5 Radar Fundamentals
5.1 Introduction

RADAR: Radio detection and ranging.

Radar systems use modulated waveform transmitted by directive antennas toward a target or object. The object will reflect a portion of the energy back to the radar. These echos are processed to extract target information such as range, velocity, angular position and other characteristics.

Radars are often classified by the types of waveforms they use and by the operating frequency.
Types of waveform:
CW: continuous wave

- Emit EM energy continuously
- Use separate transmit and receive antennas

Unmodulated radar can measure target velocity through Doppler and angular position.
Modulated CW radar can, in addition, measure target range.
It is used for target velocity search (and range search) and tracking, and in missile guidance.

PR: pulsed radar

- Use a train of pulsed waveforms with modulation

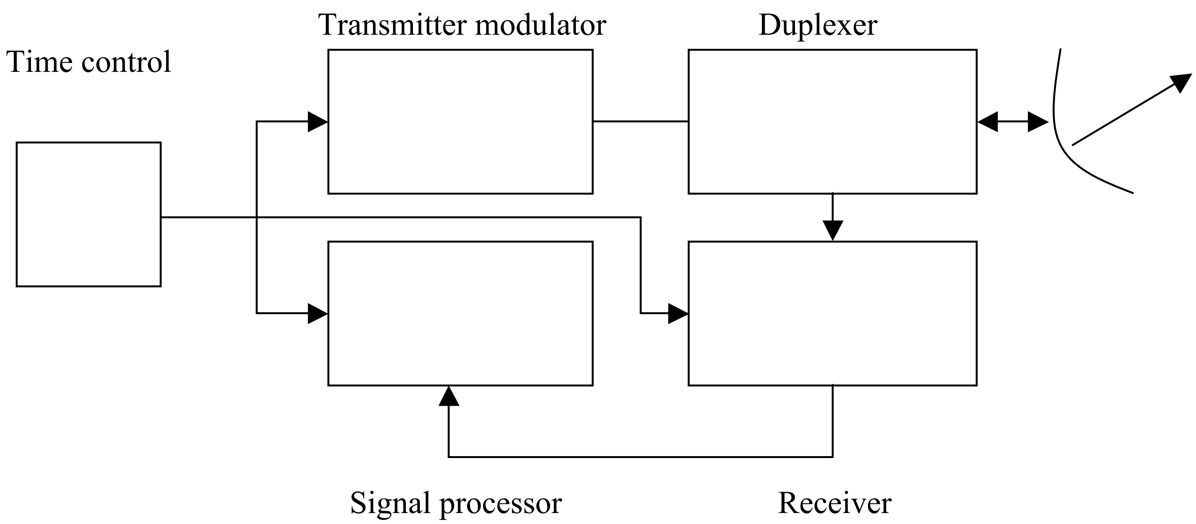
Classification based on operating frequency:

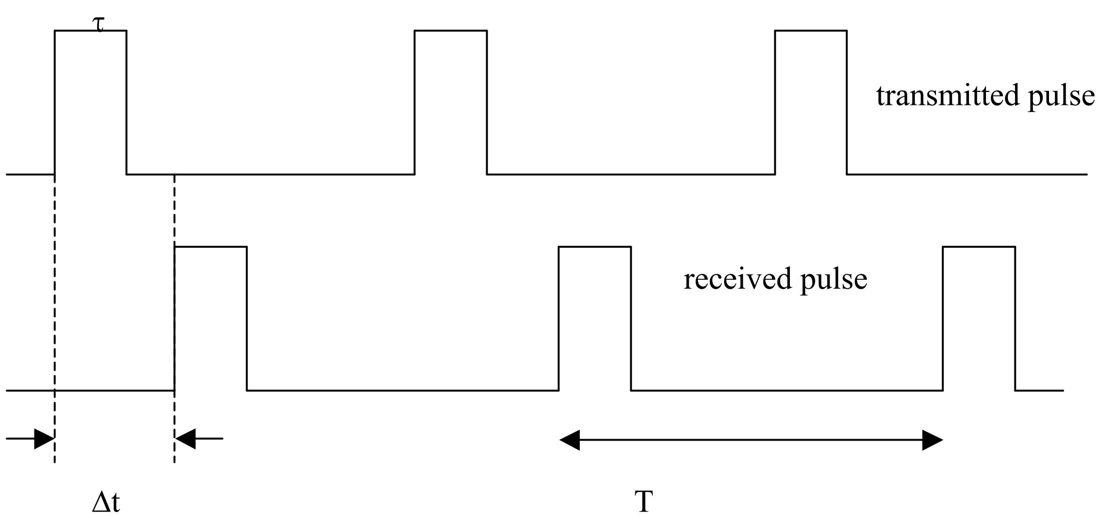
Band	ν(GHz)	Description
HF	0.003-0.03	Targets beyond horizon through reflection off the ionosphere (over the horizon backscatter)
VHF	0.03-0.3	Very long range early warning radar
UHF	0.3-1.0	
L	1.0-2.0	Ground and ship-based long range military systems, air traffic control systems
S	2.0-4.0	Ground and ship-based medium range systems (AWACS: Airborne warning and control systems)
C	4 - 7	Weather detection radar systems, fire control military radars
X	7 - 10	Used with small antennas, most military airborne antennas, radar systems that require fine target detection capability but cannot tolerate atmospheric attenuation at higher frequencies.
Ku	10 – 18	Short range application such as in police traffic radars, terrain following radars
K	18 – 26	
Ka	26 - 40	

5.2 Range

The range, R, is the distance of a target or object from the radar antenna. The two figures below show how R is measured in principle.

Pulsed Radar Block Diagram





T: IPP (Inter pulse period)
PRI: (Pulse repetition interval)

1/T: PRF (Pulse repetition frequency)

A modulated signal (pulse train) is transmitted via the antenna to a target. The echo is received and amplified and then compared with the transmitted signal. Everything is controlled by a stable time and frequency standard. Then

$$R = \frac{c\Delta t}{2}$$

During each PRI, the radar radiates only during the time duration of the pulse, τ, and listens for the echo for the rest of the PRI. We further define:

$d_t = \frac{\tau}{T}$ Radar transmitting duty cycle

P_t Radar peak transmitted power

$\overline{P} = P_t d_t$ Radar average transmitted power

$E_p = P_t \tau$ Pulse energy

$$\begin{aligned} E_p &= P_t \tau \\ &= \overline{P} T \\ &= \overline{P} \frac{1}{f_r} \end{aligned}$$

Example:
An airborne pulsed radar has peak power P_t= 10 kW and uses a PRF of 10 kHz. What is the required pulse width so that the average transmitted power is equal to 1.5 kW?

Solution:

$$\begin{aligned} \overline{P} &= P_t d_t \\ &= P_t \frac{\tau}{T} \\ &= P_t \tau f_r \\ \tau &= \frac{\overline{P}}{P_t f_r} \\ &= \frac{1.5 \bullet 10^3}{10 \bullet 10^3 \bullet 10^4} \\ &= 15 \mu s \end{aligned}$$

if the target is far away, then the range determination could be ambiguous. This is the case when $\Delta t > T$. therefore it follows that the maximum unambiguous range is given by $\Delta t = T$, or

$$\begin{aligned} R_u &= c \frac{T}{2} \\ &= c \frac{1}{2 f_r} \end{aligned}$$

radar systems are normally designed to operate between a minimum and maximum range so that ambiguities are avoided.

Radar systes are also characterized by a range resolution, ΔR, that describes its ability to detect targets in close proximity to each other as distict objects. A prerequisite is that the echo from target 1 and the echo from target 2 can be distinguished as two distinct pulses. The range difference between target 2 and target 1, $R_2 - R_1$, is given as

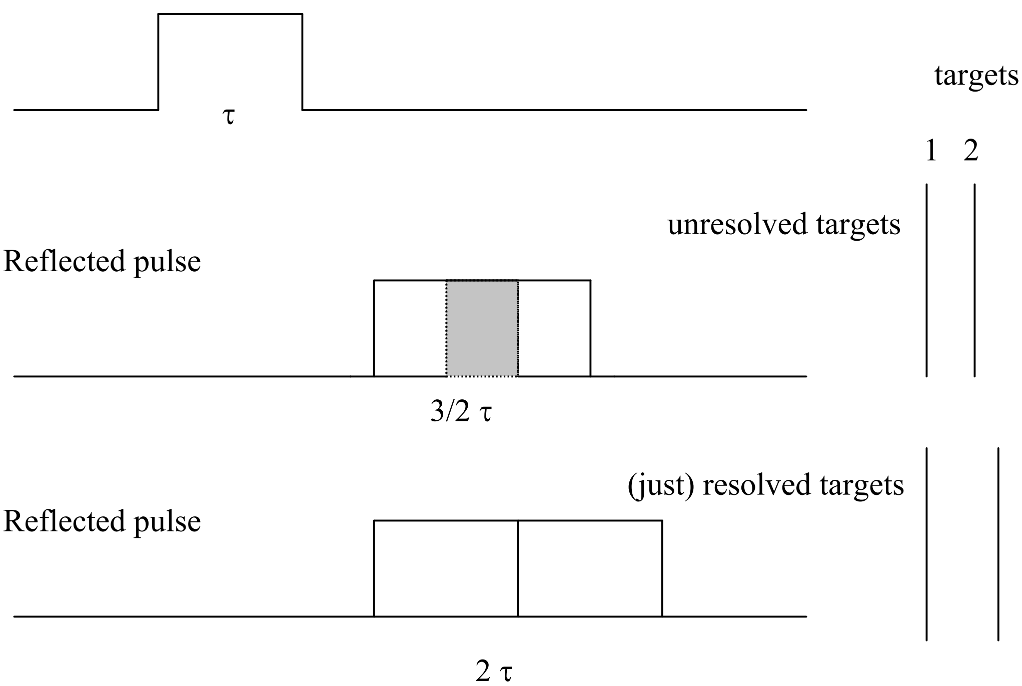
$$R_2 - R_1 = c\,\frac{(\Delta t)_2 - (\Delta t)_1}{2}$$

The differences in the rages can be clearly measured if
$(\Delta t)_2 - (\Delta t)_1 \geq \tau$

Thus:
$\Delta R = c\,\dfrac{\tau}{2}$ and with
$\tau = \dfrac{1}{B}$ with B as the bandwidth of the system

$$\Delta R = c\,\frac{1}{2B}$$

Sketched examples
Transmitted pulse



Example;
A radar system has R_u = 100 km and a bandwidth B =0.5 MHz. Compute the PRF, ΔR and τ.

$$PRF = f_r = \frac{c}{2R_u} = \frac{3 \bullet 10^8}{2 \bullet 10^5} = 1500Hz$$

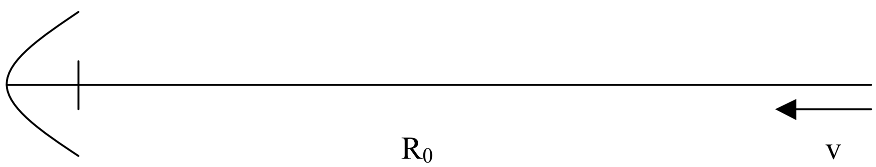
$$\Delta R = \frac{c}{2B} = \frac{3 \bullet 10^8}{2 \bullet 0.5 \bullet 10^6} = 300m$$

$$\tau = \frac{2\Delta R}{c} = \frac{3 \bullet 300}{3 \bullet 10^8} = 2\mu s$$

5.3 Doppler frequency and range rate

Radars use Doppler frequency to extract target radial velocity (range rate) as well as to distinguish between moving and stationary targets. Depending on the direction of the target's motion, the frequency is shifted to higher values (closing target) or lower values (opening target).

Consider the following scenario of a closing target with velocity v.



R_0 is the range at time t_0. Then the range to the target at any time t is:

$$R(t) = R_0 - v(t - t_0)$$

Suppose that a signal $x_t(t)$ is transmitted toward the target, Then the received signal $x_r(t)$ is:

$$\begin{aligned} x_r(t) &= x_t\left(t - \frac{2R(t)}{c}\right) \\ &= x_t\left[t - \left(\frac{2R_0}{c} - \frac{2v}{c}(t - t_0)\right)\right] \end{aligned}$$

Then with

$$\gamma = 1 + \frac{2v}{c}$$

and

$$\tau_0 = \frac{2R_0}{c} + \frac{2v}{c} t_0$$

we get:

$$
\begin{aligned}
x_r(t) &= x_t\left[t - \left(\tau_0 - \frac{2v}{c}t\right)\right] \\
&= x_t\left[t \cdot \left(1 + \frac{2v}{c}\right) - \tau_0\right] \\
&= x_t(\gamma t - \tau_0)
\end{aligned}
$$

For a stationary target with v=0 we get

$$x_r(t) = x_t\left(t - \frac{2R_0}{c}\right)$$

We see that the second from the last equation is a time-compressed version of the last equation. Remembering the scaling property of the FT we can see that the spectrum of $x_r(t)$ is expanded in frequency by a factor γ.

Let us consider a special case:

$$x_t(t) = y(t)\cos\omega_0 t$$

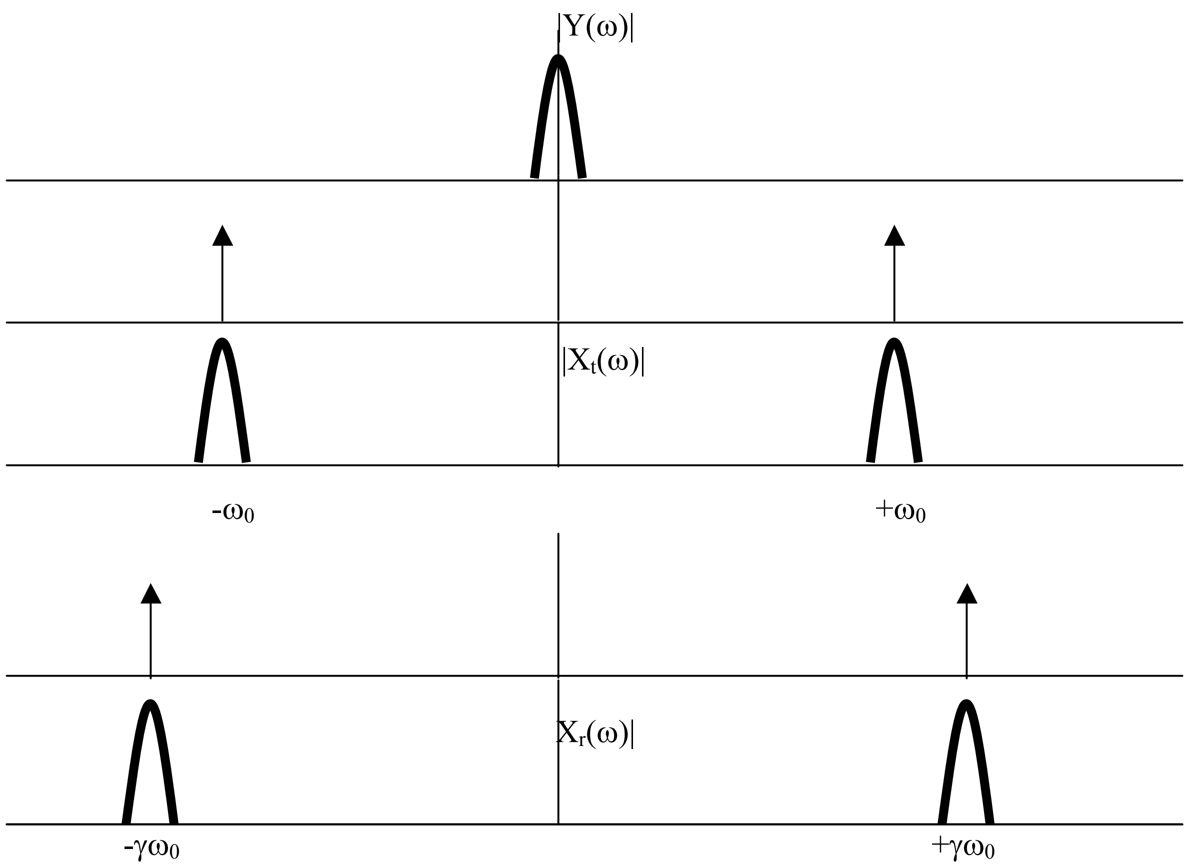
$$X_t(\omega) = \pi\left[Y(\omega + \omega_0) + Y(\omega - \omega_0)\right]$$

Then:

$$x_r(t) = y(\gamma t - \tau_0)\cos\omega_0(\gamma t - \tau_0)$$

$$X_r(\omega) = \pi\frac{1}{\gamma}\left[Y\left(\frac{\omega}{\gamma} + \omega_0\right) + Y\left(\frac{\omega}{\gamma} - \omega_0\right)\right]e^{-i\phi}$$

Let us sketch this scenario by assuming a particular spectrum for Y(ω).



The spectrum is now shifted to $-\gamma\omega_0$ and $+\gamma\omega_0$

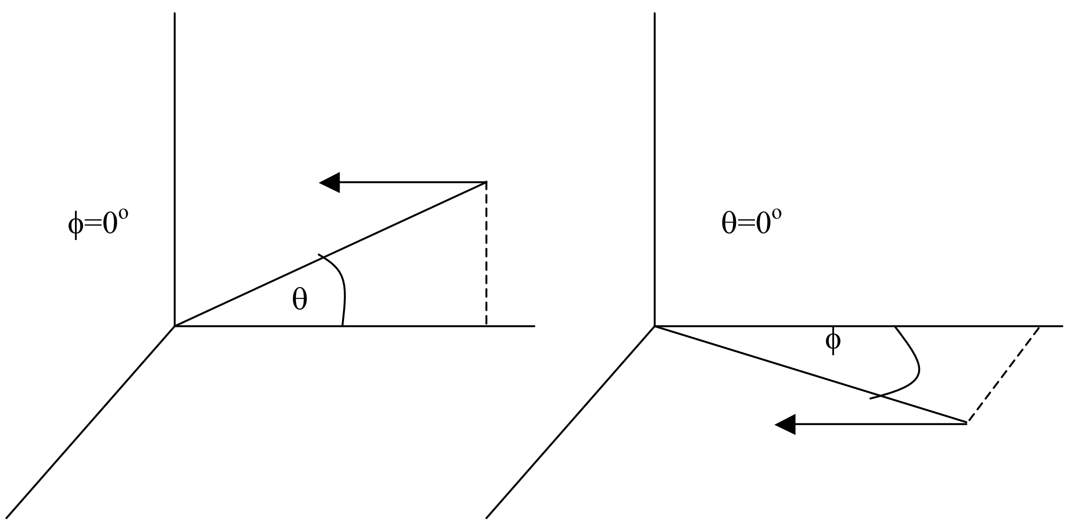
$\omega_d = \omega_0 - \gamma\omega_0$
$= (1 - \gamma)\,\omega_0$ Doppler frequency

$f_d = 2v/c \bullet f_0$
$= 2v/\lambda_0$

If the target is moving at an angle θ in elevation and an angle ϕ in azimuth relative to the line of sight then the expression for the Doppler shift in frequency becomes:

$$f_d = -\frac{2v}{\lambda}\cos\theta\cos\phi \qquad \text{Opening target}$$

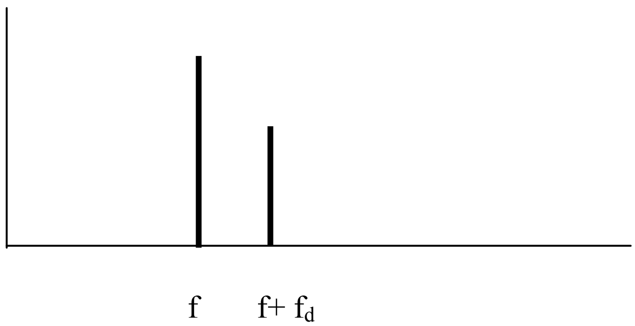
$$f_d = +\frac{2v}{\lambda}\cos\theta\cos\phi \qquad \text{Closing target}$$



Example:

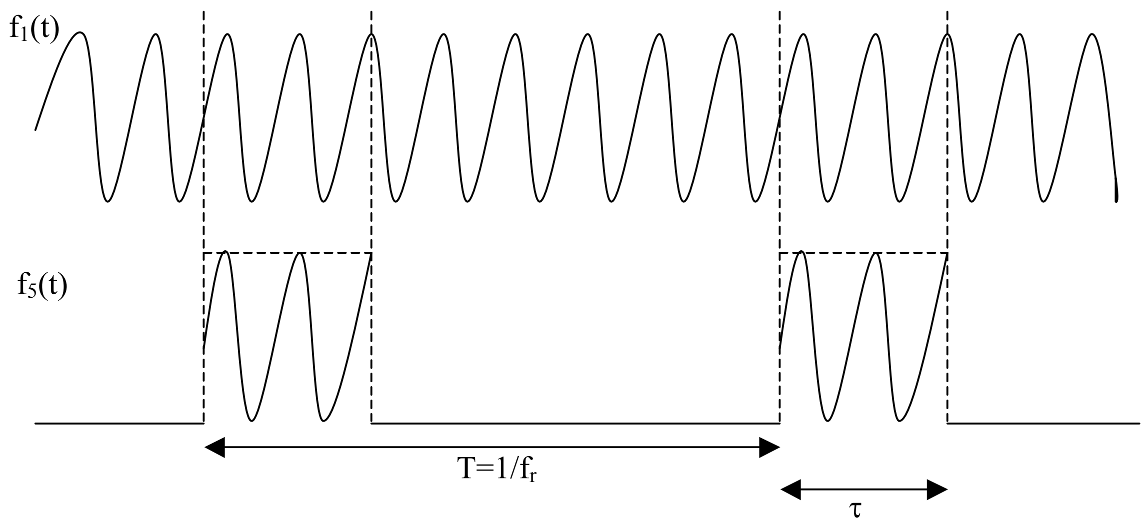
$\lambda = 0.03$ m, f= 10 GHz
v = 175 m/s : closing
$\theta = 30^o$
$\phi = 40^o$

➔ $f_d = +2 \quad 175/0.03 \;\; \cos 30^o \; \cos 40^o$
$= +7.7$ kHz



Coherence

A radar is said to be coherent if the phase changes continuously with time from one pulse to the next



If this continuity is not given, then the radar is incoherent.

Now consider a finite coherent, gated, pulsed waveform transmitted by a radar antenna. What is the spectrum?

It is best to describe the waveform as a product of functions.

1) $$F_1(\omega) = A\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$

2) $f_2(t) = \Pi\left(\frac{t}{\tau}\right)$ $$F_2(\omega) = \tau \sin c\left(\frac{\omega\tau}{2\pi}\right)$$

Now we have to find a discription of an infinite pulse train and its spectrum. We did this at the beginning of the course. For the infinite pulse train we have to sum up the individual pulses, each delayed by a certain time interval, and for the FT we need to consider the complex exponential FS.

$$f_3(t) = \sum_{n=-\infty}^{+\infty} c_n e^{in\omega_0 t} = \sum_{n=-\infty}^{+\infty} c_n e^{\frac{i2\pi nt}{T}}$$

with

$$c_n = \frac{A\tau}{T} \operatorname{sinc} \frac{\pi n\tau}{T}$$

We then get

3) $f_3(t) = \sum_{n=-\infty}^{+\infty} f_2(t - nT)$ $\qquad$ $F_3(\omega) = 2\pi \sum_{n=-\infty}^{+\infty} c_n \delta(\omega - 2\pi n f_r)$

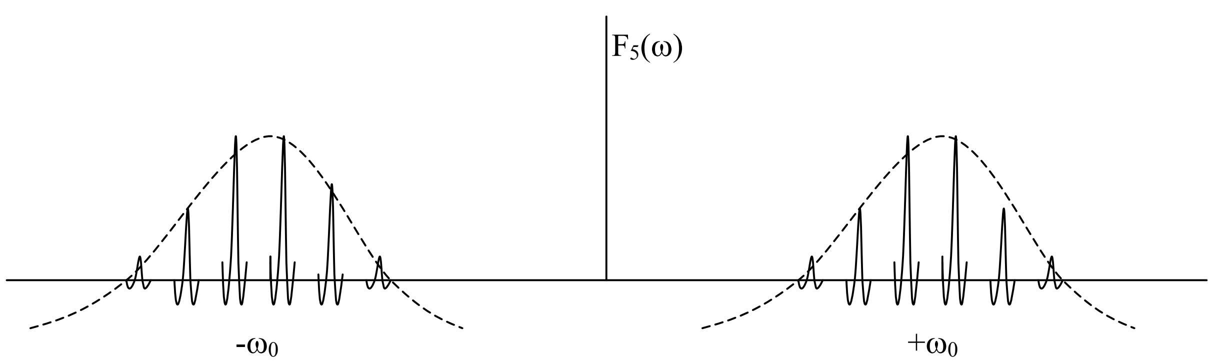
When we limit this pulse train to a finite number of pulses we describe the new pulse train $f_4(t)$ as $f_3(t)$ multiplied by a gate function of width NT.

4) $f_4(t) = \Pi\left(\frac{t}{NT}\right) f_3(t)$ $\qquad$ $F_4(\omega) = NT \operatorname{sinc}\left(\omega \frac{nT}{2\pi}\right) * F_3(\omega)$

The amplitude spectrum is a sinc function convolved with a train of amplitude-modulated delta functions of F_3.

Finally we use $f_4(t)$ to modulate $f_1(t)$ to get the function $f_5(t)$ as sketched in the figure. This is a cos-wave switched on only during the duration of each of a finite nu,ner of pulses.

5) $f_5(t) = f_1(t) f_4(t)$ $\qquad$ $F_5(\omega) = A\pi\left[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)\right] * F_4(\omega)$



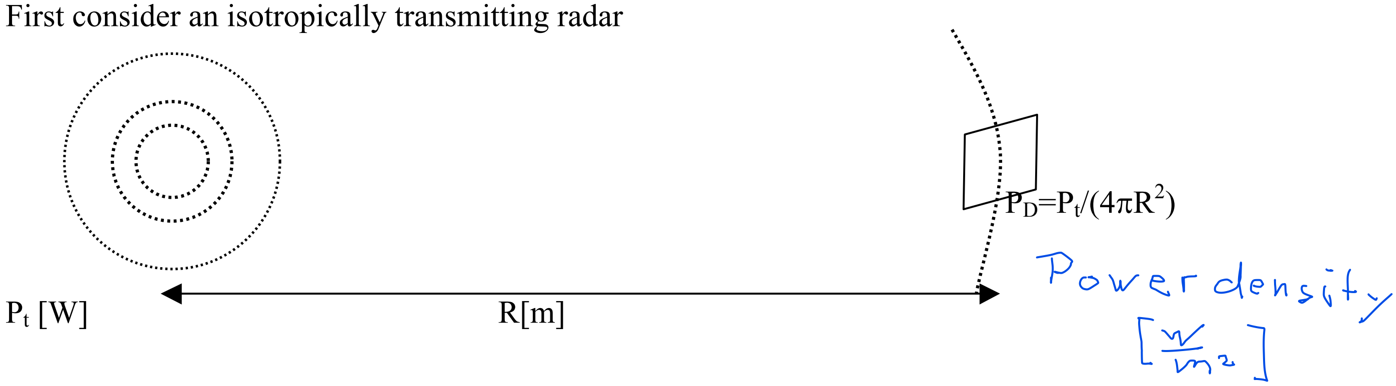
5.4 The radar equation

First we want to introduce the parameters we have to use:

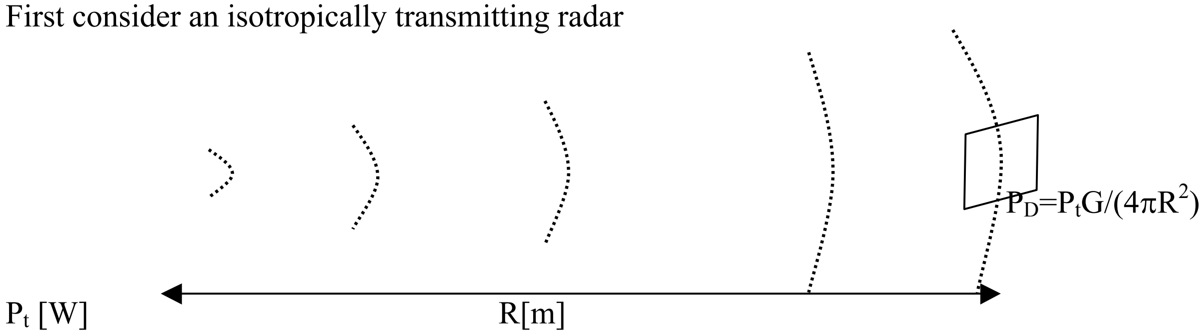
P_t peak transmitted power
P_{ref} power reflected back to radar
G gain of antenna

P_D power density
A_{eff} effective aperture of antenna
σ target cross section (RCS: radar cross section)
R range
P_r received power
SNR signal to noise ratio
F noise figure
T_e effective noise temperature
k Boltzmann's constant 1.38 10^{-23} J/K
N noise power

First consider an isotropically transmitting radar



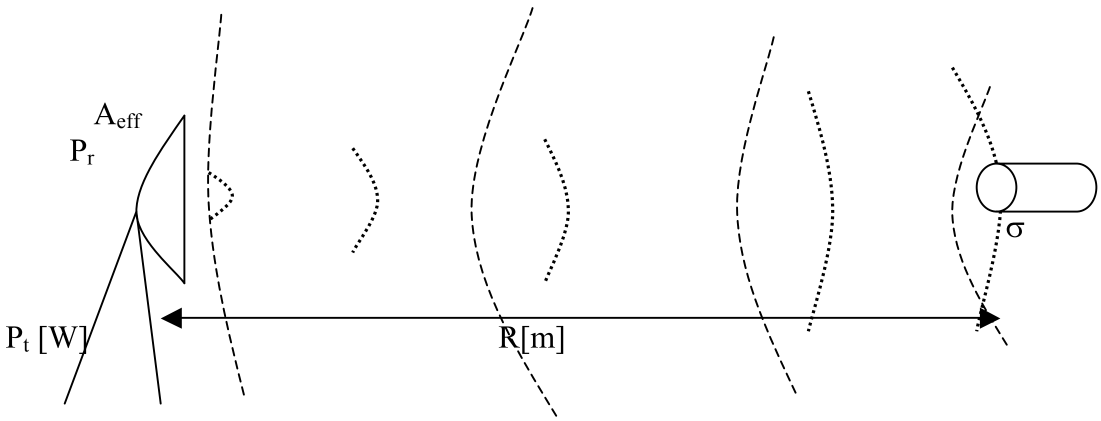
Now assume that the power is transmitted in a certain direction with an antenna with gain G.
First consider an isotropically transmitting radar



Now assume that there is a target at range R. When the radiated energy impinges on that target currents are induced on the surface and EM energy is radiated in all directions. The power reflected back to the radar antenna depends on the size of the target and on many other parameters. A measure of all these parameters is the radar cross section. It is defined as:

$$\sigma = \frac{P_{ref}}{P_D}[m^2] \quad \leftarrow P_D$$

Further assume that the radar antenna receives the reflected power. Then



And the total power delivered to the radar signal processor by the antenna is:

$$P_r = \frac{P_t G \sigma A_{eff}}{(4\pi R^2)^2}$$

$$A_{eff} = \eta A = \frac{G\lambda^2}{4\pi}$$

$$P_r = \frac{P_t G^2 \sigma \lambda^2}{(4\pi)^3 R^4}$$

The radar equation can be obtained by further considering noise. The presence of noise determines a minimum detectable signal power $P_{min} = P_{r,\,min}$. The maximum range is then given as

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 P_{min}} \right)^{1/4}$$

Note: In order to double R_{min} we must increase, e.g., P_t 16 times!

However, the noise of the receivers will limit the maximum range of the radar. The noise power spectral density, N_0, is

$N_0 = kT_e$

The noise power for a radar which is operating with a bandwidth, B, is given by

$N = kT_eB$

For a fully operational radar, the minimum detectable signal power, P_{min}, has to be greater than the noise power N. One characteristic of the radar receiver is the figure of merit, called the noise figure F.

$$F = \frac{(SNR)_i}{(SNR)_o} = \frac{P_i / N_i}{P_o / N_o}$$

Here, $(SNR)_i$ and $(SNR)_o$ are, respectively, the signal to noise ratios at the input and output of the receiver. Substituting and rearranging gives

$P_i = kT_eBF\,(SNR)_o$

For the minimum detectable signal power we thus get

$P_{i,\,min} = kT_eBF\,(SNR)_{o,\,min}$

Further substituting yields

$$R_{\max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e B F (SNR)_{o,\min}} \right)^{1/4}$$

or equivalently

$$(SNR)_{o,\min} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e B F R_{\max}^4}$$

Finally, if we include all kinds of radar losses lumped into L that reducees the overall SNR, we get the

RADAR EQUATION

$$(SNR)_{o,\min} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e B F L R_{\max}^4}$$

Example:

Compute R_{max} for C-band radar with the following parameters:

$P_t = 1.5$ MW

$f_0 = 5.6$ GHz $\rightarrow \lambda = 5.36$ cm $\qquad \lambda = \frac{c}{f_0}$

G= 45 dB $\hat{=}\ 10^{4.5} = 31623$

$T_e = 290$ K

$F = 3$ dB $\hat{=}\ 10^{0.3} = 1.995$

$L = 0$ dB $= 10^{0} = 1$

$\tau = 0.2\ \mu s$ ➔ B = 5 MHz

$(SNR)_{o,\,min} = 20$ dB $\hat{=}\ 10^{2} = 100$

$\sigma = 0.1\ m^2$

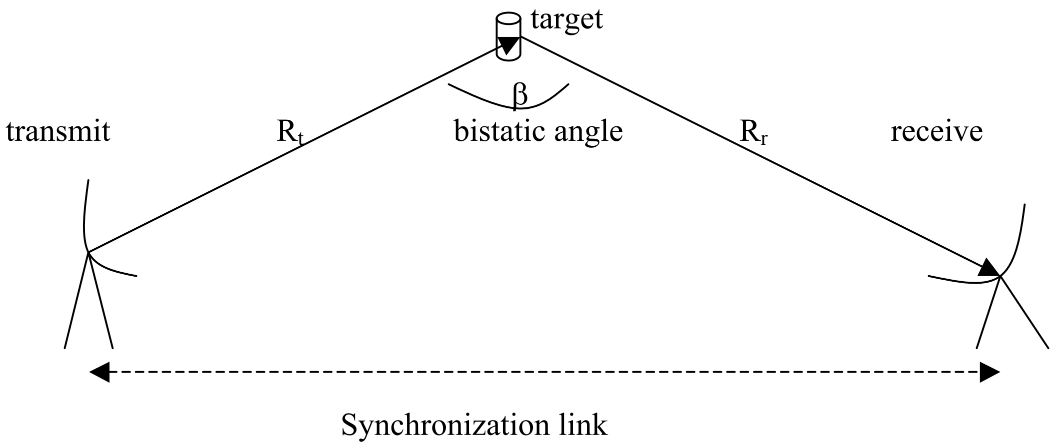
$[R^4_{max}] = 61.76 + 90 - 25.42 - 10 - 32.98 + 136.99 - 3 - 20$
$= 197.35$ dB

$R^4_{max} = 5.43\ 10^{19}\ m^4$ $\quad = \dfrac{1.5 \cdot 10^6 \cdot 10^9 \cdot (5.357 \cdot 10^{-2})^2 \cdot 0.1}{(4\pi)^3 \cdot 1.38 \cdot 10^{-23} \cdot 290 \cdot 5 \cdot 10^6 \cdot 1.995 \cdot 1 \cdot 100}$

$R_{max} = 85.85$ km

Bistatic radar

So far we have discussed only monostatic radars where the same antenna is used for transmitting and receiving. Bistatic radars are transmit and receive antenna systems located at different positions.



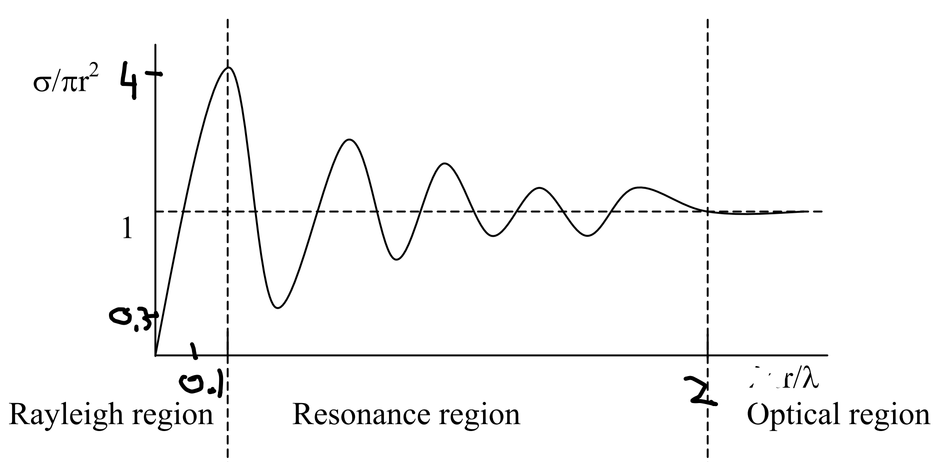
The total power delivered to the signal processor by a receiver antenna with aperture A_{eff} is similar to the monostatic radar with the difference that we now distinguish between R_t and R_r and G_t and G_r. With σ_b as the bistatic radar cross section (RCS) we get

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma_b}{(4\pi)^3 R_t^{\,2} R_r^{\,2}}$$

If we include the losses of the transmitter, receiver and the medium, L_t, L_r, L_p, respectively, we get

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma_b}{(4\pi)^3 R_t^{\,2} R_r^{\,2} L_t L_r L_p}$$

If the target has a size comparable to the wavelength, then the cross section gets quite complicated. Here is the functional dependence.



5.5 CW radar FM

Please refer to the figure at the end of this section. In order to avoid interruption in the continuous radar energy emission, two antennas are used in CW radar, one for transmission and one for reception. If the CW radar is frequency modulated (FM) then both range and Doppler information can be measured. As an example we want to discuss the linear frequency modulation (LFM) CW radar. The hand-out sheet shows two sketches, the upper one for a stationary target and the lower one for a moving target. Let's first discuss the case of a stationary target.

Our example is ffor a triangular LFM waveform. In other words, the carrier frequency, f_0, is modulated in such a way that it is increased linearly to $f_0+\Delta f$ in the time interval 0 to t_0 and subsequently decreased linearly to f_0 in the time interval t_0 to $2t_0$. This pattern of modulation is repeated with a modulation frequency

$$f_m = \frac{1}{2t_0}$$

The rate of frequency change is given as

$$\dot{f} = \frac{\Delta f}{f_0} = \frac{\Delta f}{1/2f_m} = 2f_m\Delta f$$

with a positive or negative sign depending on which side of the triangle we are. After a time, Δt, the reflected signal is received. If we mix the transmitted signal with the received signal and use a LPF at the output of the mixer, we get a signal with the difference of the frequencies at any given time. The difference of the frequencies is the beat frequency, f_b. For most of the time, f_b is given by

$$f_b = \Delta t\,\dot{f} = \frac{2R}{c}\dot{f}$$

When we combine this equation with the previous one we get

$$f_b = \frac{4Rf_m\Delta f}{c}$$

$$R = \frac{cf_b}{4f_m\Delta f}$$

The lower sketch of the figure describes the case where the target is moving. The received signal will therefore contain a Doppler shift term in addition to the frequency shift term. If we denote with f_{bu} and f_{bd} the beat frequencies during the positive and negative slope portions of the triangle, then we get

$$f_{bu} = \frac{2R}{c}\dot{f} - \frac{2\dot{R}}{\lambda}$$ ← Doppler term δf

$$f_{bd} = \frac{2R}{c}\dot{f} + \frac{2\dot{R}}{\lambda}$$

$$\frac{\delta f}{f} = \frac{v}{c} = \frac{v}{\lambda \cdot f}$$

$$\delta f = \frac{v}{\lambda}$$

$$\delta f = \frac{2\dot{R}}{\lambda}$$ for reflection

Then we get the range by adding the equations and the range rate by subtracting the equations.

$$R = \frac{c}{4\dot{f}}(f_{bu} + f_{bd})$$

$$\dot{R} = \frac{\lambda}{4}(f_{bd} - f_{bu})$$

Example;
A CW radar uses LFM to determine range and range rate of a target. The radar wavelength, λ, is 3 cm, the frequency sweep is Δf=200 kHz and t_0 = 20 ms. Compute f_{bu} and f_{bd} corresponding to a target at range R = 350 km which is approaching the radar with a radial velocity of 250 km/s.

Solution:

$f_m = 1/0.04 = 25$ Hz
$df/dt = 2\ \ 25\ \ 200\ 10^3 = 10^7$ Hz/s

➔ f_{bu} = 6.667 kHz
➔ f_{bd} = 40 kHz