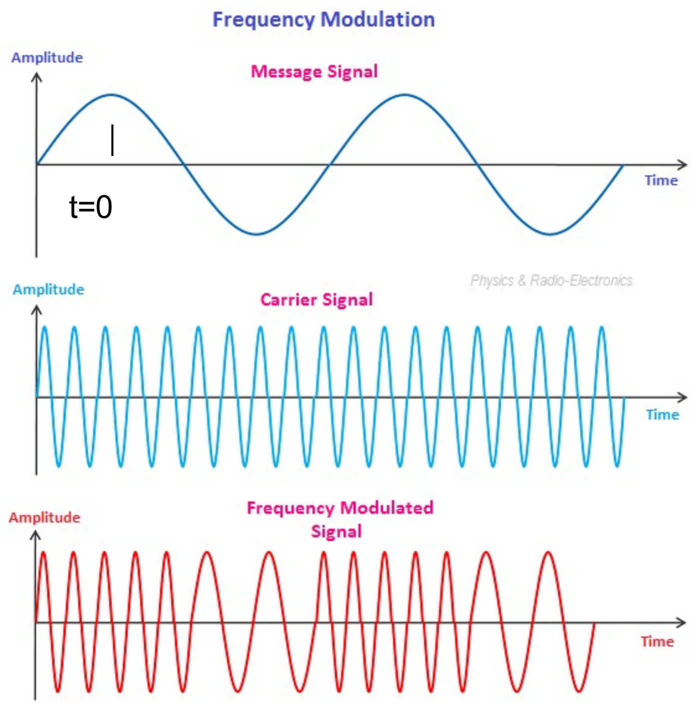
$\omega_i = \omega_c + k_\omega \cos(\omega_m t)$

$C(t) = A_0 \cos(\omega_c t + \beta \sin \omega_m t + \phi_0)$

$\beta = \frac{k_\omega}{\omega_m} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$: Modulation index

$\omega_m = 2\pi f_m$: Modulation frequency

$\Delta\omega = 2\pi\Delta f$: Max. frequency deviation, the frequency of the modulated carrier varies between $\omega = \omega_c - \Delta\omega$ and $\omega_c + \Delta\omega$



$m(t) = \cos(\omega_m t)$ Baseband signal

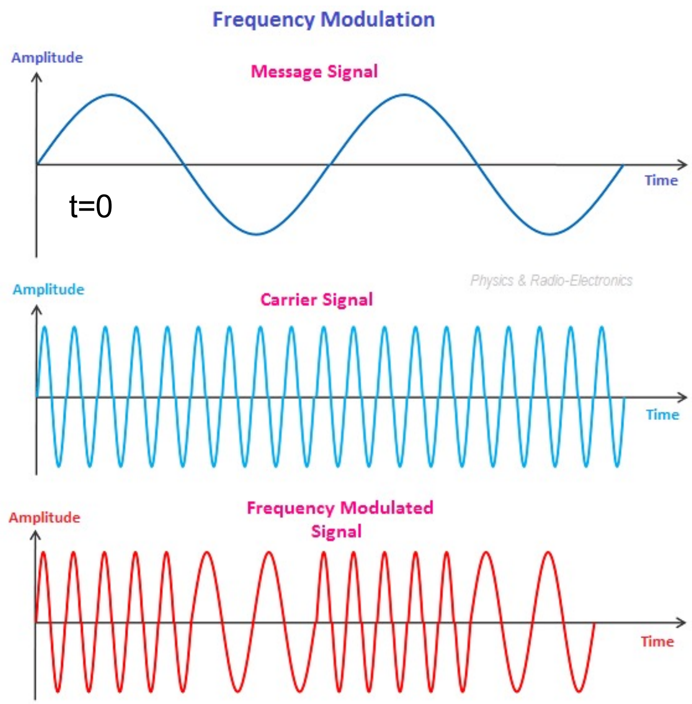
$A_0 \cos(\omega_c t + \phi_0)$ Carrier

Modulated carrier

$C(t) = A_0 \cos(\omega_c t + \beta \sin \omega_m t + \phi_0)$

How does the spectrum look?

What are the frequency components of $\cos[(\omega_c + \beta \sin \omega_m)t + \phi_0]$?



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How does the spectrum look?

What are the frequency components of $\cos(\omega_c t + \beta \sin \omega_m t + \phi_0)$?

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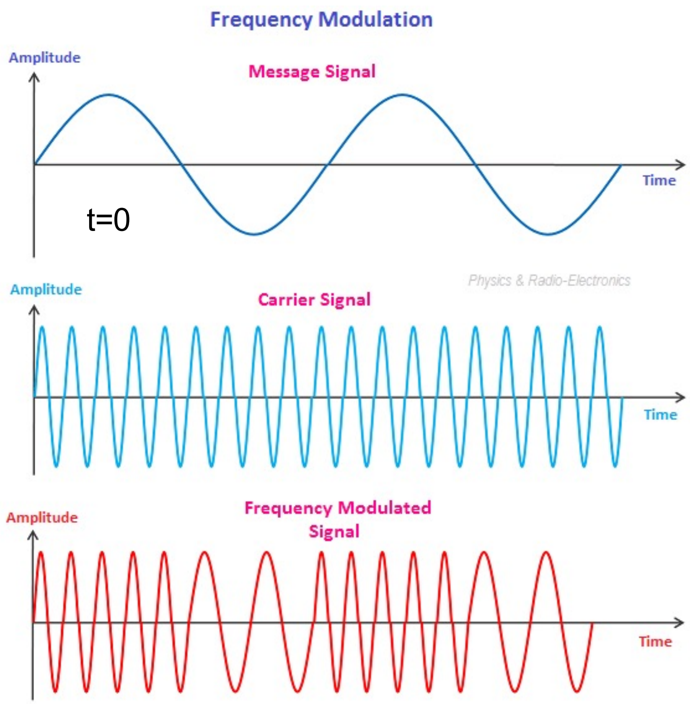


$m(t) = \cos(\omega_m t)$ Baseband signal

$A_0 \cos(\omega_c t + \phi_0)$ Carrier

Modulated carrier

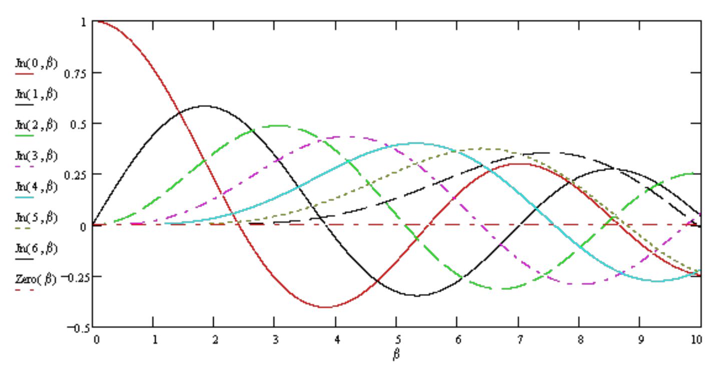
$C(t) = A_0 \cos(\omega_c t + \beta \sin \omega_m t + \phi_0)$

Spectrum can be represented by Bessel functions

$$C(t) = A_0 \sum_{n=-\infty}^{n=+\infty} J_n(\beta)(\omega_c t + n\omega_m t + \frac{n\pi}{2} + \psi)$$

$$J_n(\beta) = \sum_{m=1}^{\infty} \frac{(-1)^m \left(\frac{\beta}{2}\right)^{2m+n}}{m!(m+n)!}$$

$$J_{-n}(\beta) = (-1)^n J_n(\beta)$$



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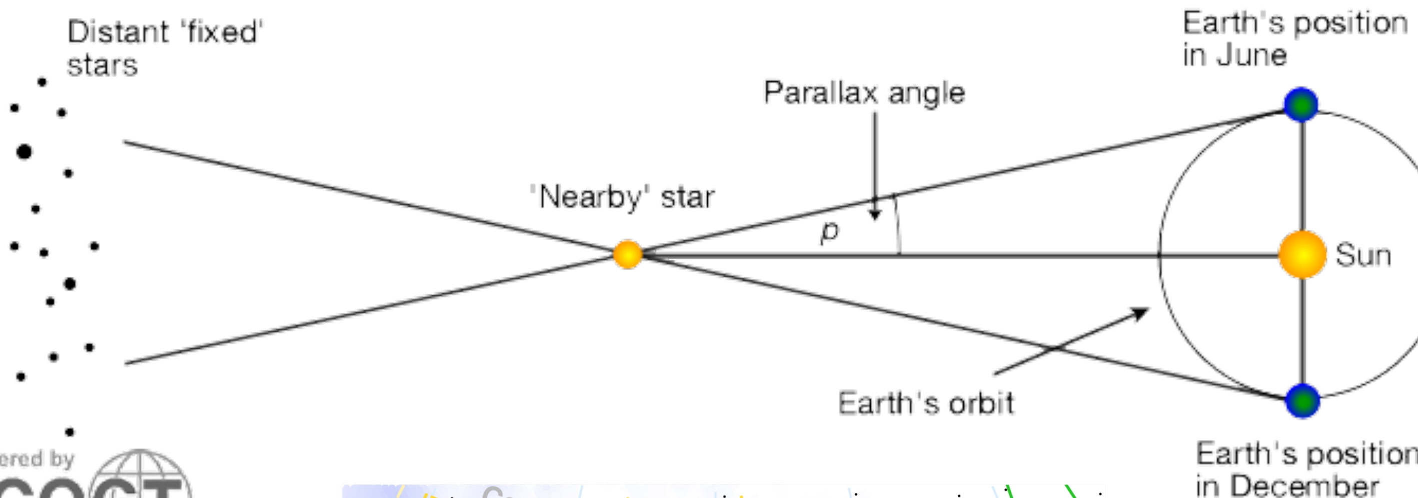
Bessel determined positions and proper motions of 50,000 stars.
He was the first to determine the parallax (distance) of a star

61 Cygni



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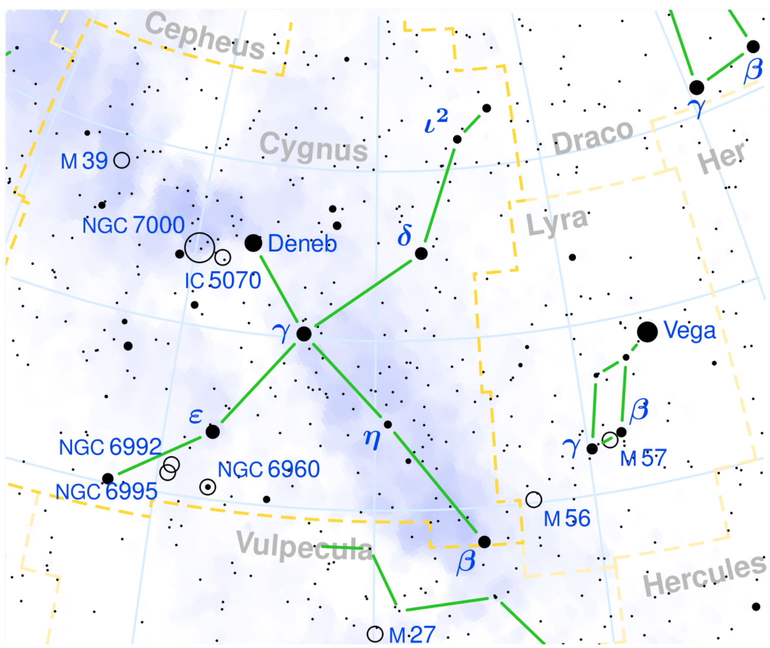

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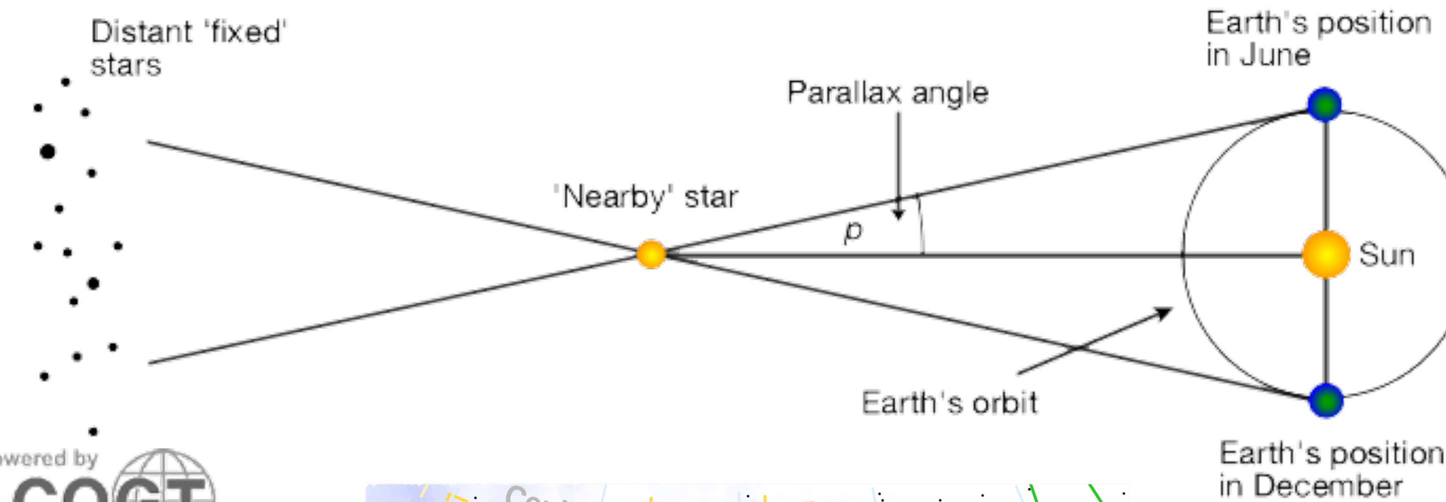

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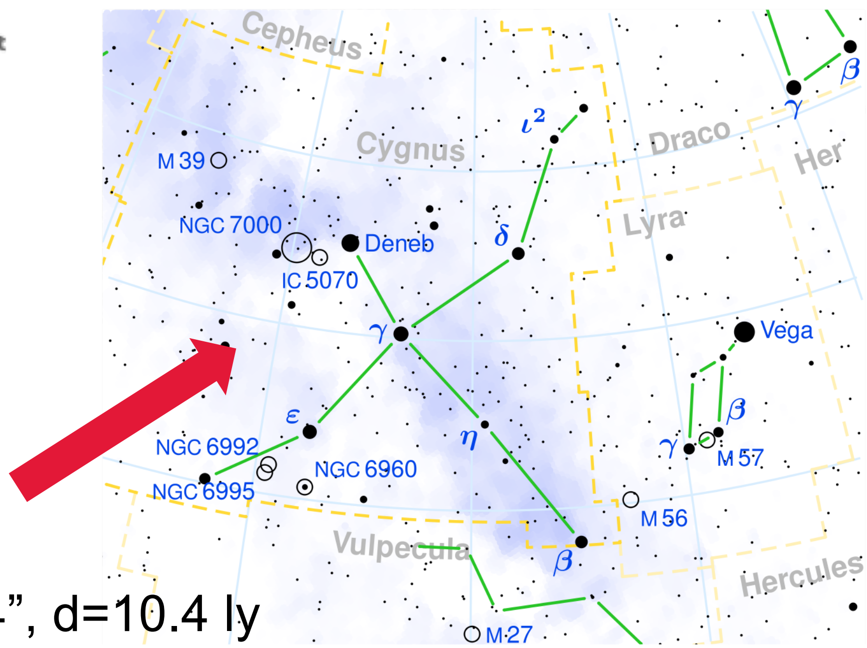

61 Cygni

Bessel : p= 0.314”, d=10.4 ly
Now: p=0.292”,d=11.4 ly

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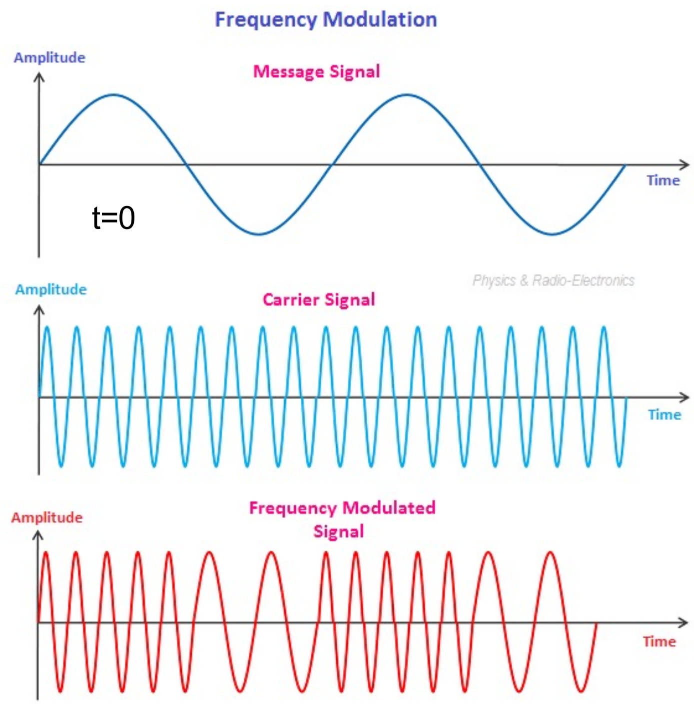


$m(t) = \cos(\omega_m t)$ Baseband signal

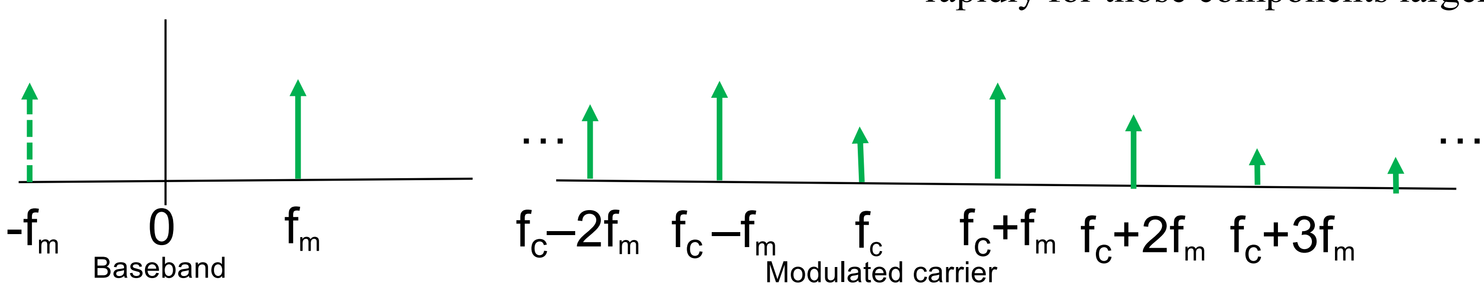
$A_0 \cos(\omega_c t + \phi_0)$ Carrier

Modulated carrier

$C(t) = A_0 \cos(\omega_c t + \beta \sin \omega_m t + \phi_0)$

FM spectrum

There is an infinite number of side frequencies. However, the amplitudes of the side spectral components decrease very rapidly for those components larger than β.



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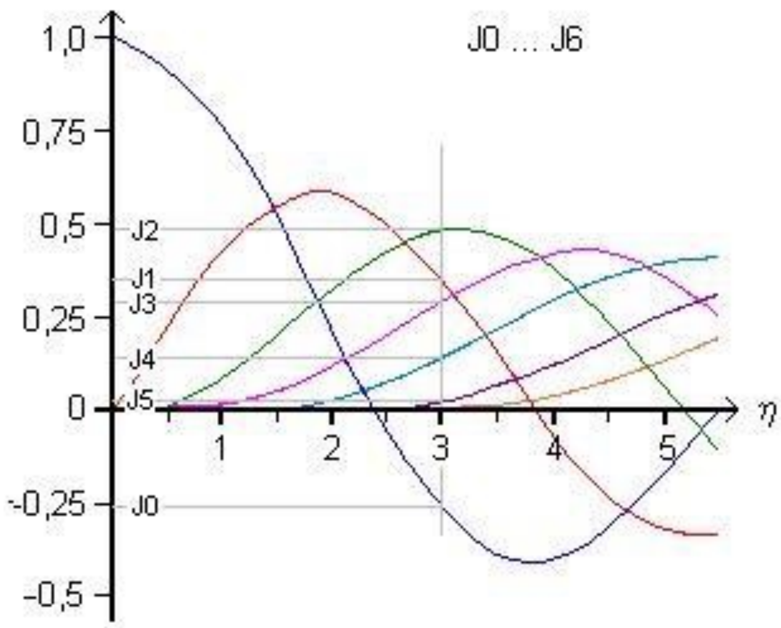
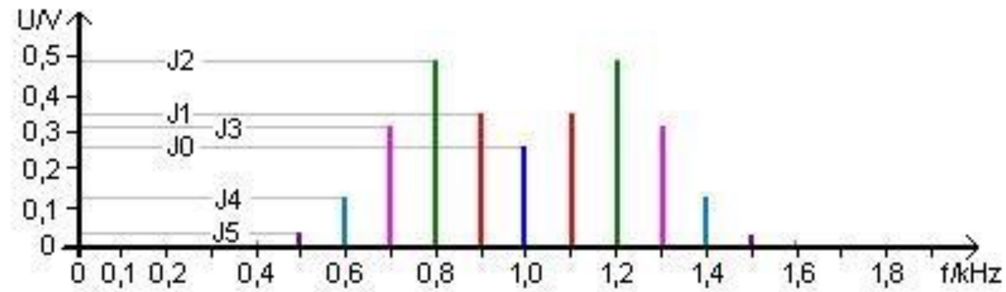
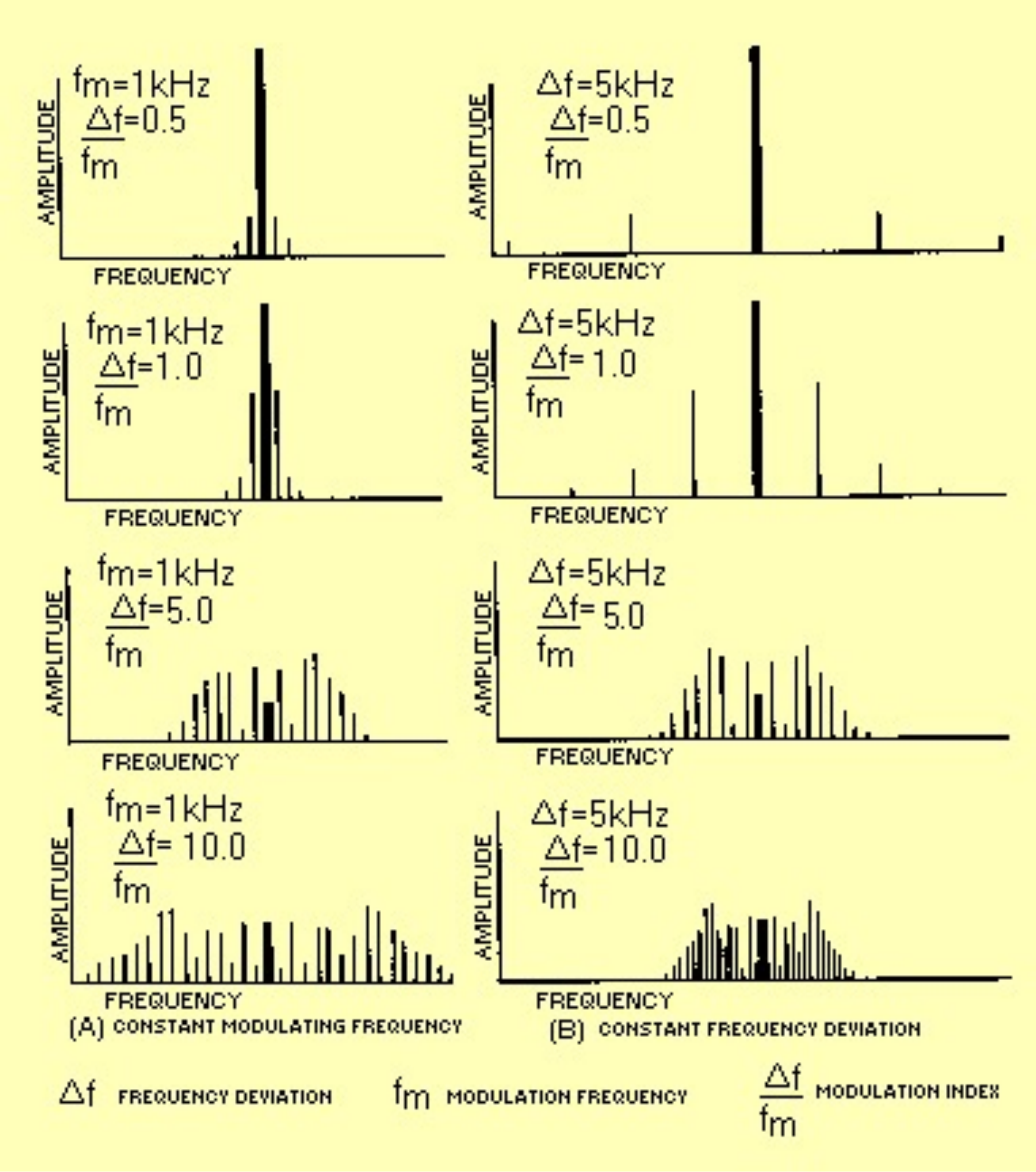
How to determine the spectrum from the Bessel functions



$\beta = 3$

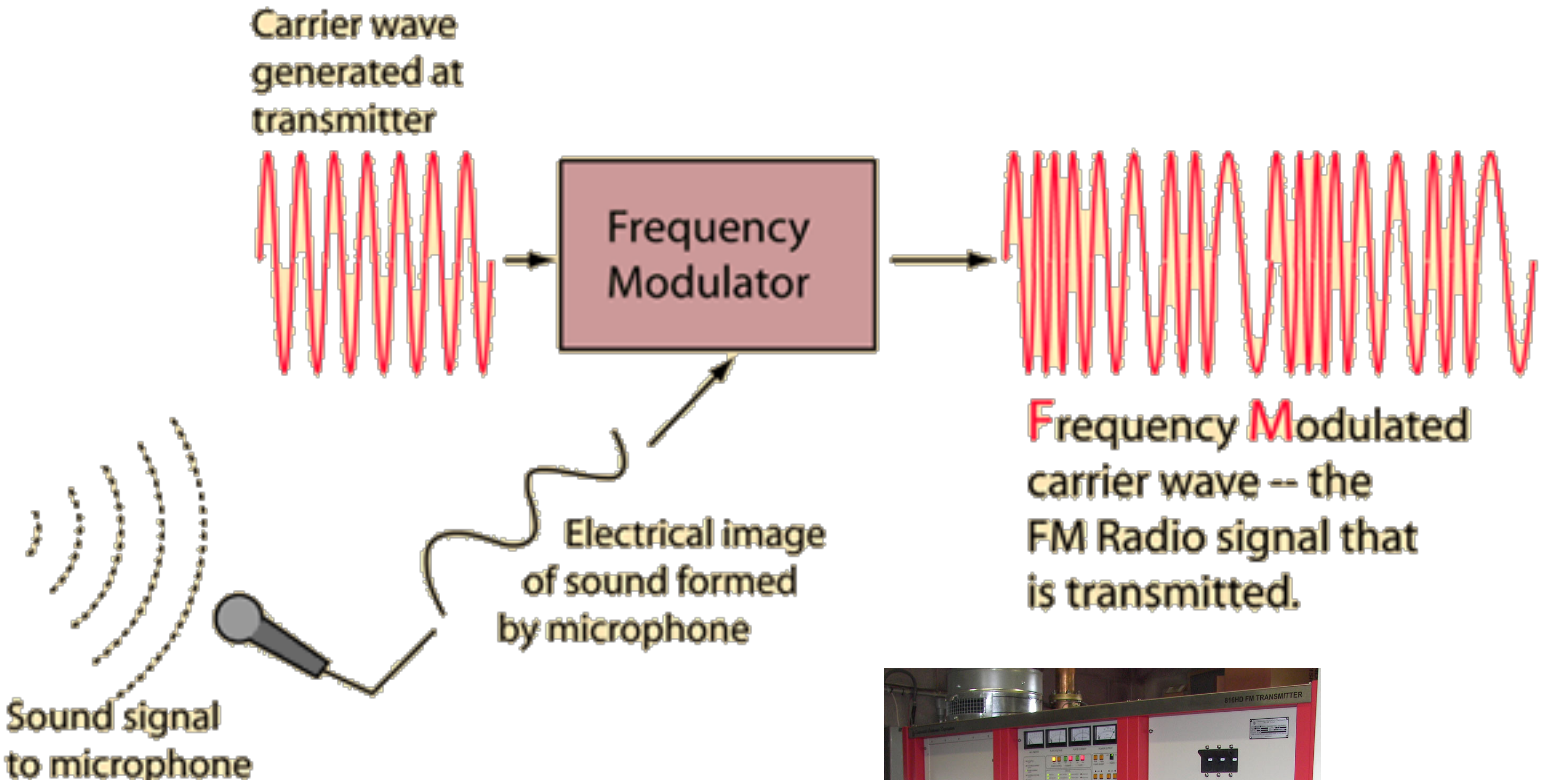


FM spectrum as a function of the modulation index



tpub.com

Sketch of FM with baseband signal



FM radio transmitter
in Buffalo, NY
Wikipedia

We do not need an infinitely wide filter bandwidth. Most of the information of a FM signal is in fact passed through a filter with a finite bandwidth, B, that depends on the modulation index β.

The bandwidth, B, is essentially the range in frequencies from
$f_c - (\beta + 1) f_m$ to $f_c + (\beta + 1) f_m$.

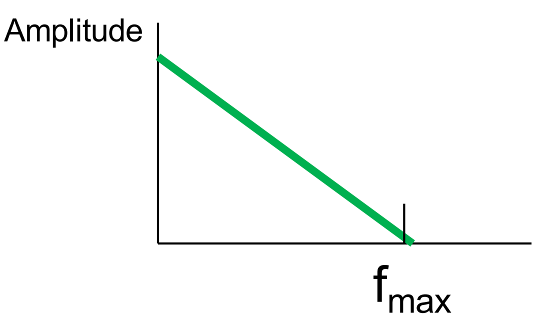
Carson's rule:

$$B = 2 (\beta + 1) f_m$$
$$B = 2 (\Delta f + f_m)$$

If m is not just a cosine wave but a baseband signal with a continuous range of frequencies, then we plot the spectrum as a generic spectrum as shown below, then:

$$\beta = \Delta F / f_{max}$$
$$B = 2 (\Delta F + f_{max})$$

$\beta = \Delta F / f_{max}$: modulation index for highest frequency in baseband spectrum. It is also sometimes called the deviation ratio, D.



ΔF is similar to Δf that is it is the frequency range over which the carrier frequency is modulated.

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Example 6-1

A 100 MHz carrier is frequency modulated by a 1 kHz tone which produces frequency deviations of up to 75 kHz.

$\beta = \Delta f / f_m$
$= 75/1 = 75$
$B = 2(\Delta f + f_m)$
$= 2(75 + 1)$
$= 152$ kHz

Example 6-2

A video signal of bandwidth 4.2 MHz is used to frequency- modulate a carrier with $\Delta F = 10.75$ MHz.

$D = \beta = \Delta F / f_{max}$
$= 10.75/4.2 = 2.56$
$B = 2(\Delta F + f_{max})$
$= 2(10.75 + 4.2)$
$= 29.9$ MHz

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Signal-to-noise ratio

For FM, the S/N is not equal to C/N.
There are three factors that cause an improvement of S/N over C/N.

1. The *processing gain K_R* of the detector as a function of β.
   $S/N = K_R \bullet C/N$
   $K_R = 3(\beta + 1)\beta^2$ (This is valid only if $[C/N] \geq 10$ dB)

Example 6-3

For a video signal with $\beta = 2.56 \Rightarrow K_R = 70$ or 18.5 dB

2. *Pre-emphasis and de-emphasis*: If the noise at the output of an FM demodulator where conversion back to baseband takes place, increases with increasing frequency, the signal spectrum can be adjusted to the noise spectrum (pre-emphasis) at the transmitter and then re-adjusted at the receiver (de-emphasis). The result is a S/N independent of frequency.

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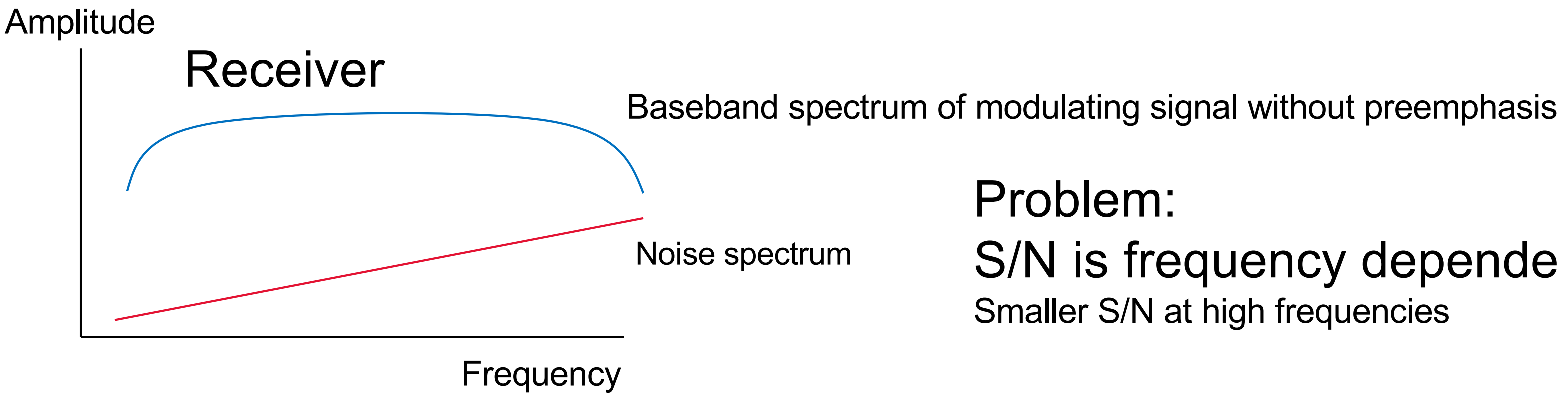
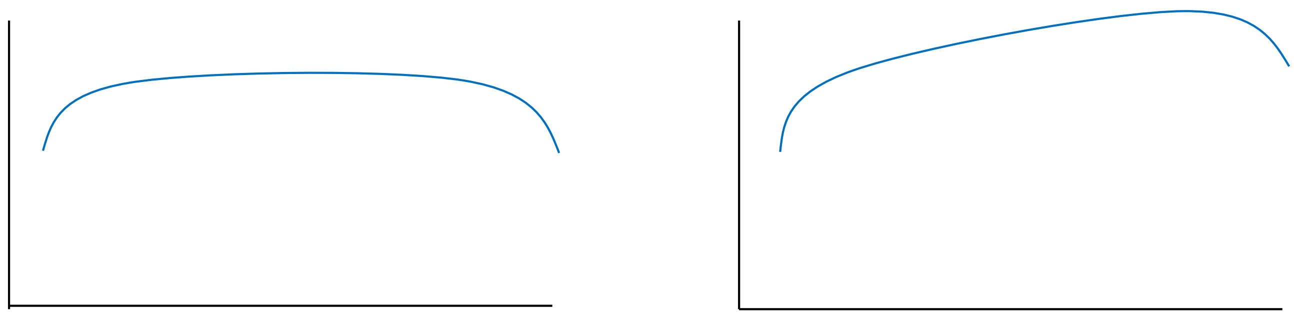


Problem:

S/N is frequency dependent:

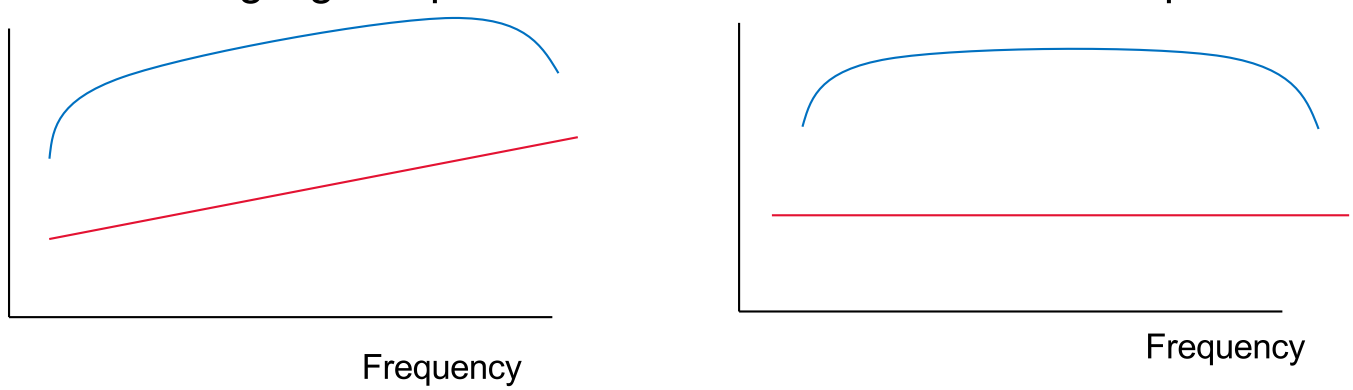
Smaller S/N at high frequencies

Modulating signal spectrum before …. and after pre-emphasis



Transmitter

Modulating signal spectrum before …. and after de-emphasis



Receiver

Pre-emphasis and de-emphasis can increase the S/N by an emphasis improvement factor, P with

[P]$\leq 4\ dB$ for telephony
[P] $\leq 13\ dB$ for TV

3. *Noise weighting:* By modifying the noise spectrum and adjusting it to the response of the output device and/or the response to the human ear, S/N can be further improved by [W]:

[W]$\leq 2.5\ dB$ for telephony
[W] $\leq 12\ dB$ for TV

The S/N ratio is then given by:

$$S/N = C/N \cdot K_R \cdot P \cdot W$$

$$[S/N] = [C/N] + [3\,(\beta + 1)\,\beta^2\,] + [P] + [W]$$

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Example 6-4

Example for S/N of a typical FDM system:

$[C/N] = 25$ dB
$[P] = 4$ dB
$[W] = 2.5$ dB
$\Delta F = 281$ kHz
$f_{max} = 108$ kHz

$\beta = \Delta F / f_{max} = 2.60$
$3(\beta + 1)\beta^2 = 73.0$

$$[S/N] = [C/N] + [3(\beta + 1)\beta^2] + [P] + [W]$$
$$= 25 + 18.63 + 4 + 2.5$$
$$= 50.13 \text{ dB}$$
$$S/N = 103{,}000$$

Peripheral information only

For worst channel (WC) the S/N is computed this way:

$\Delta F_{rms} = 35$ kHz. (rms of frequency deviations)
$b = 3.1$ kHz. (bandwidth of individual channel)
$B_{IF} = 778$ kHz. (intermediate frequency bandwidth from Carson's rule, $B_{IF} = B = 2(\Delta F + f_{max})$)

$$[S/N]_{WC} = [C/N] + [(B_{IF}/b)(\Delta F_{rms}/f_{max})^2] + [P] + [W]$$
$$= 25 + 14.21 + 4 + 2.5$$
$$= 45.71 \text{ dB}$$

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6.2.3 Phase modulation (PM)

Modulated carrier

$$C(t) = A_0 \cos(\omega_c\, t + \phi(t))$$
$$= A_0 \cos(\omega_c t + k_p\, m(t) + \phi_0)$$

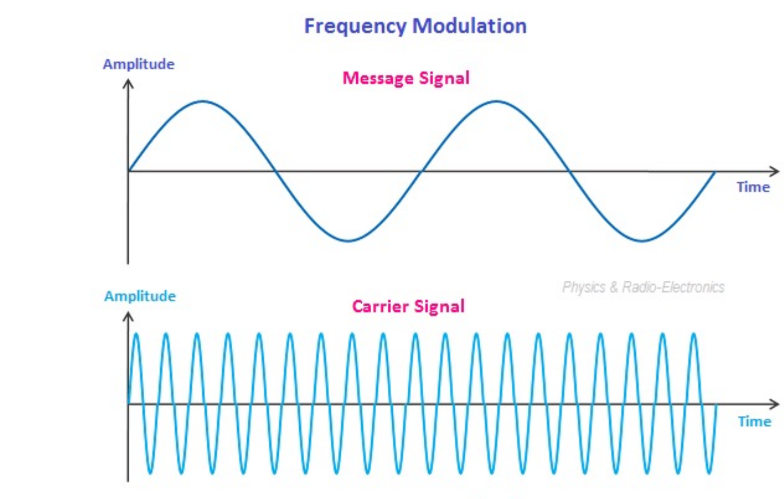
ω_c: carrier angular frequency

$\theta(t) = \omega_c t + k_p\, m(t)$: phase,

k_p : frequency modulator=constant

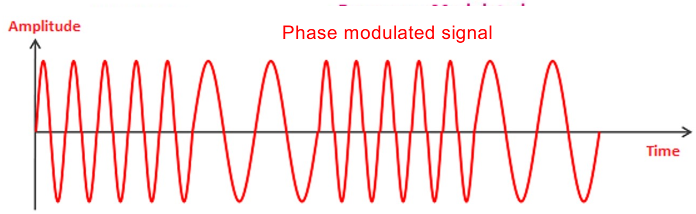
$\omega_c + k_p \frac{d}{dt} m(t) = \omega_i(t)$: instantaneous frequency,

Graphical example for a special case: $m(t) = \cos \omega_m t$



$m(t) = \cos(\omega_m t)$ Baseband signal

$A_0 \cos(\omega_c t + \phi_0)$ Carrier



Modulated carrier

$C(t) = A_0 \cos(\omega_c t + k_p\, m(t) + \phi_0)$

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The spectrum is very similar to that of FM

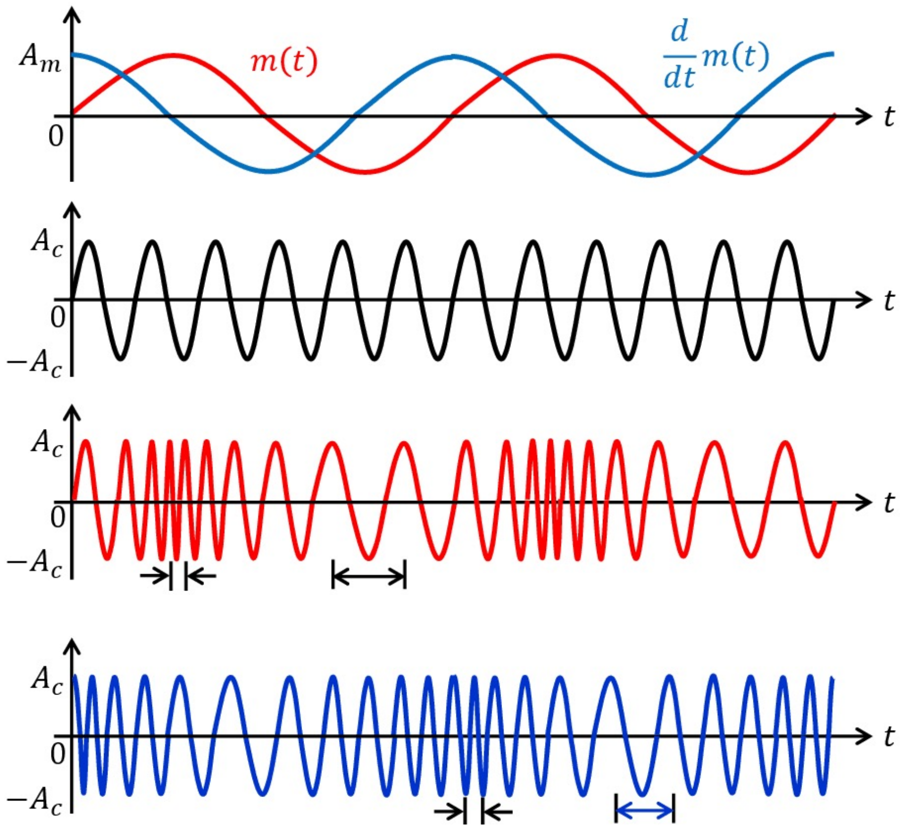
FM and PM comparison

	Frequency modulation	Phase modulation
Phase	$\theta(t) = \omega_c t + k_\omega \int_0^t m(\tau)d\tau$	$\theta(t) = \omega_c t + k_p m(t)$
Frequency	$\omega_i(t) = \omega_c + k_\omega m(t)$	$\omega_i(t) = \omega_c + k_p \frac{d}{dt} m(t)$

$\omega_i(t) = \frac{d}{dt}\theta(t)$: Instantaneous frequency is the derivative of phase

Frequency modulation: m(t) drives the frequency variation
Phase modulation: m(t) drives the phase variation

Again FM and PM **Modulation** with focus on m(t)



Message signal and it's derivative

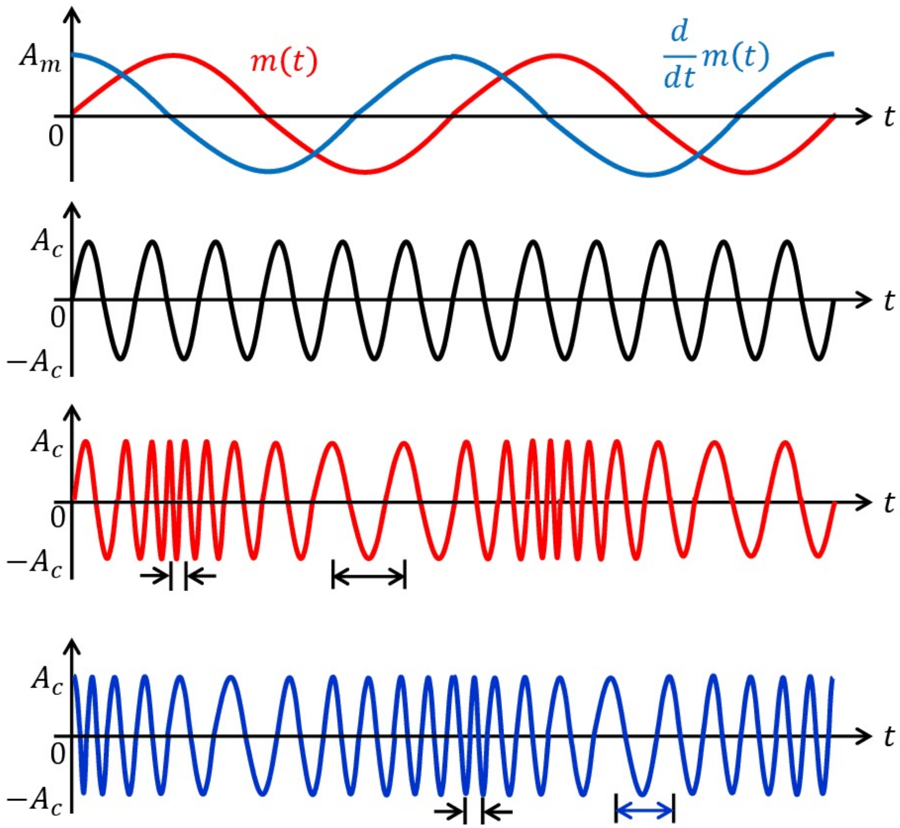
Unmodulated carrier wave

Max. and min. freq. variation of the carrier correspond to $m(t)|_{max}$ and $m(t)|_{min}$.

Max. and min. freq. variation of the carrier correspond to $\frac{d}{dt}m(t)\Big|_{max}$ and $\frac{d}{dt}m(t)\Big|_{min}$.

What is FM and what is PM?

Again FM and PM **Modulation** with focus on m(t)



Message signal and it's derivative

Unmodulated carrier wave

FM

Max. and min. freq. variation of the carrier correspond to $m(t)|_{max}$ and $m(t)|_{min}$.

PM

Max. and min. freq. variation of the carrier correspond to $\frac{d}{dt}m(t)\Big|_{max}$ and $\frac{d}{dt}m(t)\Big|_{min}$.

What is FM and what is PM?

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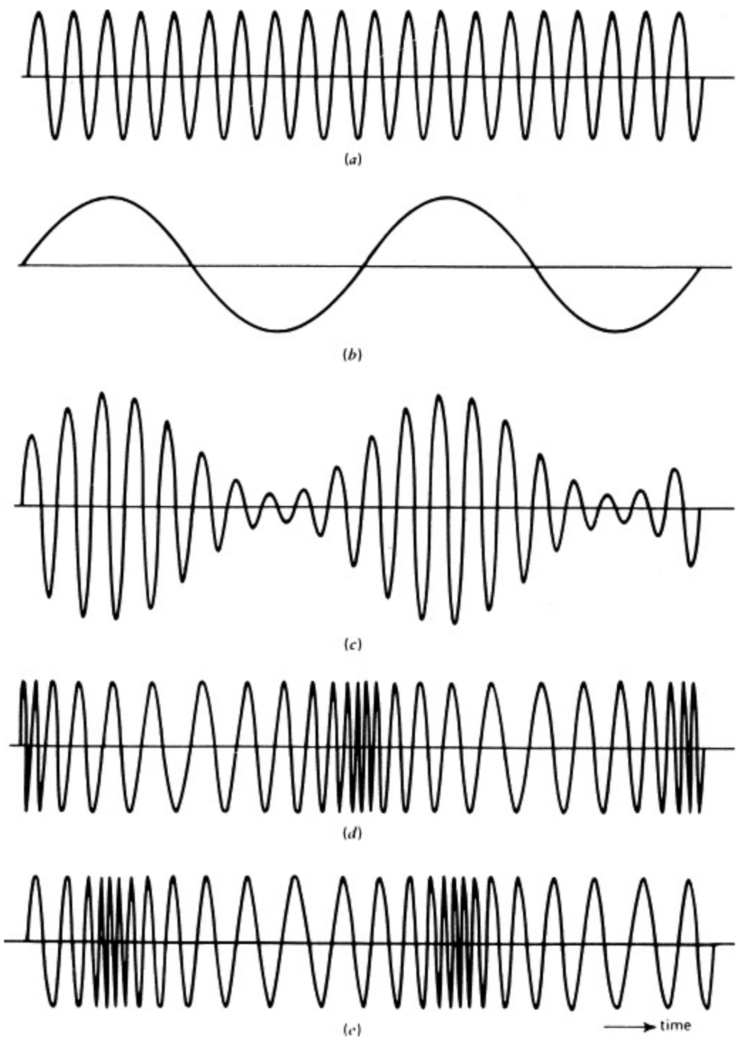
Graphical differences between AM, PM and FM carriers



Carrier

Baseband signal

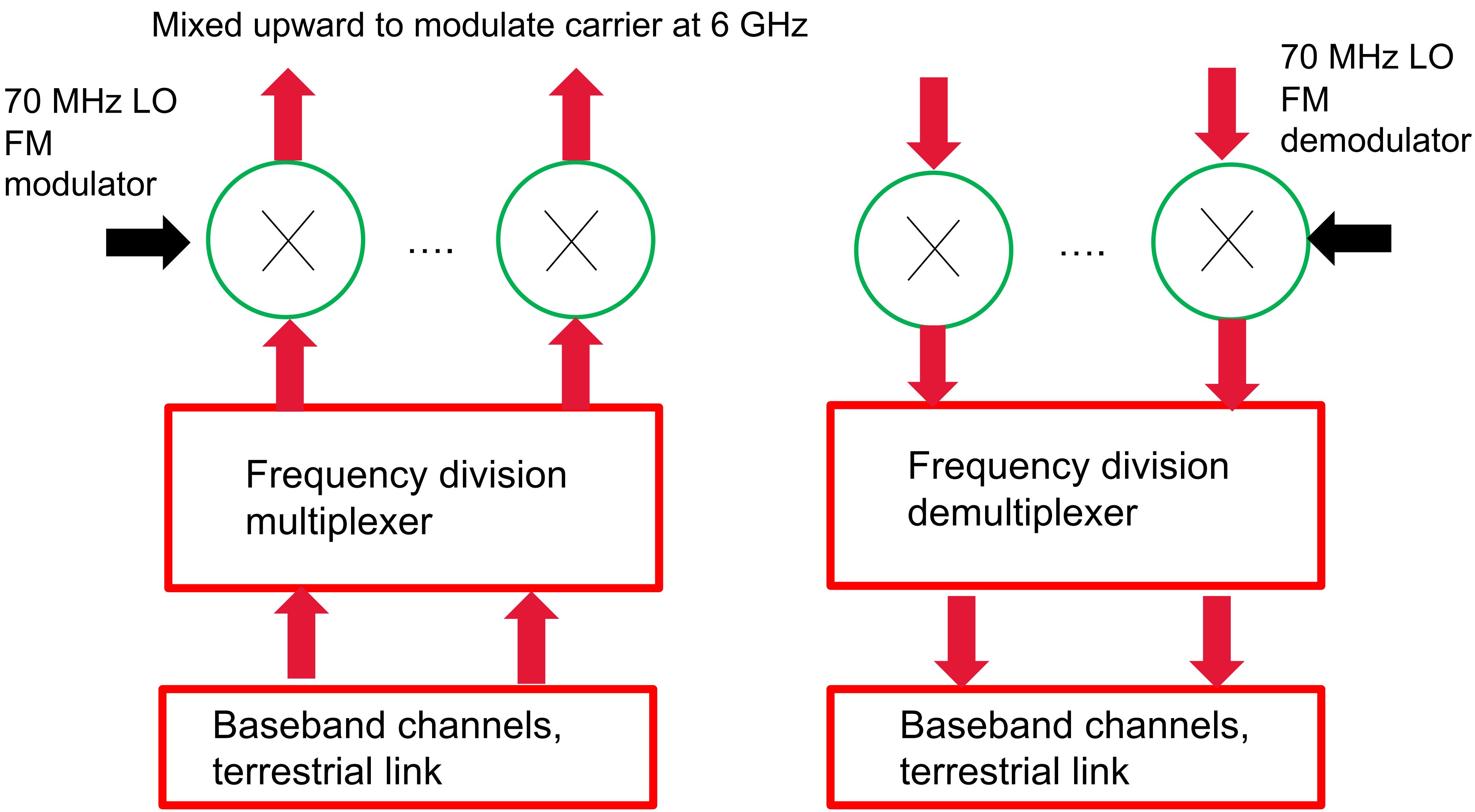
Amplitude modulated carrier

Phase modulated carrier

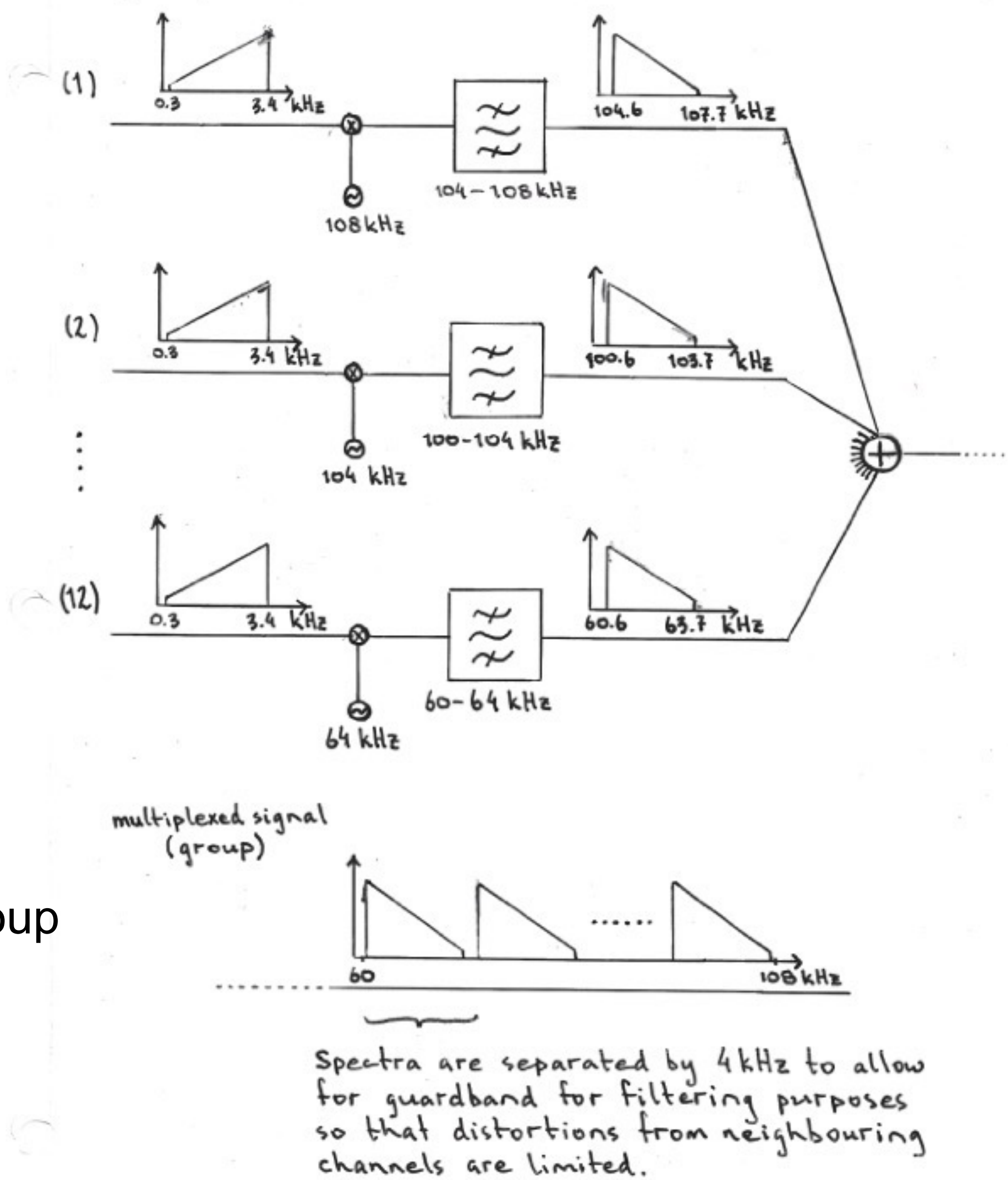
Frequency modulated carrier

	FM	AM
1	S/N >C/N	S/N~C/N
2	S/N can be improved through: 1. processing gain 2. Pre-emphasis and de-emphasis 3. Noise weighting	No improvements possible
3	Bandwidth is in theory ∞, in practise depends on modulation index and is given through Carson's rule	Bandwidth is given by 2 x f_c-f_m and is independent of modulation index
4	FM transmitters and receivers are relatively complex	AM transmitters and receivers are relatively simple
5	FM is power efficient	AM is not that power efficient

6.3 Frequency division multiplexing (FDM)

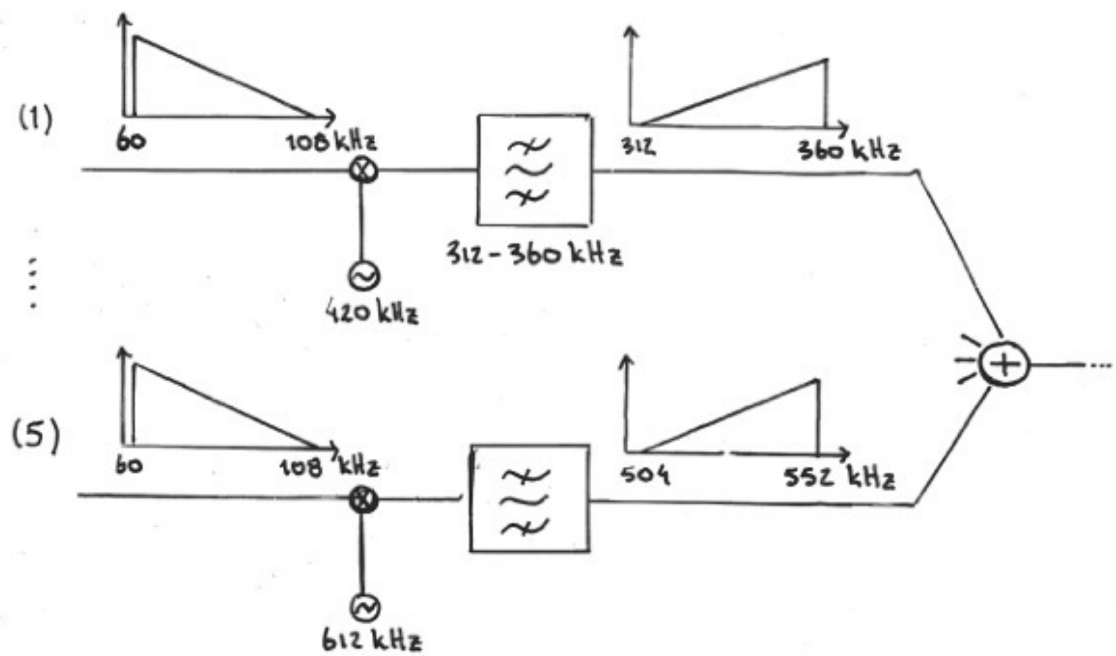


12 baseband
(voice) channels

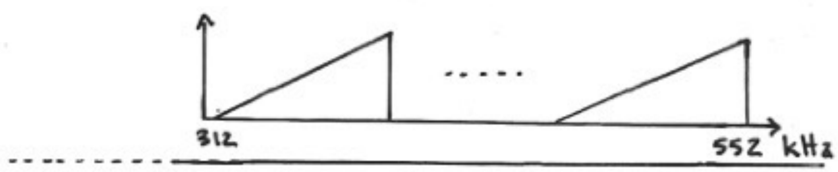


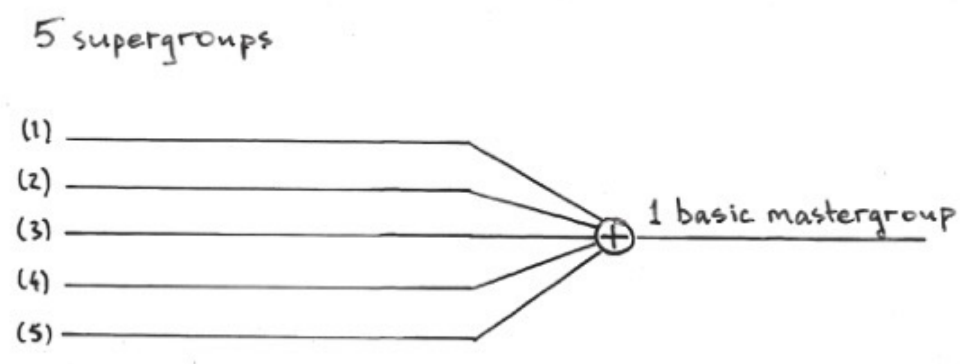
1 group

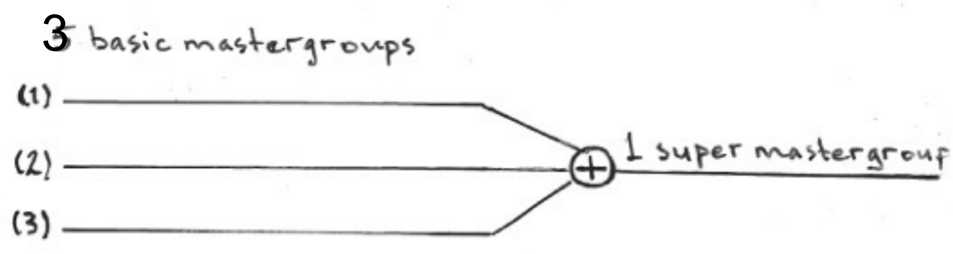
5 groups



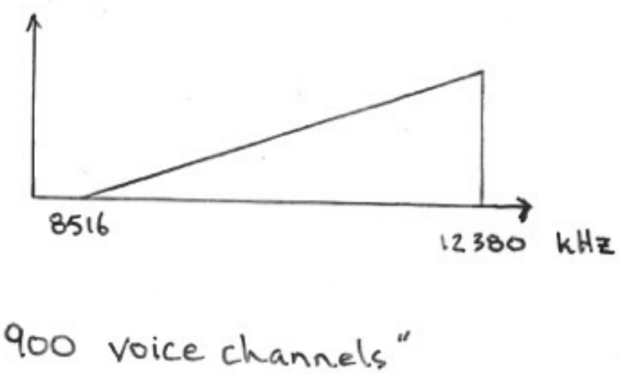
1 supergroup







1 super mastergroup



Hirarchical structure of FDM according to CCITT

(**CCITT** Comité Consultatif Internationale de Télégraphique et Téléphonique (International Telegraph and Telephone Consultative Committee) is an agency of the International Telecommunications Union (ITU). The ITU is an agency of the UN. The CCITT is an agency coordinating telephone and data communications systems on a worldwide basis and dealing with regulatory matters and with technical standards.)

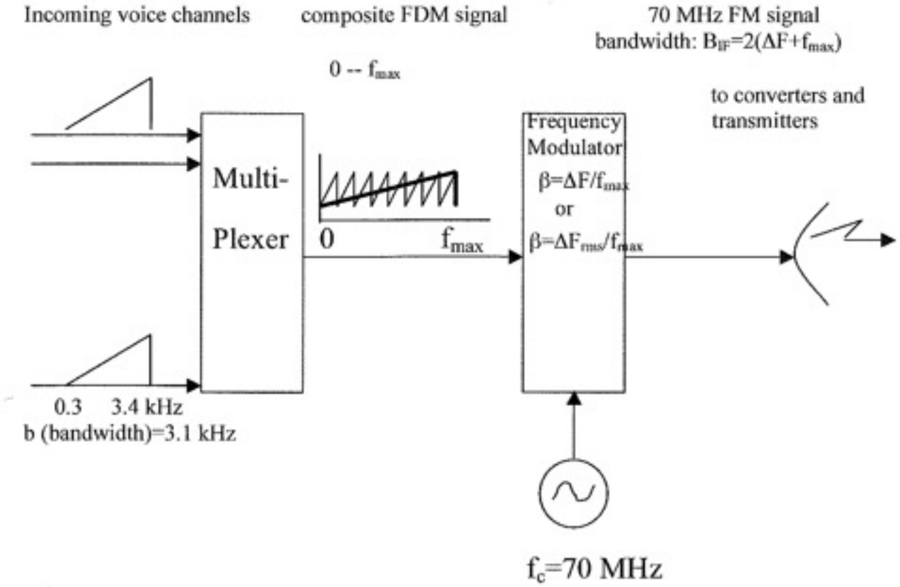
12 (voice) cahnnels → 1 group
5 groups → 1 supergroup
5 supergroups → 1 basic mastergroup
3 mastergroups → 1 super mastergroup

The super mastergroup is the highest baseband unit.

----

900 voice channels are frequency division multiplexed. However, lots of other schemes are in use.

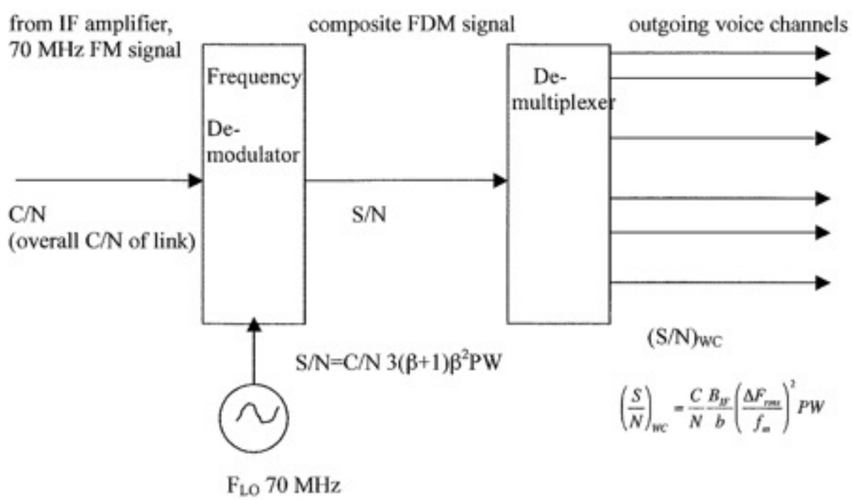
Transmitting and receiving ends of a typical FDM system



ΔF_{rms} : rms frequency deviation per channel

ΔF : peak frequency deviation w.r.t. all channels

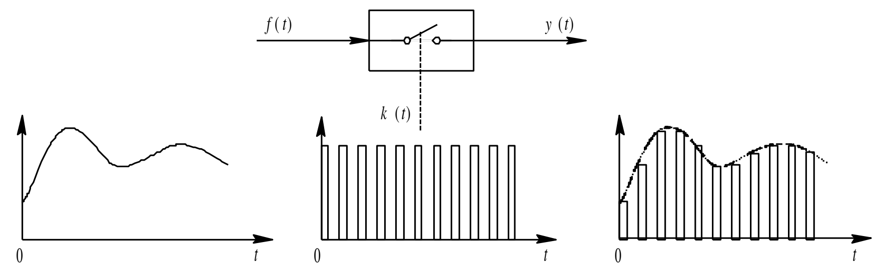
f_m : center frequency of individual channel



6.4 Digital baseband signal

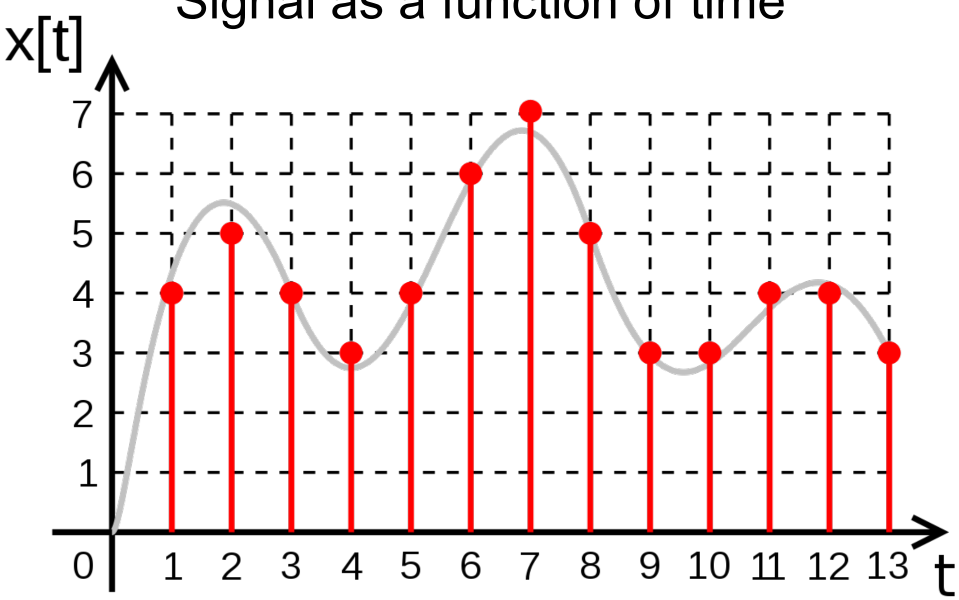
A/D conversion:
analogue signal converted into a digital signal

Sampler

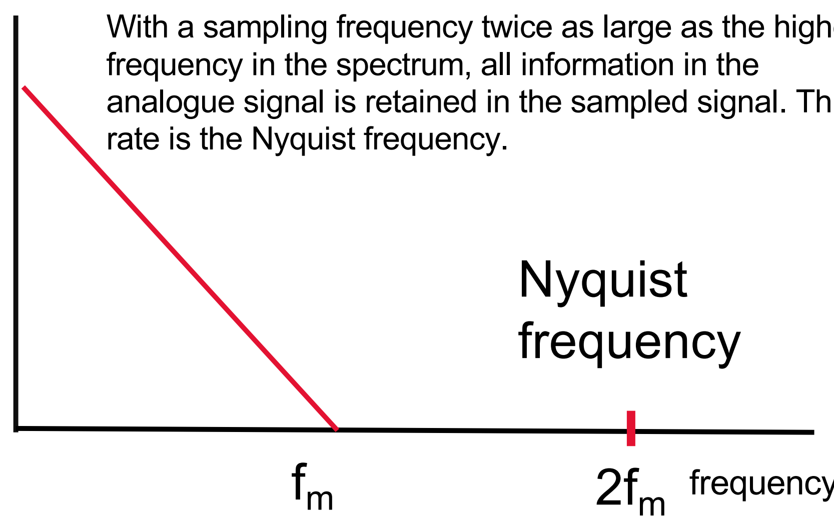


A digital signal represents data as a sequence of discrete values.

Signal as a function of time



Spectrum (generalized)



Example 6-3

Decimal# 0,1,2,3,4,5,6,7,8,9,10
Binary# 000,001,010,011,100,101,110,111,1000,1001 1010

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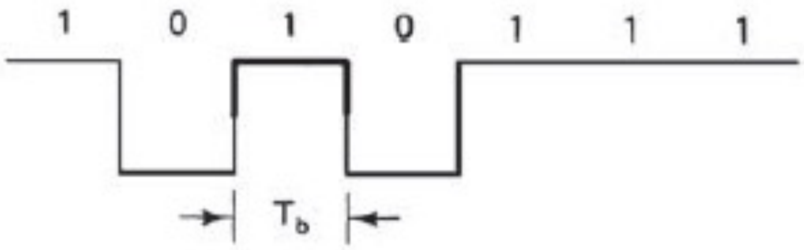
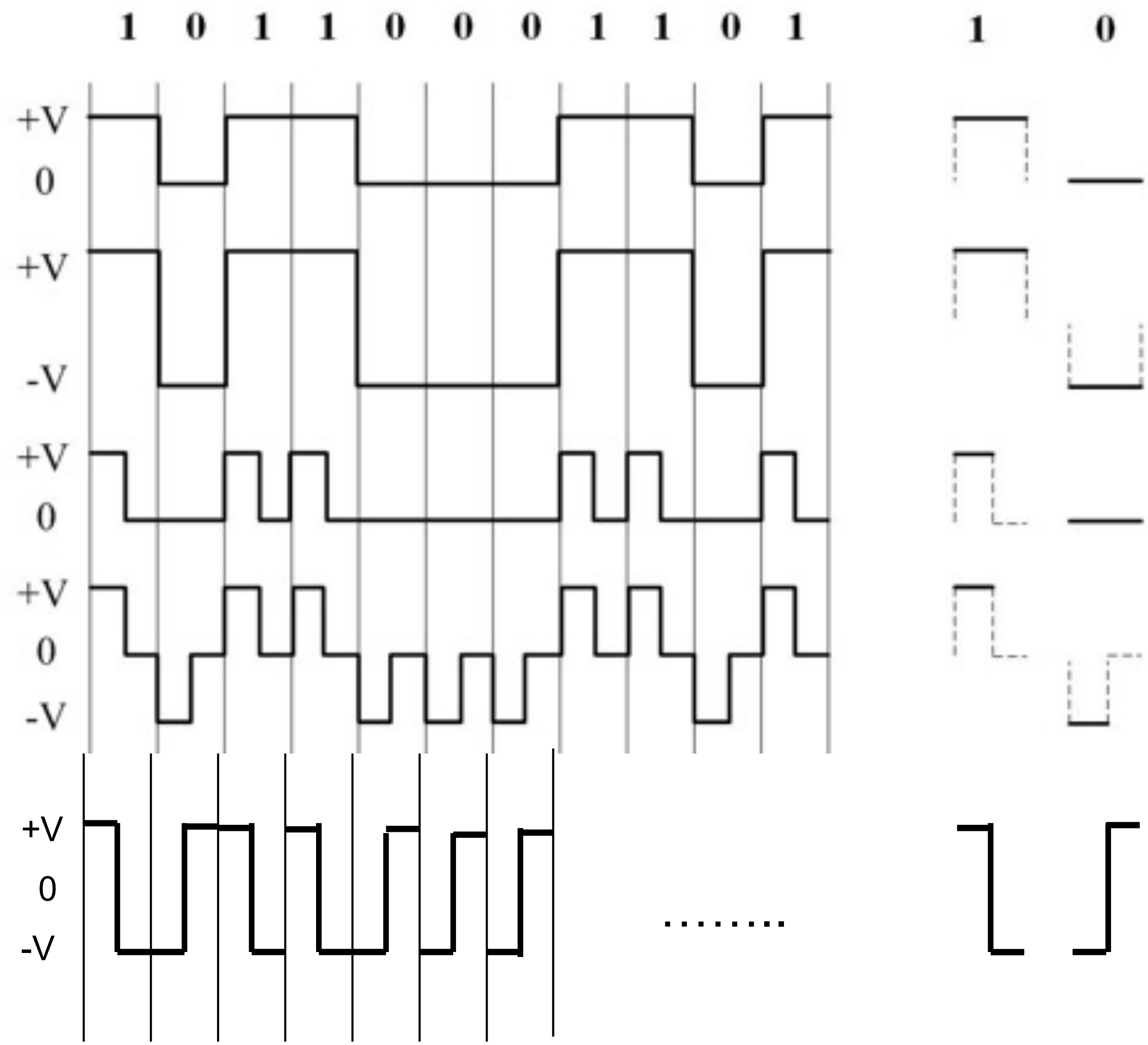
The *digital* information is transmitted as a waveform. The most fundamental such waveform is a string of 1's and 0's



A unit of information is a *bit*. The duration of a bit is the *bit period* , T_b given in seconds or s. The inverse of the bit period is the bit rate, R_b given in bits per second of b/s.

$$R_b = \frac{1}{T_b}$$

Five different forms of coding



Coding	Disadvantages
Unipolar NRZ	DC component Synchronisation problem for long sequences of 1's , 0's
Bipolar NRZ	DC component if 1's and 0's are unbalanced Synchronisation problem for long sequences of 1's , 0's
Unipolar RZ	DC component, pulse width=1/2 T_b Synchronisation problem for long sequences of 1's , 0's
Bipolar RZ	DC component if 1's and 0's are unbalanced pulse width=1/2 T_b
Manchester encoding	pulse width=1/2 T_b

NRZ: non return to zero during T_b
RZ: return to zero

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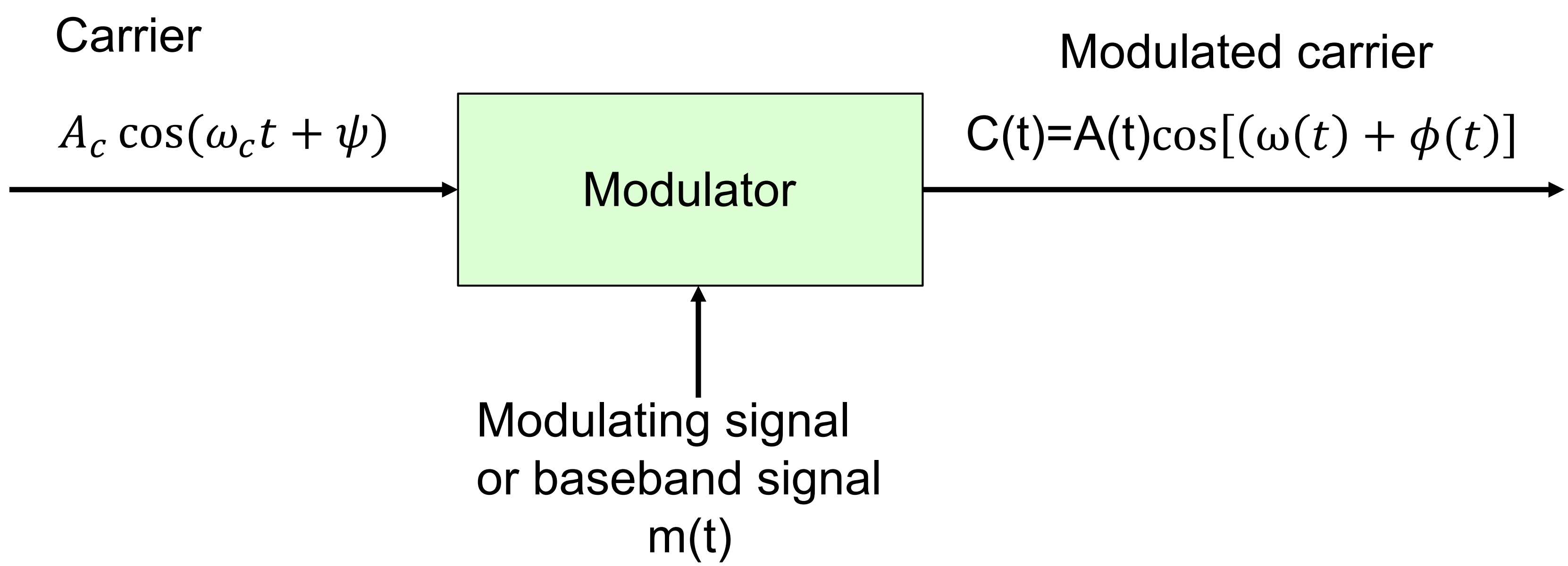
6.5 Digital modulation



Contains message to be transmitted

In contrast to analogue modulation, the time dependent variable assumes generally only 2 or 4 (QPSK) different values

ASK: Message is carried in A(t): C(t)=A(t) $\cos[\omega_c t + \phi_0]$

Amplitude shift keying or on-off keying (OOK)

FSK: Message is carried in $\omega(t)$: C(t)=$A_c \cos[\omega(t) + \phi_0]$

Frequency shift keying

PSK: Message is carried in ϕ(t): C(t)=$A_c \cos[\omega_c t + \phi(t)]$

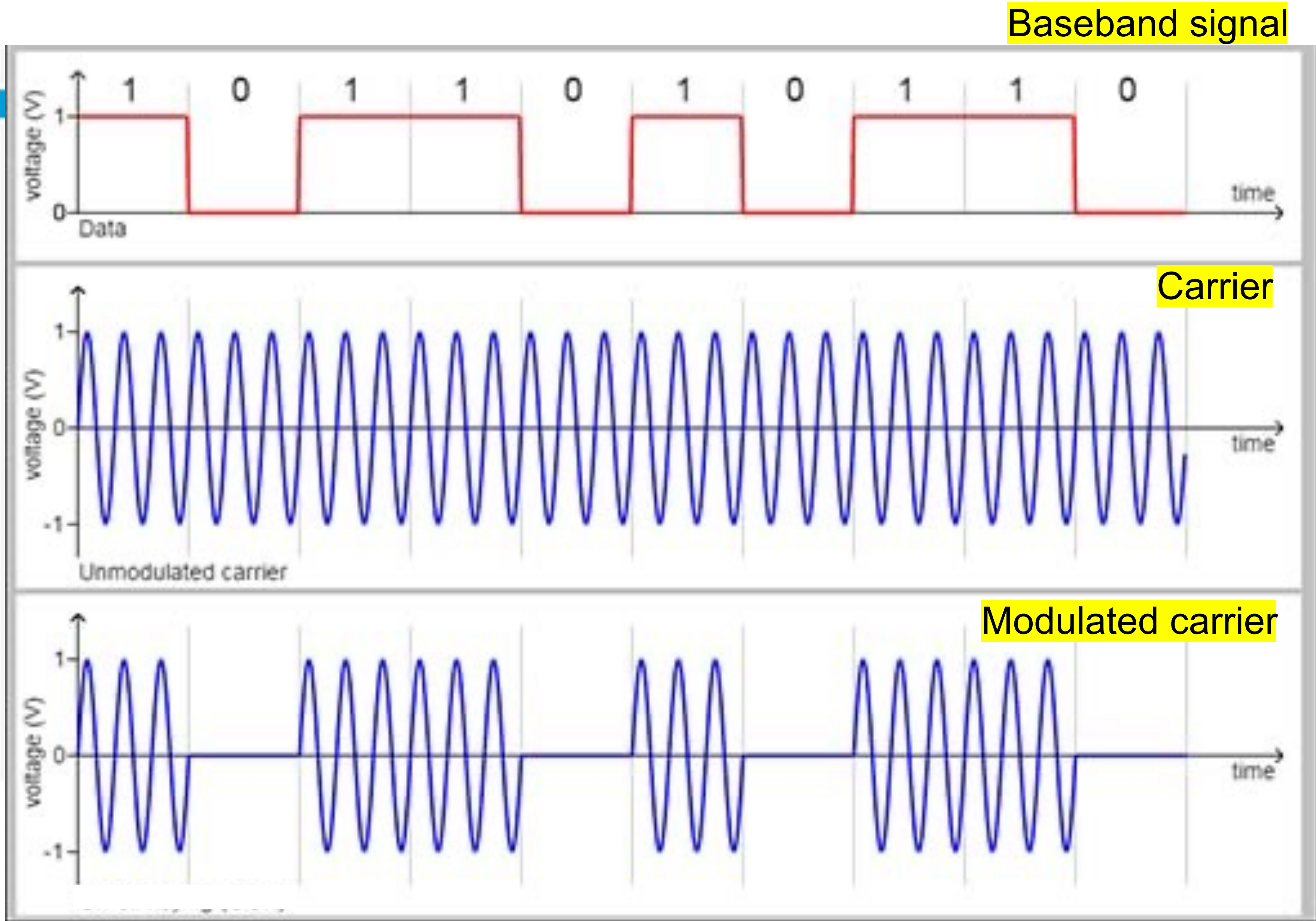
Phase shift keying

BPSK:Binary phase shift keying
QPSK: Quadrature phase shift keying

6.5.1 Amplitude shift keying (AM) or on-off keying (OOK)

0 C(t)= 0

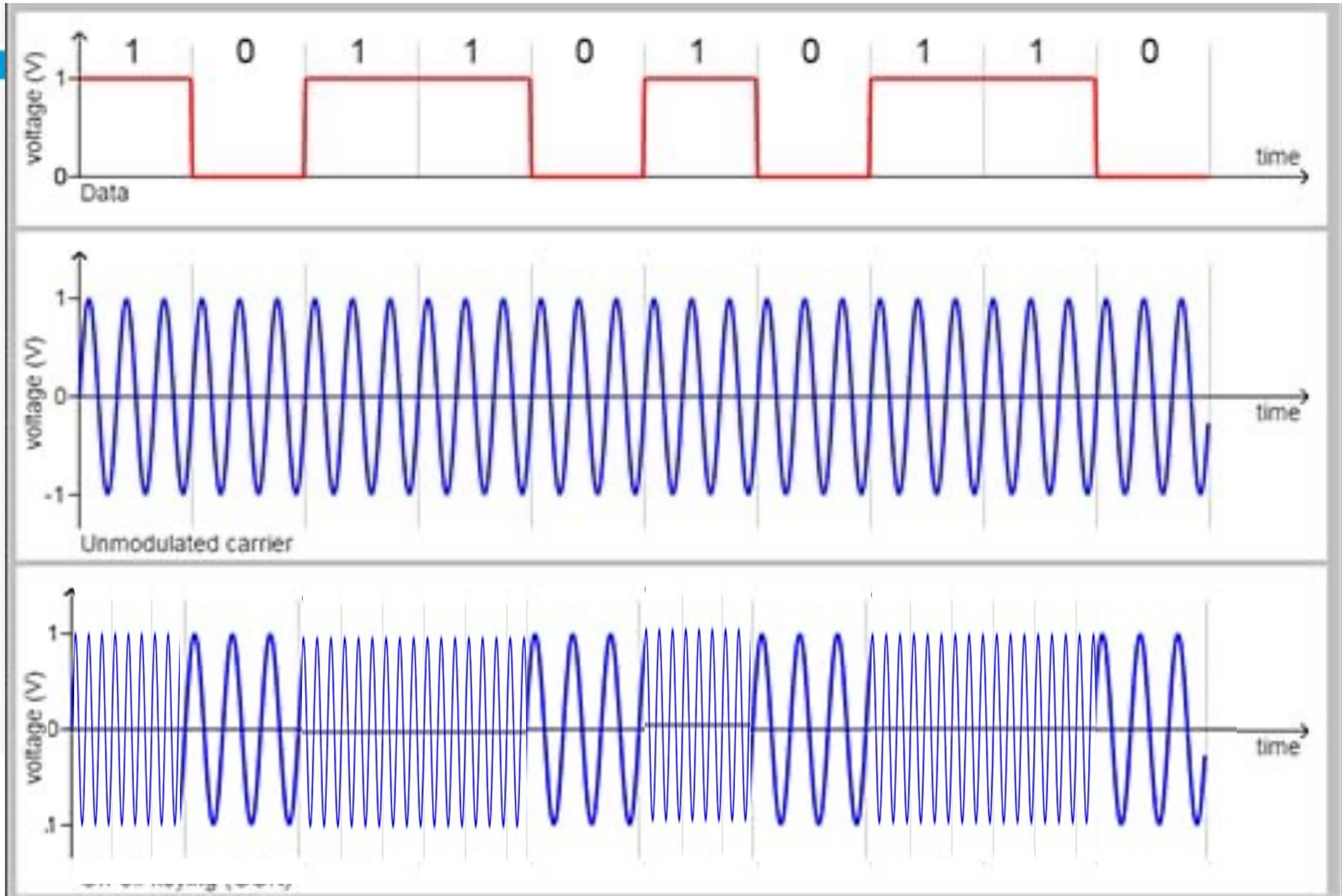
1 C(t)=$A_c \cos[\omega_c t + \phi]$



6.5.2 Frequency shift keying (FSK)

0 $C(t)=A_c \cos[\omega_c t + \phi]$

1 $C(t)=A_c \cos[(\omega_c+\Delta\omega)t + \varphi]$

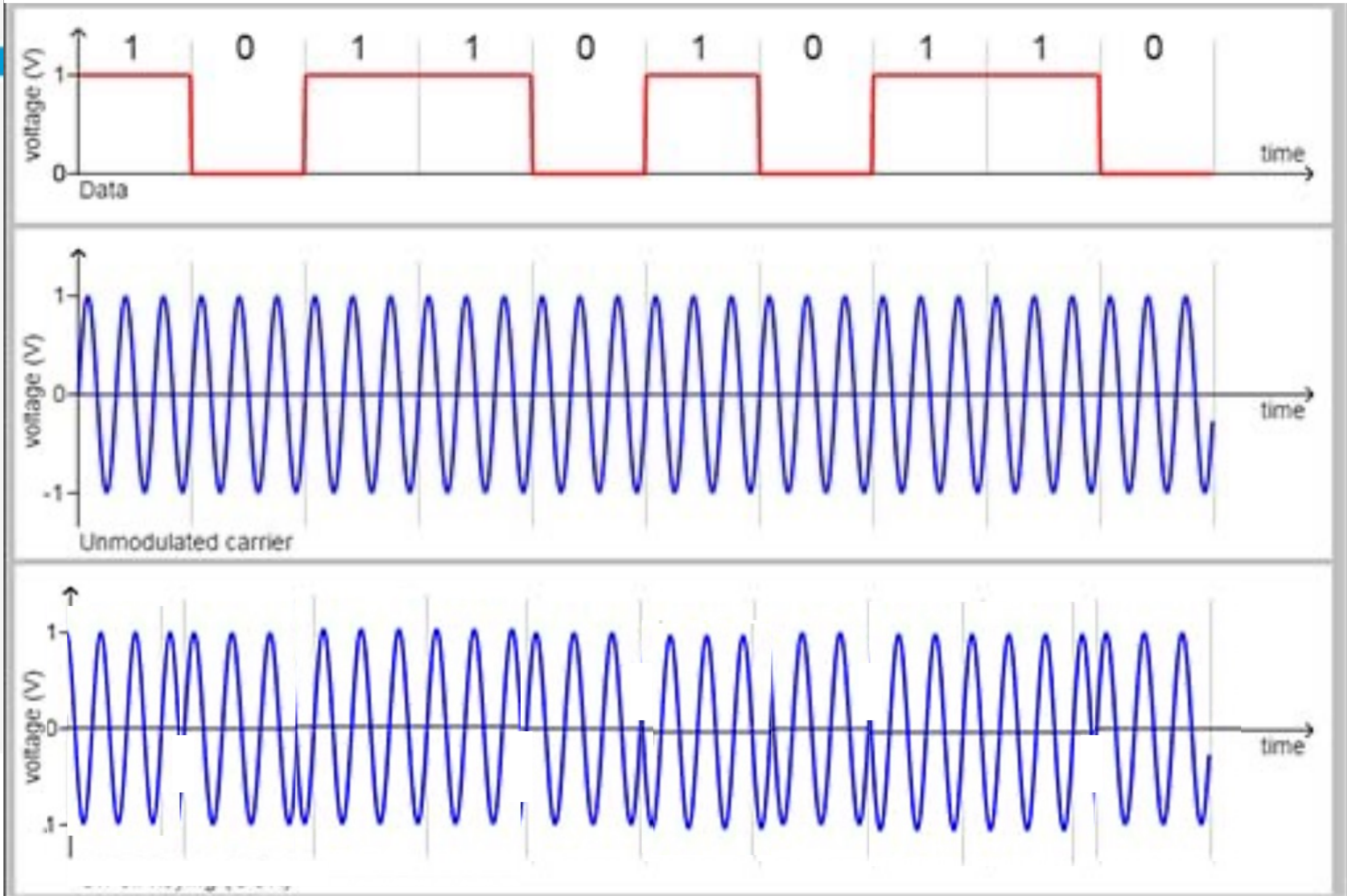


6.5.3 Phase shift keying (PSK)

Binary phase shift keying (BPSK)

0 C(t)=$A_c \cos[\omega_c t + \phi]$

1 C(t)=$A_c \cos[\omega_c t + \pi + \phi]$



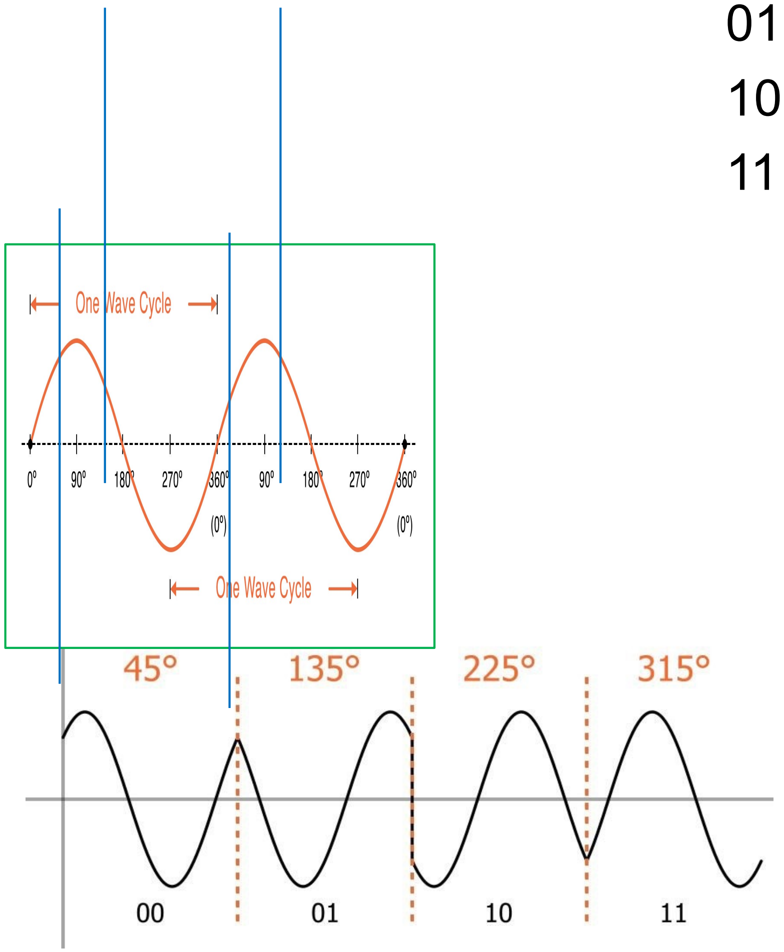
0: 1:

Quadrature phase shift keying (QPSK)

Four phase shifts are used for 00, 01, 10 and 11

00	C(t)=$A_c \cos[\omega_c t + 0 + \phi]$
01	C(t)=$A_c \cos\left[\omega_c t + \frac{\pi}{2} + \phi\right]$
10	C(t)=$A_c \cos[\omega_c t + \pi + \phi]$
11	C(t)=$A_c \cos\left[\omega_c t + \frac{3\pi}{2} + \phi\right]$

$\phi = +\frac{\pi}{4}$



Advantage of QPSK (or 4-PSK):

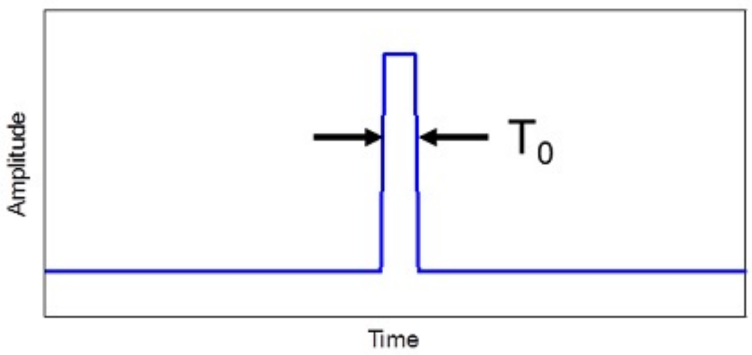
1) Higher data rate than with BPSK (doubled) while bandwidth remains about the same.

2) 4-PSK can be extended to n-PSK as far as equipment is able to distinguish between increasingly smaller phase differences.

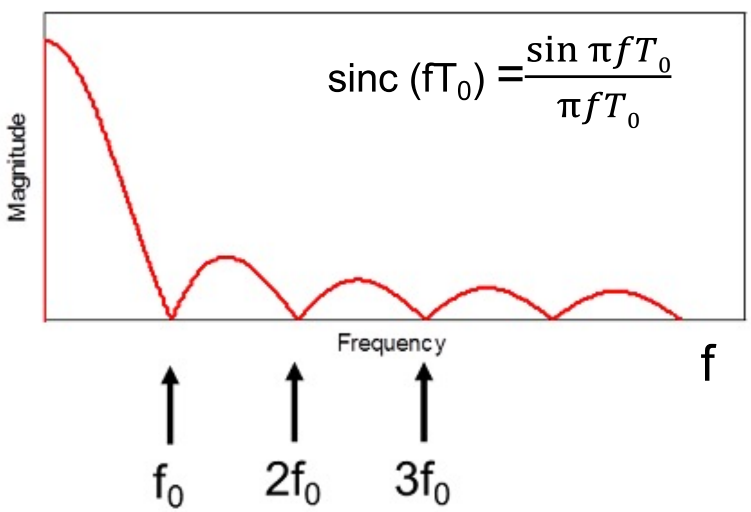
QPSK: four distinct phases are utilized. That means that essentially two BPSK signals are transmitted at once ➔ rate of transmission is doubled.

6.5.4 Spectra

All digital modulation relies on the basic characteristic of a square pulse



Single square pulse of width, T_0

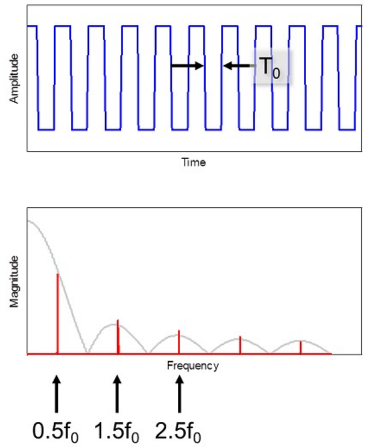


The Fourier transform of that pulse is a sinc function : $\mathrm{sinc}\,(x) = \frac{\sin x}{x}$

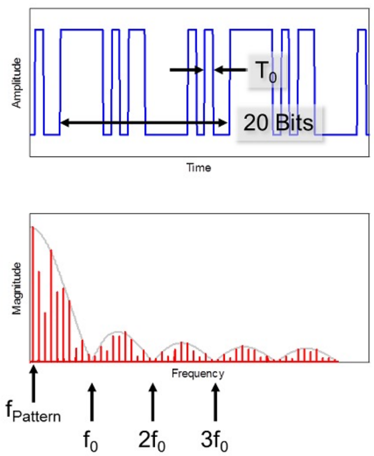
The magnitude of that function is given as the spectrum on the left. It is a continuum spectrum consisting of an infinite number of spectral components.

$f_0 = 1/T_0$

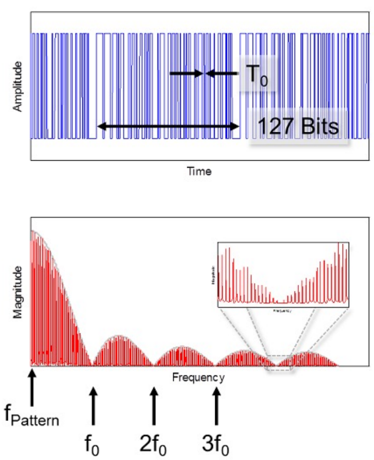
https://www.edn.com/the-basics-of-digital-signal-spectra/



In infinitely long string of pulses with a period $2T_0$ has a spectrum with components at f= n ·$1/2T_0$ The envelope of the frequency components is again a sinc function. Note, that in this example for n=2,4,6… The component amplitudes are zero

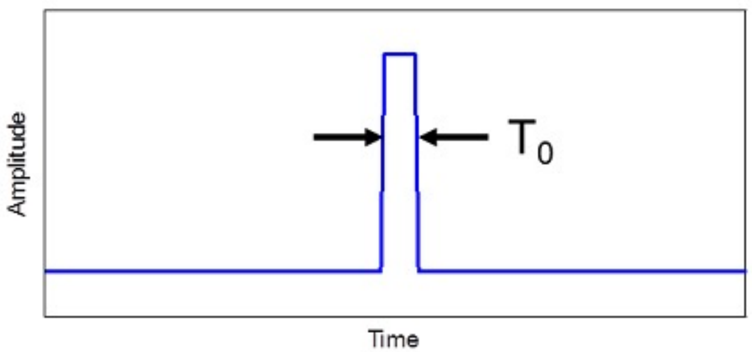


For a string of pulses with different durations but all equal to an integer number times T_0, the area under the envelope fills up. In this case the lowest frequency component is at $f=1/20T_0$

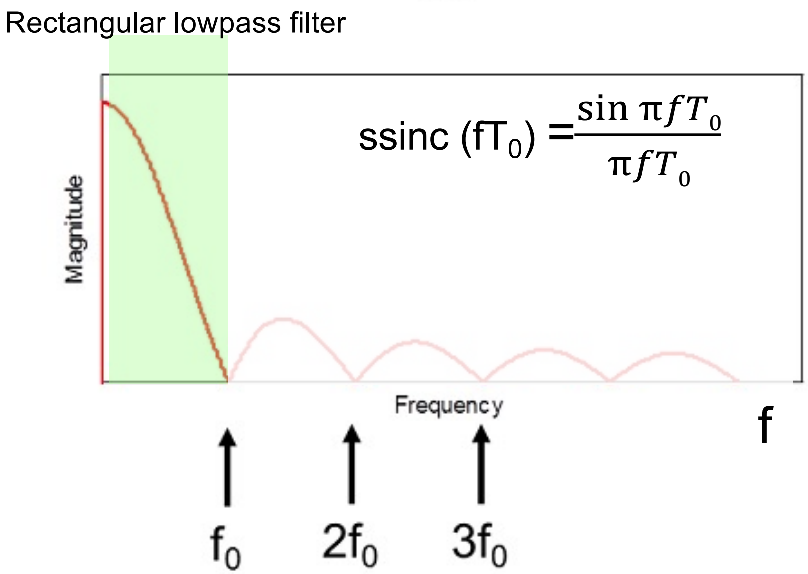


The longer the data pattern, the denser the components under the sinc function.

Back to the single square pulse



Single square pulse of width, T_0



$f_0 = 1/T_0$

The Fourier transform of that pulse is a sinc function : sinc $(x) = \frac{\sin x}{x}$

The magnitude of that function is given as the spectrum on the left. It is a continuum spectrum consisting of an infinite number of spectral components. If we pass the pulse through a low-pass filter then the higher frequency would be absent in the out put spectrum. The output spectrum would only reproduce a smooth version of the square pulse with sidelobes.

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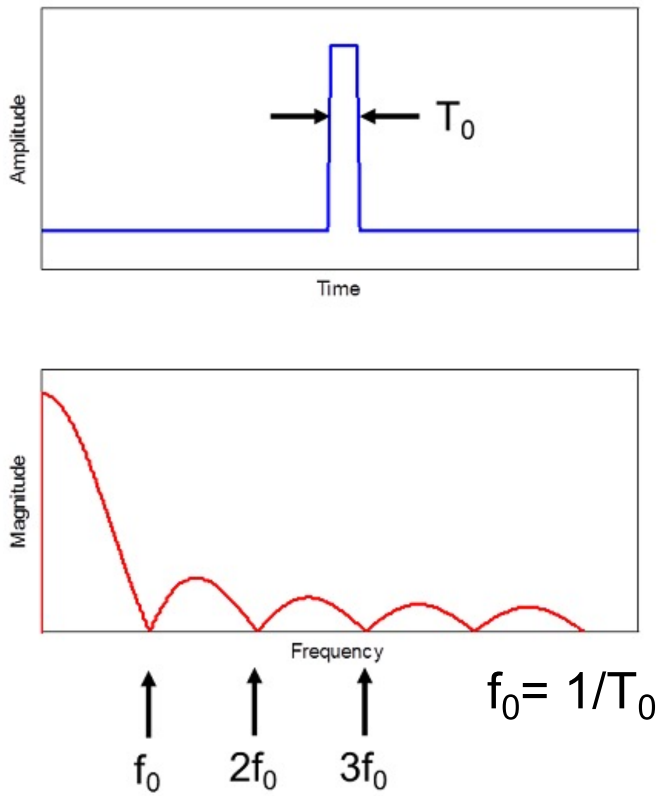
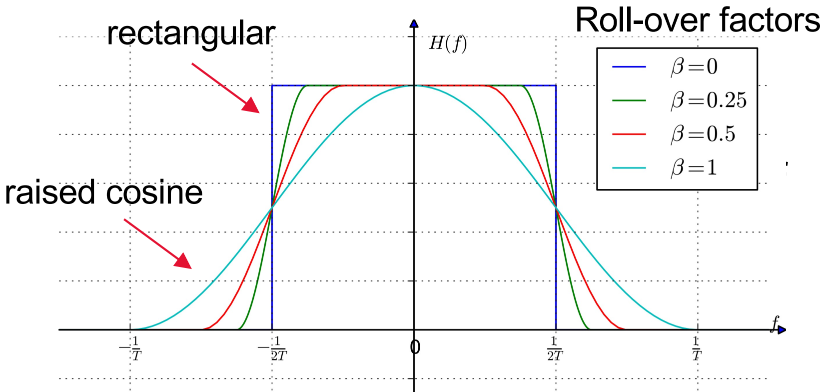
https://www.edn.com/the-basics-of-digital-signal-spectra/

Raised cosine filter



$f_0 = 1/T_0$



Frequency response of raised cosine filters with different roll-over factors

wikipedia/

Instead of lowpass filters
with a rectangular bandpass,
in digital modulation
raised cosine filters are frequently
Used. Different roll-over factors
characterize the raised cosine filters

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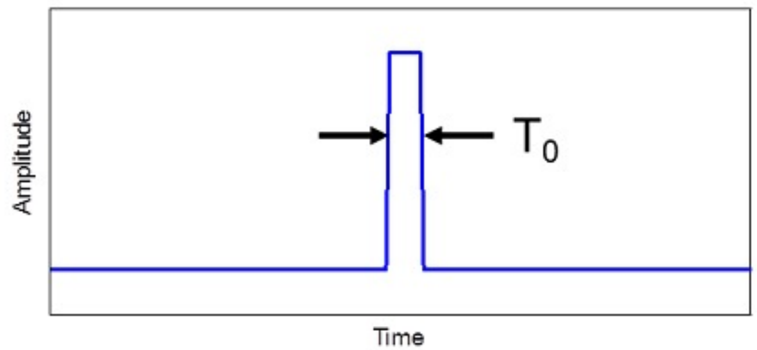
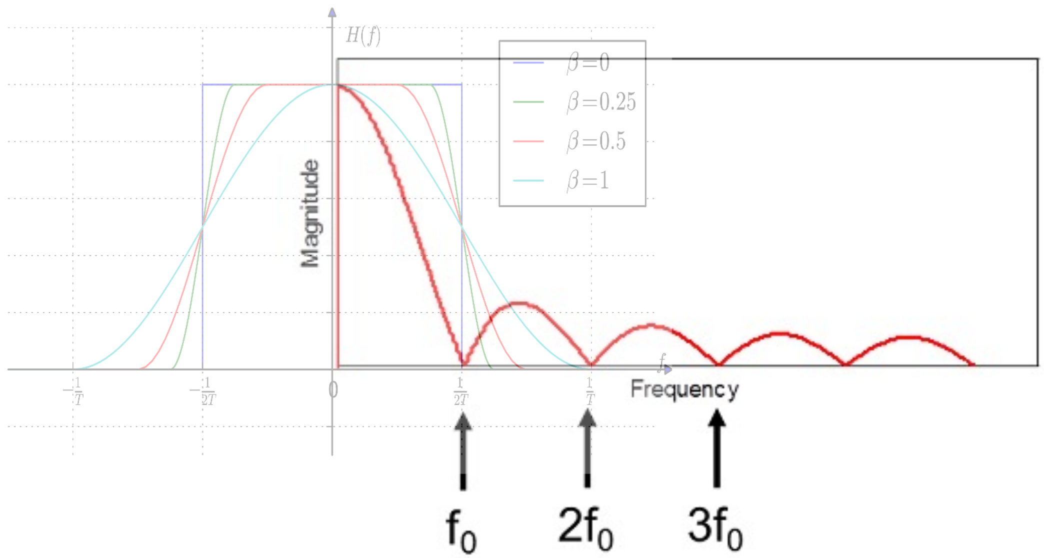
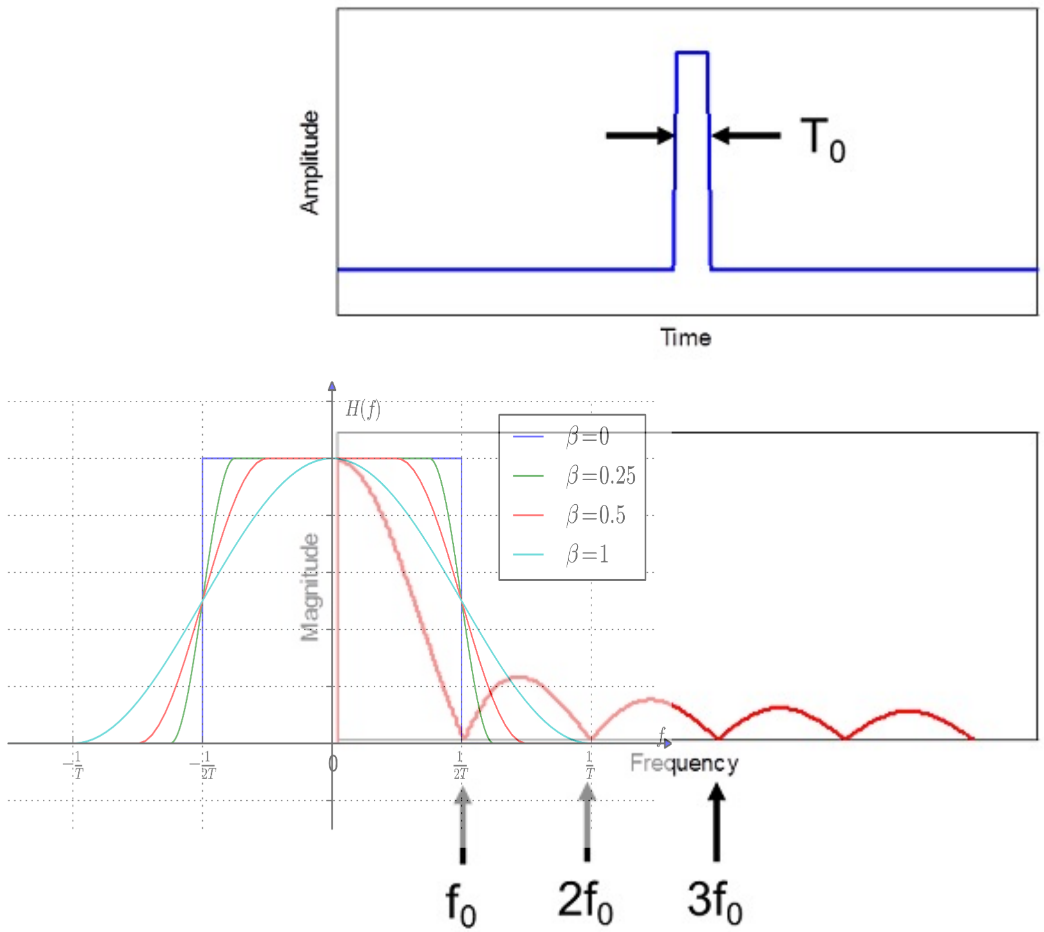
Raised cosine filter

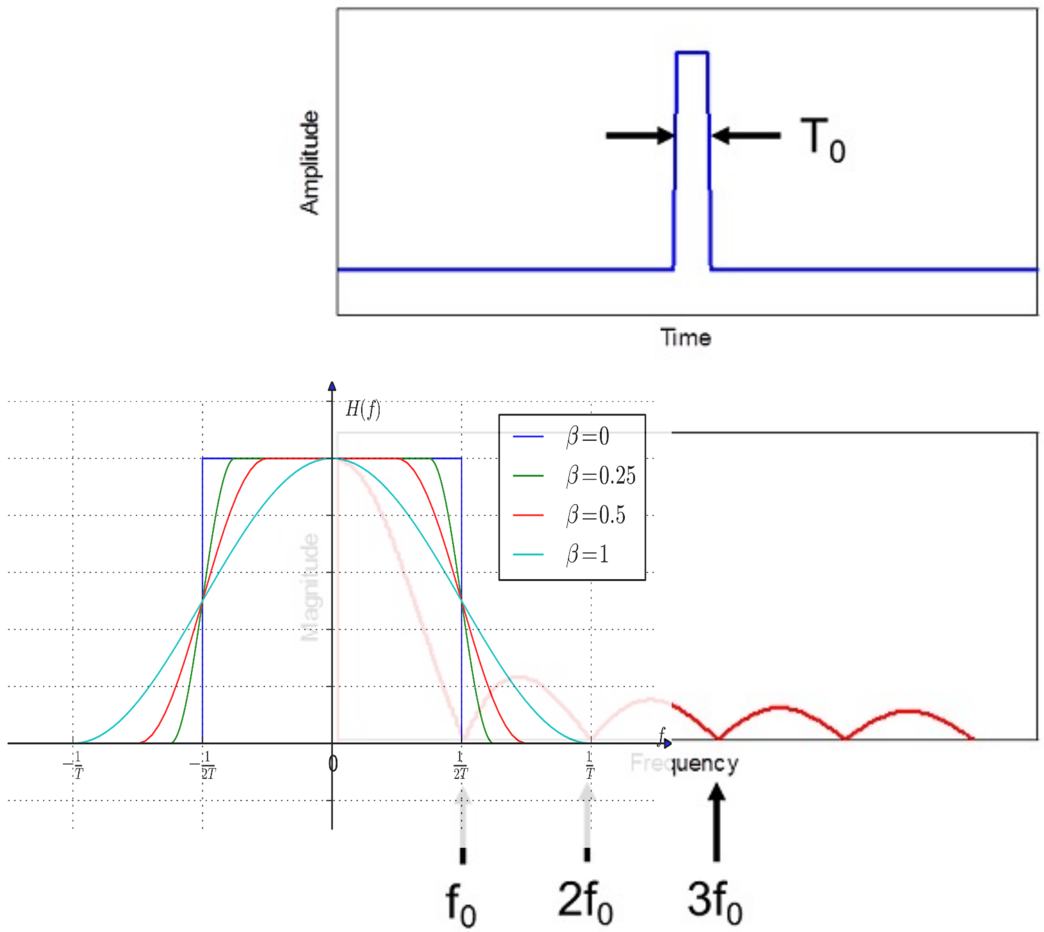




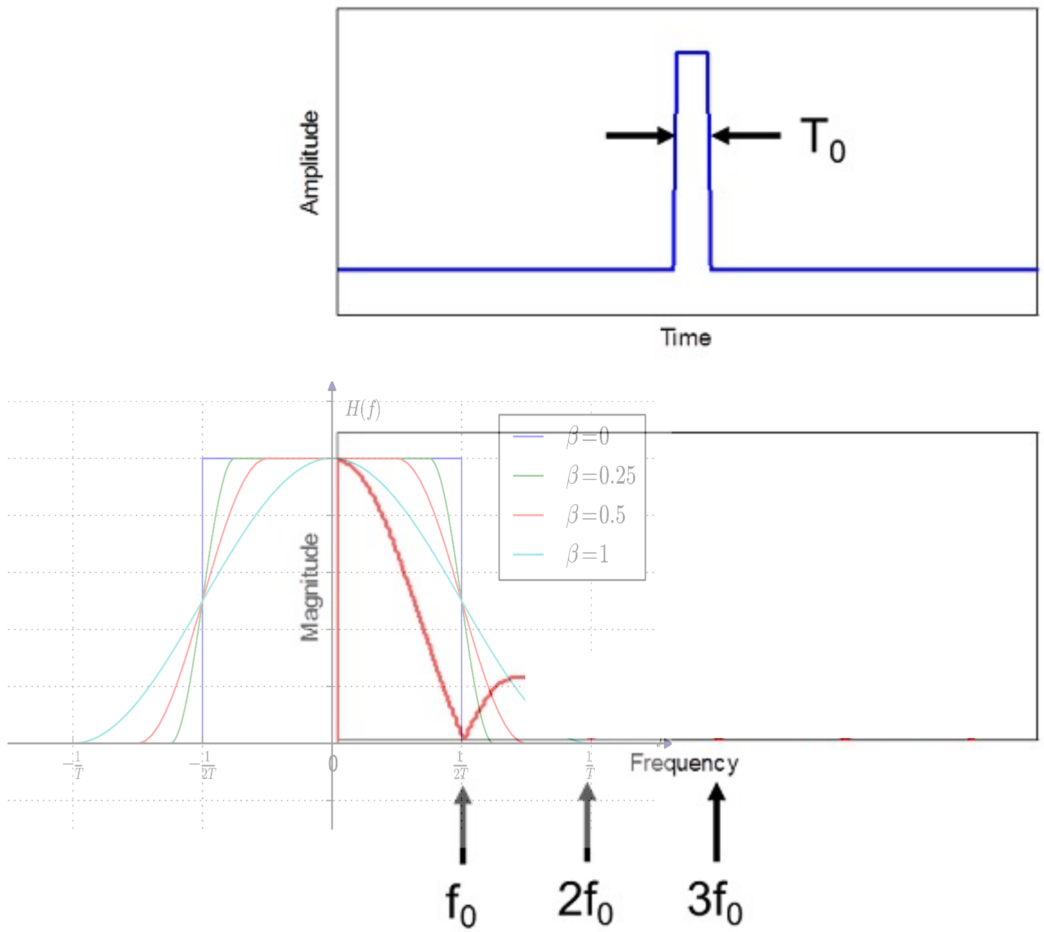
Raised cosine filter is a low-pass filter that smoothly cuts out higher frequencies



Raised cosine filter is a low-pass filter that
Smoothly cuts out higher frequencies



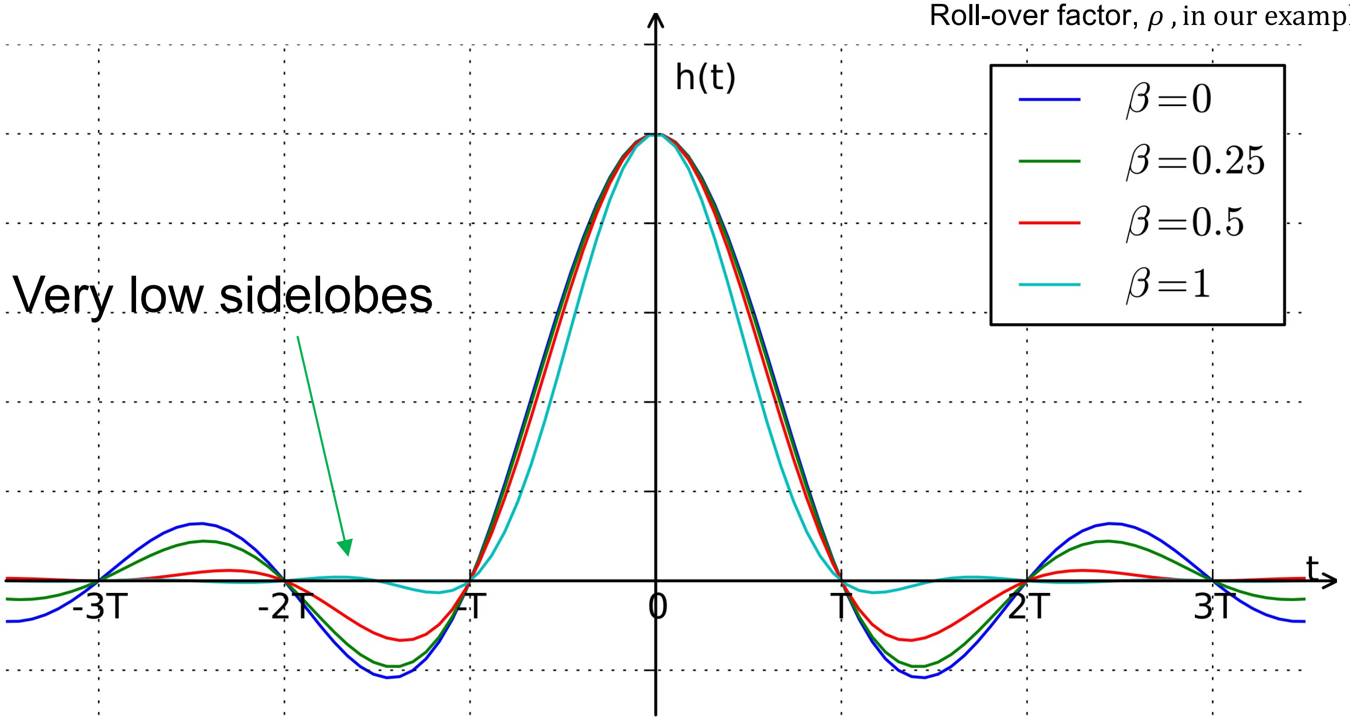
Raised cosine filter is a low-pass filter that
Smoothly cuts out higher frequencies



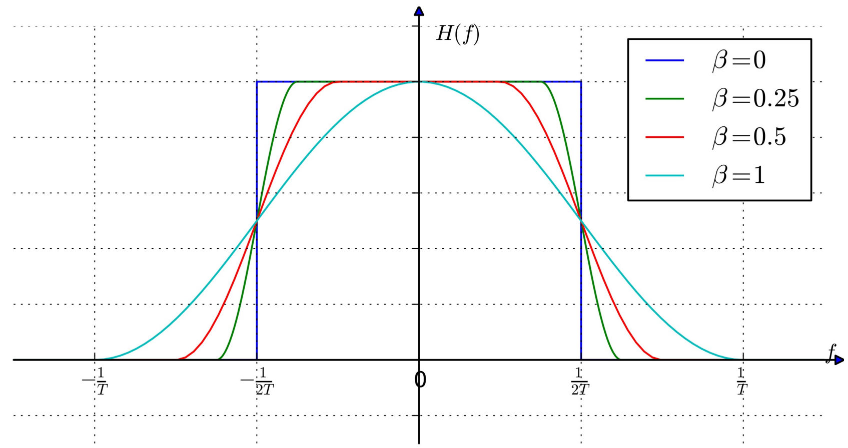
Raised cosine filter is a low-pass filter that
Smoothly cuts out higher frequencies

The output is a rounded pulse like a sinc function
with lower sidelobes depending on the roll-over factor

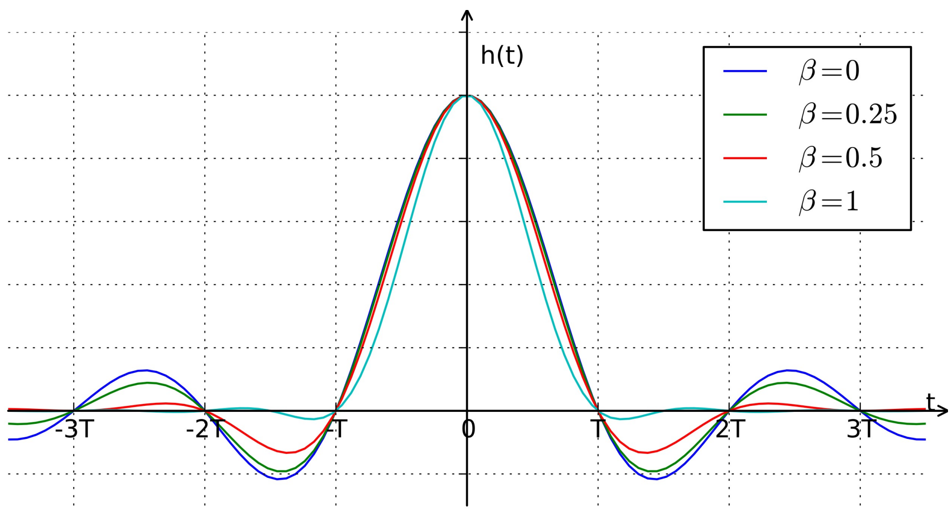
Roll-over factor, ρ, in our examples



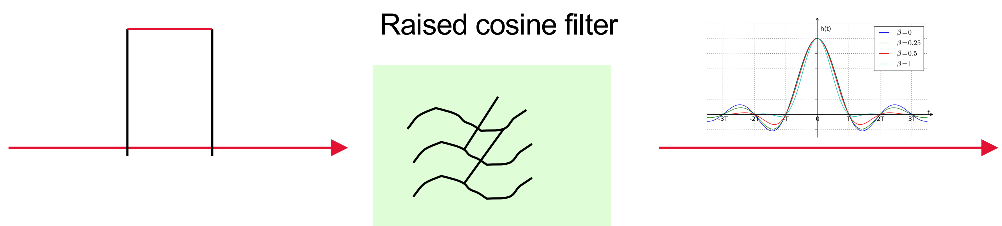
$$H(f) = \begin{cases} 1, & |f| \leq \frac{1-\beta}{2T} \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}\left[|f| - \frac{1-\beta}{2T}\right]\right)\right], & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$



With a rectangular lowpass filter, the rectangular pulse would be converted to a pulse with a sinc function shape. It has ripples or sidelobes in the time Domain. These sidelobes could interfere with neighbouring pulses.



Sidelobes in time cause intersymbol interference (ISI). With appropriate roll-over factors, the sidelobes and thus ISI can be Minimized.



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For BPSK:

$B = (1 + \rho)\ 1/(2T_b)$
$= \frac{1}{2}(1 + \rho) \bullet R_b$

ρ: roll over factor, defines raised cosine filter characteristics, $0 \le \rho \le 1$

$B_{IF} = 2B$
$= (1 + \rho) \bullet R_b$

Example 6-5

$R_b = 100$ kbps, $\rho = 1$
➔ $B_{IF} = 200$ kHz

For QPSK the information rate is doubled:

$B = \frac{1}{2}(1 + \rho)\ 1/(2T_b)$
$= \frac{1}{4}(1 + \rho) \bullet R_b$

ρ: roll over factor, defines raised cosine filter characteristics, $0 \le \rho \le 1$

$B_{IF} = \frac{1}{2}(1 + \rho) \bullet R_b$

Example 6-6

$R_b = 100$ kbps, $\rho = 1$
➔ $B_{IF} = 100$ kHz

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6.5.4 Bit error rate (BER)

BER characterizes the quality of digital transmission

$$\text{BER} = \frac{N_{err}}{N}$$

N_{err}: Number of bits received in error
N: Total number of bits transmitted

Example 6-5

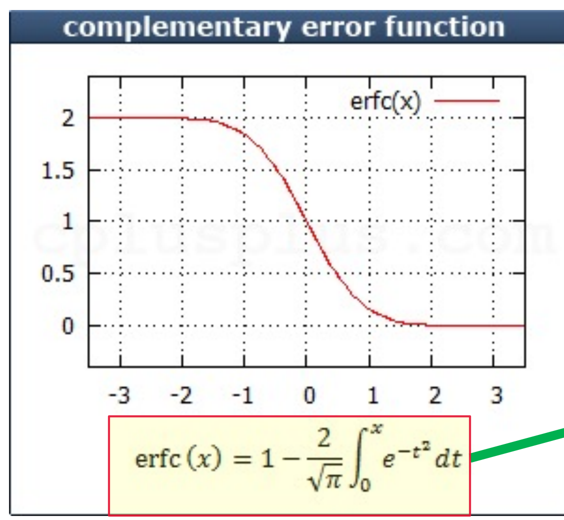
100101110100 Transmitted bit sequence
100101100101 Received bit sequence

$N_{err} = 2$
$N = 12$

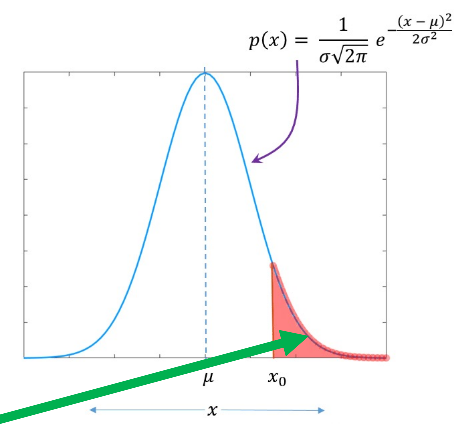
→ BER = 0.167

$$\text{BER} = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{N_0}}$$

$$\frac{E_b}{N_0} = \frac{\text{Energy per bit}}{\text{Noise power spectral density}}$$



Gaussian probability distribution



$$E_b = \frac{P_R}{P_R} = \frac{received\ power\ (W)}{bit\ rate\ (bits/s}$$

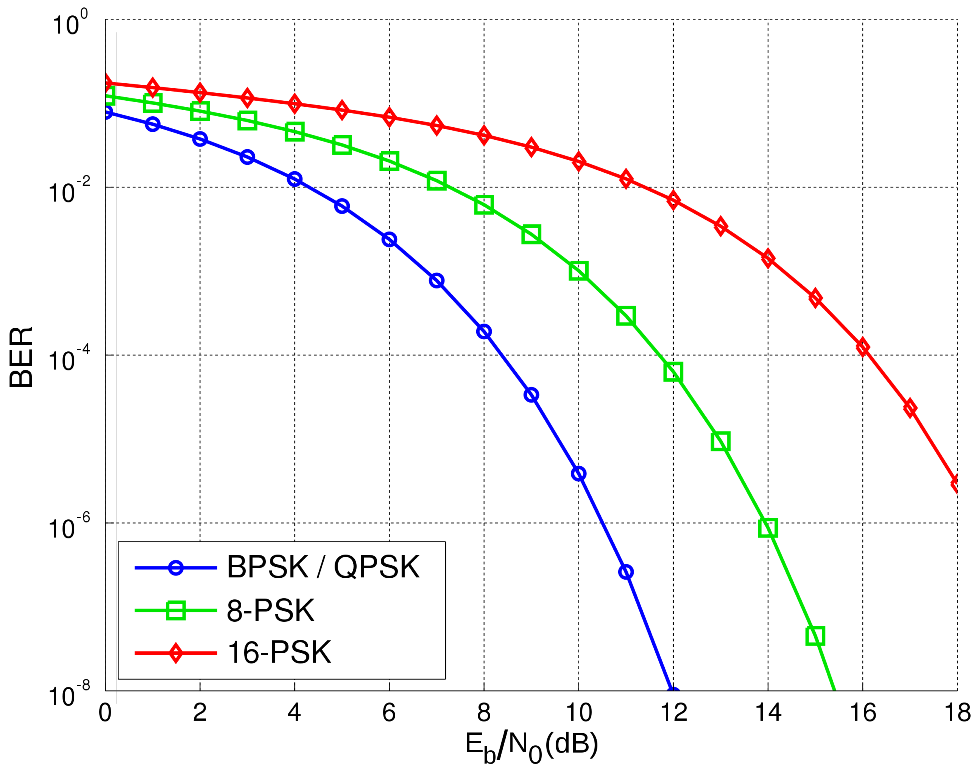
$$\frac{E_b}{N_0} = \frac{P_R}{R_B\ N_0}$$

$$\frac{E_b}{N_0} = \frac{C}{N_0} \cdot \frac{1}{R_B}$$

If $B_N = R_B$

Then with N = $N_0 \cdot B_N$

$$\frac{E_b}{N_0} = \frac{C}{N}$$



Example 6-6

$[C/N] = [E_b/ N_0] = 1.56$ dB ➔ BER = 5 10^{-2}
= 6 dB ➔ BER = 2 10^{-3}

Improvement of BER is possible through coding. By encoding extra bits (redundant bits) it is possible to detect and correct certain errors in the decoding process. However, transmission rate is affected.

For $[E_b / N_0] \geq$ 5 dB, coding gain can be achieved.
Example: $[E_b / N_0]=$ 6 dB ➔ BER $\leq 10^{-7}$ can be achieved!

6.6 Time division multiplexing (TDM)

The most common analogue multiplexing technique is frequency division multiplexing (FDM).

The most common digital multiplexing technique is time division multiplexing (TDM).

Transmitting and receiving ends of a typical TDM system

Example: US Bell T1 24 channel TDM