ASSIGNMENT ONE

Proportion and Scaling of a 10-fold taller human

If a human being were 10 times taller than normal (and all other proportions were increased by the same amount), would the human be able to bear its weight? Or would the proportions have to change, and if so, how and to what extent?

Hints

• I suspect that the ability of the leg bones to support the additional weight would be a limiting factor. I also wonder whether the neck bones would be able to bear the weight of the head.

• Simplifying the geometry (to either cylinders or rectangles) may be useful in your analyses.

• 'Real' measurements of the compressive strength of bone may be useful; hopefully such measurements are available on the web.

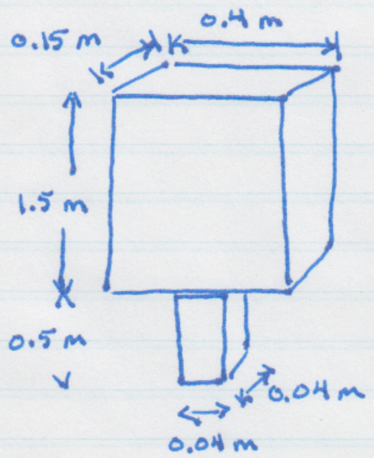
• Please try and avoid invoking abnormal geometries like a cone. Instead, try and stay as close as you can to a 'human' shape.

Guidelines

I expect that students may wish to work together on the assignment, that is fine, but, be sure that your assignment is in your own words. Remember that you have to explain your answers with sufficient clarity, so that a non-physicist like Dr. Lew will understand them. He often finds diagrams helpful and is obsessed with ensuring that the units work, so showing the units is obligatory. Excessive length is not encouraged.

Assignment One Key (3 october 2009)

For the sake of simplicity, I've modeled a human as a rectangular structure on a narrow rectangular support.



VOLUME: $(0.15\ m)(0.4\ m)(1.5\ m)$ ("torso")
$+\ (0.04\ m)(0.04\ m)(0.5\ m)$ ("leg")
$0.0908\ m^3$

mass: assume a density of $1\ g/cm^3$
or $(10^2\ cm)^3 = (1\ m)^3$ $10^6\ cm^3 = 1\ m^3$
$\longrightarrow$ density of $10^3\ kg/m^3$

$0.0908\ m^3 \cdot 10^3 \frac{kg}{m^3} = 90.8\ kg$

(internal check: 91 kg is in the range of human weights)

The force ($F = m \cdot a$, where a is the gravitational acceleration $9.807\ N/kg$)

$F = 90.8\ kg \cdot 9.807\ N/kg = 890.5\ N$

The static load on the "leg" is N/m^2

AREA: $(0.04\ m)(0.04\ m)$
$= 0.0016\ m^2$

$\frac{890.5\ N}{0.0016\ m^2} = 556.6 \cdot 10^3\ N/m^2$

The compressive failure point for bone is ~ 200 MPa
($\times 10^6\ N/m^2$)

So, the "safety" factor is:
$\frac{200 \times 10^6\ N/m^2}{556.6 \times 10^3\ N/m^2} = 360$

(very high)

Now, we "grow" the human 10-fold

VOLUME: $(1.5\ m)(4\ m)(15\ m)$
$+ \ (0.4\ m)(0.4\ m)(5\ m)$ (10³-fold greater)

$90.8\ m^3 \longrightarrow$ times $10^3 \frac{kg}{m^3} = 90.8 \times 10^3\ kg$

times $9.807\ N/kg$

$890.5 \times 10^3\ N$

Static load on the "leg"

AREA: $(0.4\ m)(0.4\ m) = 0.16\ m^2$ divide by area

The force is: $5.566 \times 10^6\ N/m^2$

Our safety factor is $\frac{200 \times 10^6\ N/m^2}{5.566 \times 10^6\ N/m^2} = 36$

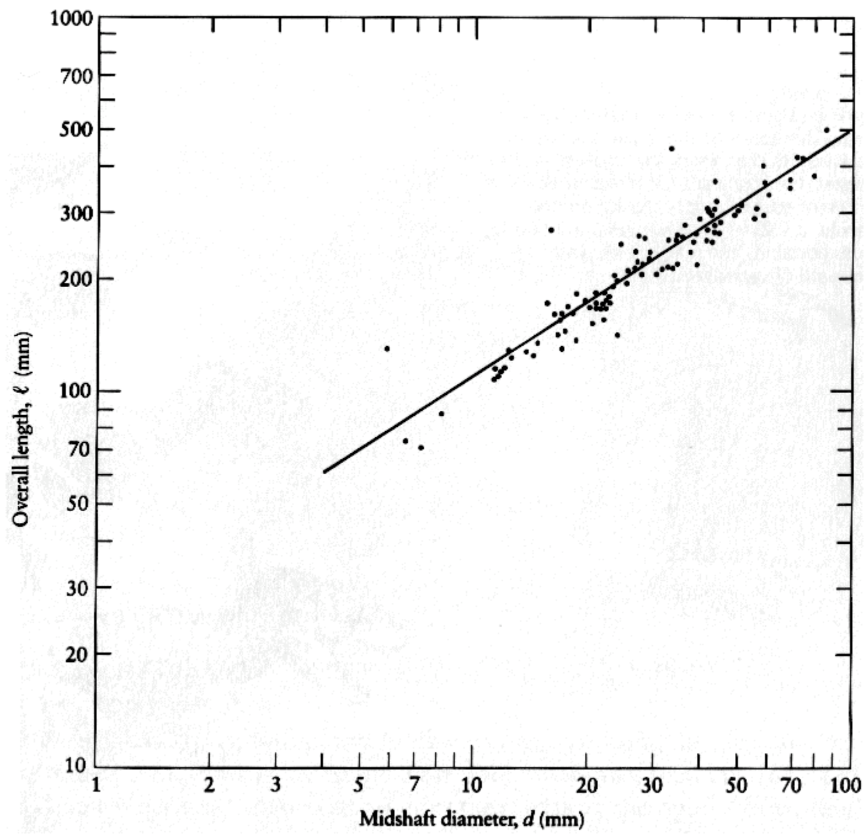
Safety has declined 10-fold

To maintain the 360 "safety" factor, the area needs to be increased 10-fold, from $0.16\ m^2$ to $1.6\ m^2$, or, leg dimensions of $(1.6\ m^2)^{0.5} \longrightarrow 1.26\ m \times 1.26\ m$

That is, if dimensions are increased 10-fold, the structural support dimensions must be increased 30-fold (ca.) $\frac{1.26}{0.04} = 31.5$

Note that $10 = 30^{0.66}$ (see next page)

The best example of mass/bone diameter scaling I could find was a comparison of humerus bone length (which is probably a reasonable estimator of height) for antelopes, ranging from 3 kg to 750 kg. This comes from McMahon and Bonner (1983) *On Size and Life* published by the Scientific American Books (pp. 125). The scaling of humerus length (l, y-axis) *versus* bone diameter (d, x-axis) is $l = 24.09 \bullet d^{0.66}$. This is reminiscent of the area *versus* mass scaling of a cube (area is proportional to mass$^{0.66}$).



There do seem to be consistent relationships between various metrics of biological scaling in the context of *size and proportion*.

Of course, biomechanical considerations are far more complex, demanding a more refined analysis of structural components that limit the physical strength of the organism, something that can only be revealed by experimental data on mechanisms of structural failure.