

2. Neutron Stars

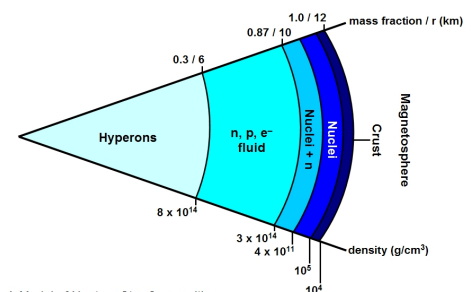
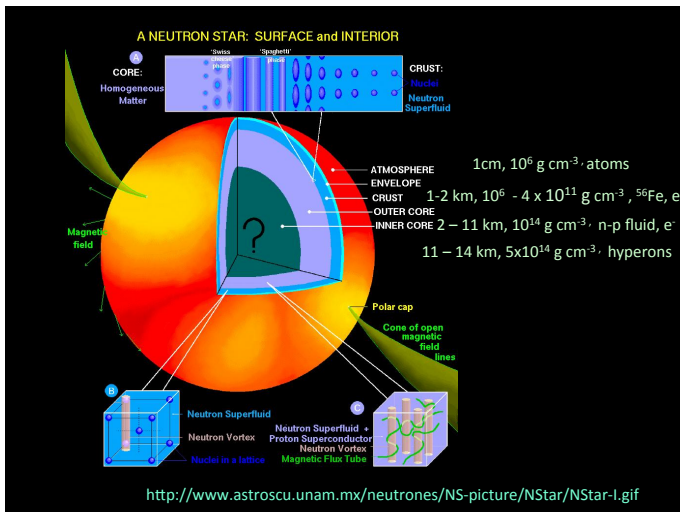
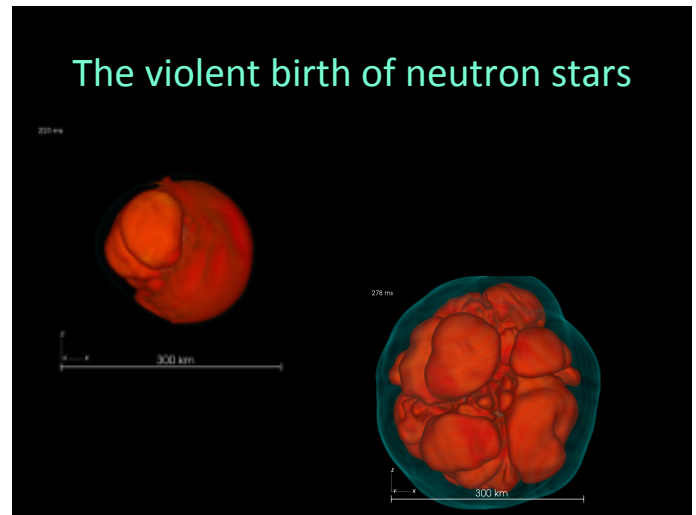
Neutron stars were first predicted in 1934 by Baade and Zwicky as the end stage of a core-collapse supernova explosion of a massive star. The electron capture process during core collapse $p+e \rightarrow n+\nu_e$ gives an indication of the large amount of neutrons produced in the center. The detection of neutrinos from SN 1987A with total energy of $\sim 10^{53}$ erg confirms this process. The core-collapse is stopped when the density exceeds the density of atomic nuclei of $\sim 3 \times 10^{14}$ g cm⁻³.

• 2.1 Structure and magnetosphere of neutron stars

Oppenheimer and Volkoff solved the general relativistic equation of the neutron gas to get the structure of the neutron star. As white dwarfs are in balance with gravity through electron degeneracy pressure, neutron stars are in balance with gravity through neutron degeneracy pressure.

How does that work?

$\Delta p \Delta x \geq \hbar/2\pi$: That means squeezing the neutrons to smaller space increases the momentum range and thus the energy (Fermi energy) range and with it the range in pressure (neutron degeneracy pressure).



A Model of Neutron Star Composition

(Values from - Astrophysics I: Stars, Bowers & Deeming, 1984)

Neutron stars have radii of about 10 to 15 km, and masses of about 1 to 2 solar masses. At their interior density can reach up to 10-15 times *nuclear density at saturation*, $\rho_0 = 2.8 \times 10^{14}$ gr cm⁻³. At densities of about $1/2 \rho_0$, protons and neutrons are no longer bound to nuclei and they form a non-ideal liquid. The *equation of state*, the relation between pressure and energy density at the interior of the neutron star, and the composition of nuclear matter at densities above $\sim 2 \rho_0$ are rather uncertain. For densities below ρ_0 matter consists mostly of *nucleons* (protons and neutrons) plus electrons and muons. Since the chemical potential of electrons and neutrons increase with density, heavier particles, e.g. *hyperons* (hadrons having a strange quark), can in principle be produced when density increases toward the neutron star centre. At high densities, π and K mesons (*pions* and *kaons*; a quark-antiquark pair) may also appear; the core of the neutron star could then consist of pion or kaon Bose-Einstein condensates. At extreme densities it may be energetically more favourable to have a fluid made of deconfined *up* (u), *down* (d) and *strange* (s) *quarks*.

- 2.2 Neutron star mass-radius relation for equations of state
- Equation of state: how is **pressure** related to **energy density** (and in principle also to the composition and the temperature)?
- For a non-interacting ideal Fermi gas:
- $R_{NS} = 10$ km
- $M_{NS} = 0.7 M_{sol}$
- (Tolman, Oppenheimer and Volkoff, 1939)

Equations of state with interactions between nucleons

In a neutron star the typical distance between neutrons is 10^{-13} cm = 1fm. With a mass of $1.4 M_{\text{sol}}$ we have $1.7 \cdot 10^{57}$ nucleons (neutrons mostly). With $R_{\text{NS}}=12$ km, distance is ~ 1 fm. The nucleons have a radius of 0.8 fm. \rightarrow nucleons are basically touching each other. Interaction (strong force, weak force) have to be taken into account.

$$GR: R > R_g = \frac{2GM}{c^2}$$

$$\text{Causality: } R < 3 \frac{GM}{c^2}$$

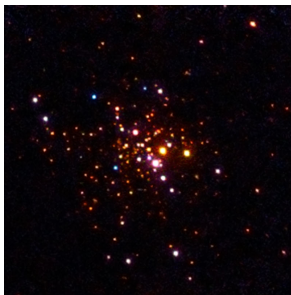
$$\text{Rotation: } R_{\text{max}}$$

$$F = G \frac{Mm}{R^2} = \frac{Mv^2}{R} = M\omega^2 R$$

$$\omega^2 = G \frac{M}{R^3}$$

$$P_c = \frac{2\pi}{\omega_c}$$

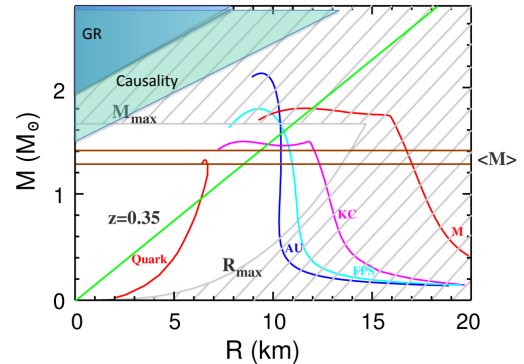
$$P_c = 2\pi \sqrt{\frac{R^3}{GM}} = 0.461 \text{ms} \left(\frac{R}{10 \text{km}} \right)^{\frac{3}{2}} \left(\frac{1.4 M_{\text{sol}}}{M} \right)^{\frac{1}{2}}$$



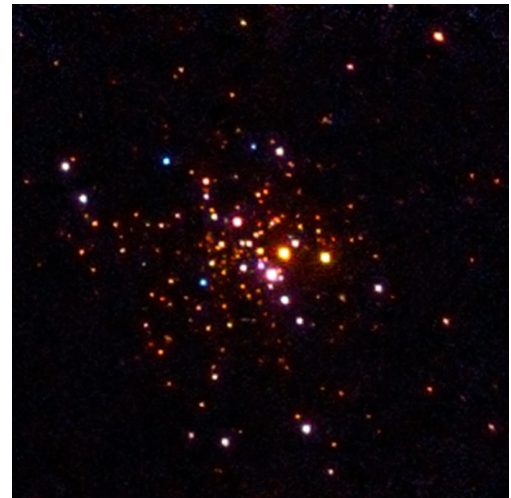
The researchers used a wide range of different models for the structure of these collapsed objects and determined that the radius of a neutron star with a mass that is 1.4 times the mass of the Sun is between 10.4 and 12.9 km (6.5 to 8.0 miles). They also estimated the density at the center of a neutron star was about 8 times that of nuclear matter found in Earth-like conditions. This translates into a pressure that is over ten trillion trillion times the

New results from Chandra and other X-ray telescopes have provided one of the most reliable determinations yet of the relation between the radius of a neutron star and its mass. Neutron stars, the ultra-dense cores left behind after massive stars collapse, contain the densest matter known in the Universe outside of a black hole. This image contains data from a long Chandra observation of 47 Tucanae, a globular cluster where one of the eight neutron stars in the study is found. Lower-energy X-rays are red, those with intermediate energies are green, and the highest-energy X-rays are shown in blue. (Credit: NASA/CXC/Michigan State/A. Steiner et al.)

Mass-radius relation for neutron stars with different equations of state



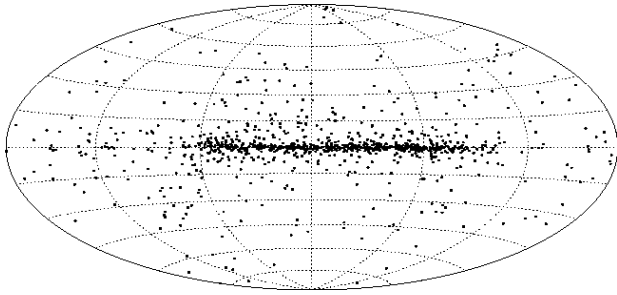
<http://www.rug.nl/research/kapteyn/onderzoek/areas/he-compact>



2.3 Characteristics of pulsars

- 1967: Observational discovery of neutron stars in the form of pulsars. Discovered through a project to find compact objects like quasars through interplanetary scintillation. PhD student J. Bell found series of regular pulses, $P=1.3$ s. Origin: outside solar system. Little Green Man? No! Also, three more "pulsars" were discovered.
- 1968: Gould and Pacini: Pulsars are highly magnetized neutron stars.
- 1974: Hewish was awarded the Nobel Prize

Location in the Galaxy of 1026 pulsars (galactic coordinates)



Lorimer 2005

Distance to pulsars

The distance, D , to pulsars can be obtained through

- parallax observations
- measurement of pulse dispersion

$$\Delta t = 4150 \left(\frac{1}{v_{low}^2} - \frac{1}{v_{high}^2} \right) DM$$

$$DM = \int_0^D n_e(l) dl$$

- association with objects of known distance

Spin period

- All stars rotate. When massive stars collapse to form neutron stars, conservation of angular momentum, L , with I being the moment of inertia and ω the angular rotation frequency, leads to an increase in spin and a shortening of the rotation period, P , of the collapse product, the neutron star. Consider an inner core with $r=20,000$ km and $M=1M_{sol}$ and $P=30$ d, undergoing core-collapse to $r=10$ km. Then P would decrease to $P=0.65$ s.

$$L = I\omega = I \frac{2\pi}{P}$$

$$I = \frac{2Mr^2}{5}$$

$$L = \frac{4}{5}\pi M \frac{r^2}{P}$$

Period derivative and energy loss

- The rotational kinetic energy of a neutron star with moment of inertia, I , for a solid sphere of radius, r , mass, M , and angular frequency, ω , is: $E_{rot} = \frac{1}{2} I \omega^2$
- For a Chandrasekhar mass and $P=0.033$ s (Crab pulsar), we get:

$$\omega = \frac{2\pi}{P}$$

$$I = \frac{2}{5} Mr^2 = \frac{2 \cdot 1.4 \cdot 2 \cdot 10^{33} \text{ g} \cdot (10^6 \text{ cm})^2}{5} = 10^{45} \text{ g cm}^2$$

$$E_{rot} = \frac{2\pi^2}{P^2} I = \frac{2\pi^2 \cdot 10^{45} \text{ g cm}^2}{(0.033\text{s})^2} = 1.8 \cdot 10^{49} \text{ erg}$$

- The rotational energy is decreasing by:

$$\dot{E}_{rot} = I\omega\dot{\omega}$$

$$\dot{\omega} = 2\pi(-P^{-2}\dot{P})$$

$$\dot{E}_{rot} = I \frac{2\pi}{P} \frac{2\pi(-\dot{P})}{P^2} = -\frac{4\pi^2 I \dot{P}}{P^3}$$

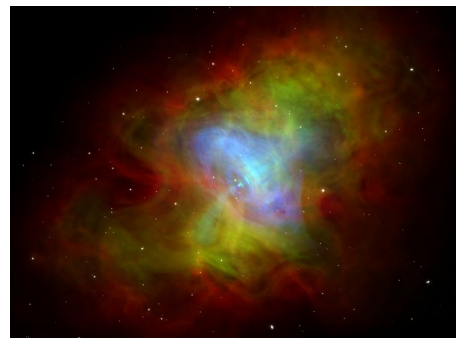
$$P = 0.033\text{s}$$

$$\dot{P} = 10^{-12.4} \text{ s s}^{-1}$$

$$\dot{E}_{rot} = -4 \cdot 10^{38} \text{ erg} \cdot \text{s}^{-1} = 10^5 L_{sol}$$

- This is comparable to the entire radio output of the Galaxy. It powers with its 30 Hz radiation the Crab Nebula.

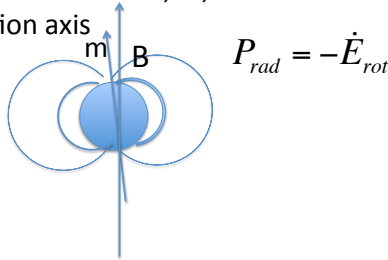
Crab Nebula



Credit: J. Hester (ASU), CXC, HST, NRAO, NSF, NASA

Magnetic field

- We can estimate the magnetic field, B, with the assumption that the pulsar loses energy through magnetic dipole radiation, P_{rad} , with the magnetic moment, m, axis inclined w.r.t. the rotation axis



$$P_{rad} = \frac{2}{3} \frac{(\ddot{m})^2}{c^3}$$

$$m = Br^3$$

$$\dot{m} = -i\omega m_0 e^{-i\omega t}$$

$$\ddot{m} = \omega^2 m_0 e^{-i\omega t} = \omega^2 m$$

$$P_{rad} = \frac{2}{3} \frac{m_0^2 \omega^4}{c^3} = \frac{2}{3c^3} (Br^3 \sin \alpha)^2 \left(\frac{2\pi}{P}\right)^4$$

$$B \sin \alpha = \sqrt{\frac{3c^3 I}{8\pi^2 r^6} P \dot{P}}$$

$$B \approx 3.2 \cdot 10^{19} G \left(\frac{P \dot{P}}{1s}\right)^{1/2}$$

With $P=0.033$ s and $P_{dot} = 4.2 \times 10^{-13} \text{ s s}^{-1}$, $B_{crab} = 4 \times 10^{12} \text{ G}$.

Also, if a typical star with $B=100 \text{ G}$ collapses from 10^6 km to 10 km → $B=10^{12} \text{ G}$. The energy density is $B^2/8\pi = 4 \times 10^{22} \text{ erg/cm}^3 = 4 \times 10^{15} \text{ J/cm}^3$!

Lifetime

$$B \sin \alpha = \sqrt{\frac{3c^3 I}{8\pi^2 r^6} P \dot{P}}$$

$$P \dot{P} = \frac{8\pi^2 r^6 (B \sin \alpha)^2}{3c^3 I} = \text{const}$$

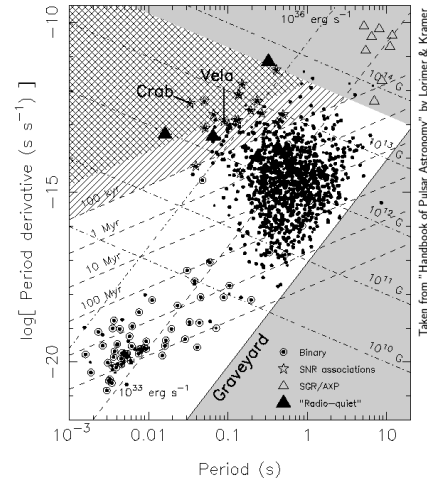
$$P dP = P \dot{P} dt$$

$$\int_{P_0}^P P dP = \int_0^\tau (P \dot{P}) dt$$

$$\frac{P^2 - P_0^2}{2} = P \dot{P} \tau$$

$$P_0^2 \ll P^2$$

$$\tau = \frac{P}{2\dot{P}}$$



P - P_{dot} explained through magnetic dipole model

- Young pulsars have strong B
- Young pulsars have short P
- B decays with time
- Pulsars lose energy through magnetic dipole radiation → P_{dot} first large then getting smaller:

$$B \propto \sqrt{P \dot{P}}$$

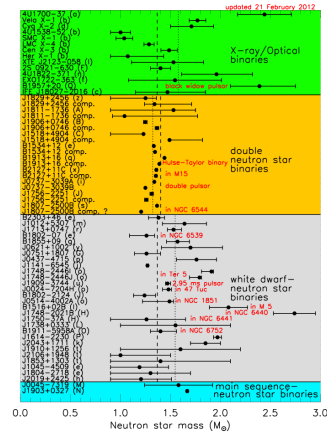
- Millisecond pulsars are old recycled pulsars mostly in binary systems. Accretion from companion spun them up to $P \sim 1.5$ to 30 ms.
- They have small B and small \dot{P}

Velocities

- Pulsars have usually high velocities up to 1000 km s^{-1} . Progenitors' velocities are smaller, typically 10 to 50 km s^{-1} apart from systemic velocity around the galactic center.
- The high velocities come from slight asymmetric explosions which give the neutron star a kick at birth. This high velocity allows some pulsars to gain high galactic latitudes. 50% of neutron stars may end up in intergalactic space.

Masses

- Masses can be determined for pulsars in binary systems – 4% of all pulsars vs. 50% of stars are in binary systems.
- With Newtonian physics the total mass can be determined
- With GR the mass of individual members of the binary system can be determined
- Mass of PSR 1913+16 = $1.4414 \pm 0.0002 M_{\text{sol}}$



Lattimer 2012

$$\langle M_{\text{NS}} \rangle = 1.35 \pm 0.04 M_{\text{sol}}$$

Thorsett and Chakrabarty 1999

Pulse characteristics

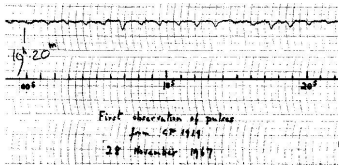
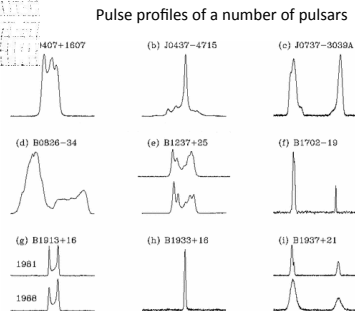
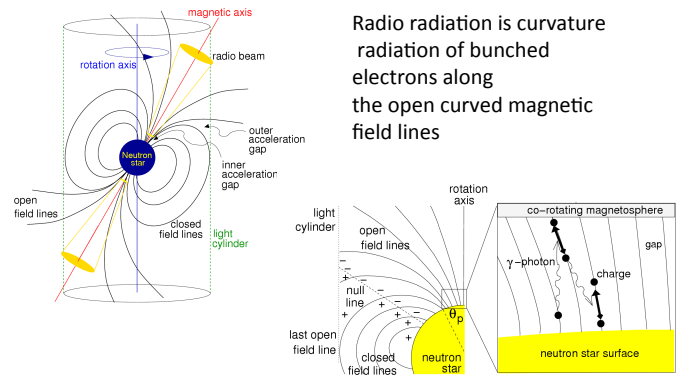


Chart recording of first pulsar pulses



Pulse profiles of a number of pulsars

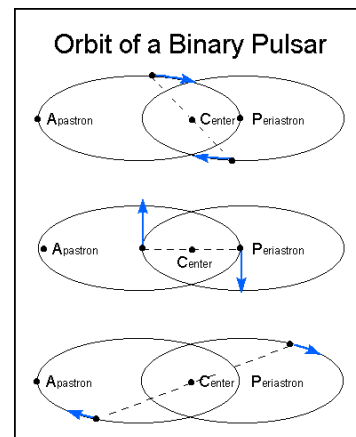
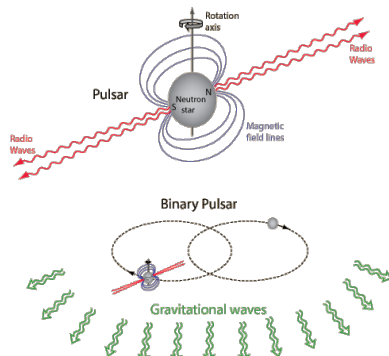
Pulsar emission mechanism



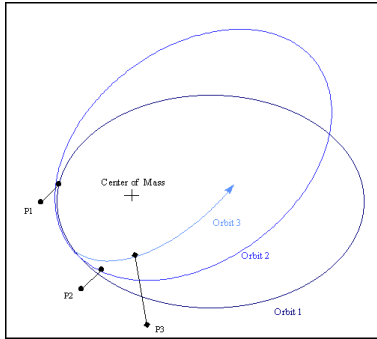
Radio radiation is curvature radiation of bunched electrons along the open curved magnetic field lines

2.4 Pulsars as clocks for tests of general relativity

Changes of the pulsed signal of a binary pulsar can give information on the changes of the orbital ellipse of the pulsar and the curved space-time in the binary system.



Advance of periastron



Three effects measured to test GR

- Periastron advance
- $dP_{orb}/dt \rightarrow$ Gravitational wave emission
- Shapiro delay of pulses in curved space

➔ GR confirmed at 10^{-4} level.

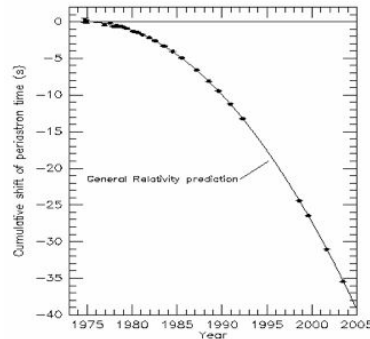
PSR 1913+16

$P_{orb} = 7.75h$, decreases by 1 s per 14,000 yr.

$a_{proj} = 7 \times 10^5$ km. $P = 87$ ms.

Orbital ellipse rotates by 4 deg/yr

1.5 Mill km orbit shrinks by 3m/yr.



Weisberg, Taylor 2004

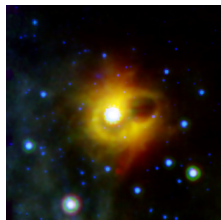
Taylor and Hulse: Nobel Prize, 1993

J0348+0432

- Neutron star – White dwarf binary
- $P = 39$ ms, $P_{orb} = 2.5$ h
- Taylor Hulse experiment is being repeated with new binary system

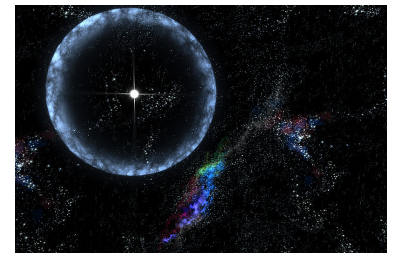
Magnetars

- A neutron star with a magnetic field, $B \sim 10^{15}G$.
- Only ~ 20 known
- Originates presumably in a core collapse SN when a dynamo effect converts heat and rotational energy into additional B increase.
- Strong B \rightarrow strong x-ray pulses
- strong gamma-rays pulses
- P: 1 to 10s
- After 10,000 yrs, B decays
- Then no X and gamma ray pulses
- Pulses can have effect on our atmosphere



SGR 1806-20

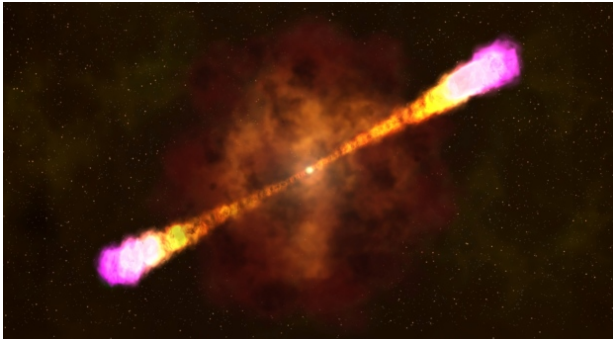
- 27 December 2004 burst of gamma rays from 50,000 ly away passed through solar system and had an effect on our atmosphere.



Artist's rendition

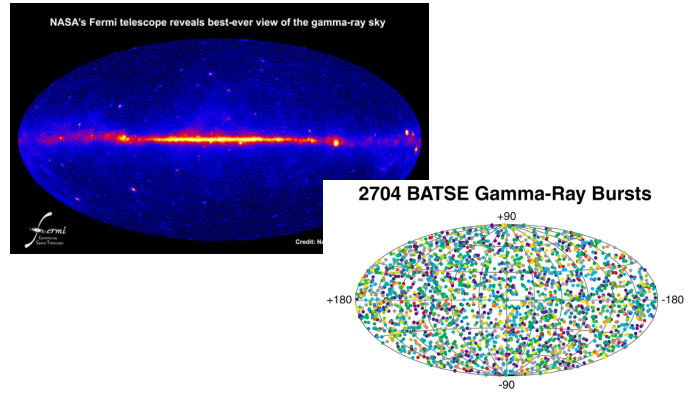
Gamma-ray bursts

the most luminous explosions in the universe



Artist's expression, NASA

The gamma ray sky --in continuum and pulsed

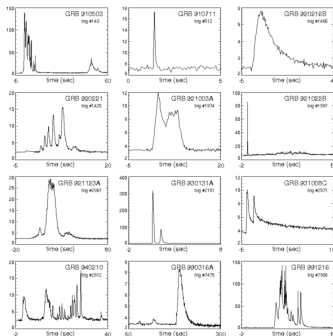


GRBs

- Discovered in the 1960's by US military satellites on the look-out for Soviet nuclear bomb testing in the atmosphere.
- Occurrence: $\sim 1/d$
- $E \sim 10^{44} \text{ J} = 10^{51} \text{ erg} = 200 M_{\text{earth}} c^2$ (beamed)
- Location: Galaxies billions of pc away from us.
- Energy is pummelled out at the north and south poles to form relativistic jets. Jets interact internally and externally. When pointing into our direction, we see GRB.

- Long GRBs: duration $>2s$ to few min. followed by afterglow at lower frequencies. Linked to galaxies with rapid star formation. Some are associated to SN Ib/c.
- Short GRBs: duration $\sim 0.2s, <2s$, afterglow recently detected in some. Linked to galaxies with low or no star formation. NS-NS or NS-BH collision.

GRB profiles



- If GRB occurred in our Galaxy and beamed to us, atmosphere could be destroyed.
- 1 GRB/ 10^6 yr. 1% beamed toward Earth?
- \rightarrow Extinction of species in earth's past through GRBs?