ASSIGNMENT ONE (TWO QUESTIONS)
deadline: 30 September 4:30 pm at 229 Farquharson

Question One. Biological Scaling within the mammalian clade.
How large would a human have to be to boil (that is, achieve an internal temperature of 100º Celsius)?

Hints

• It has something to do with joules sec^{-1} (Watts) (metabolic heat generation) and radiant energy (heat loss by radiation).

• Simplifying the geometry (to either cylinders, rectangles or even squares) may be very helpful.

• Please ignore convective and evaporative heat loss (they will introduce significant complications).

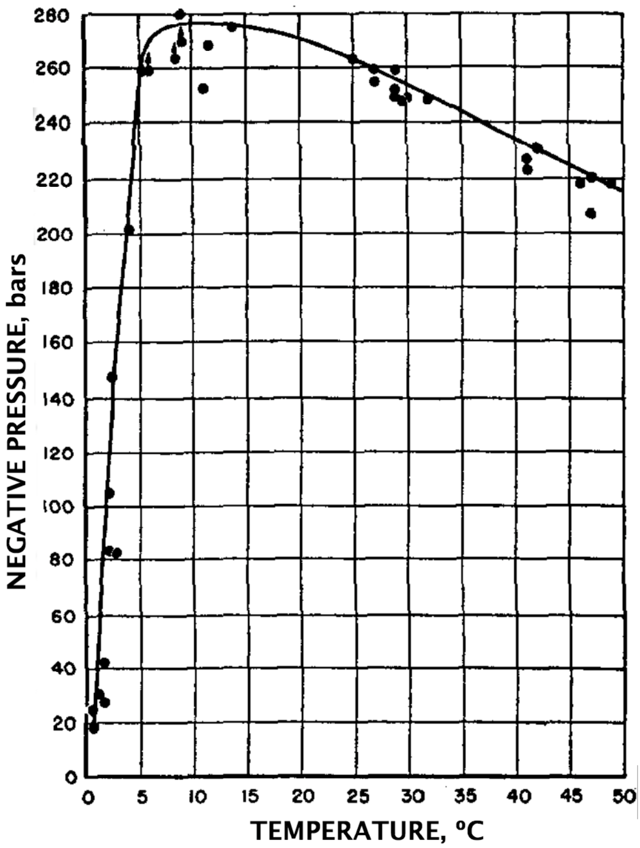
• You may have to consider issues like heat capacity and conductivity, use values for water (which is similar enough to human tissue).

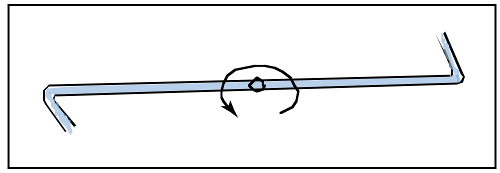
• It will be simpler to ignore heat transfer via the cardiovascular system (and breathing), although blood flow does affect heat mixing — which you may want to invoke.

Guidelines
I expect that students may wish to work together on the assignment, that is fine, but be sure that your assignment is in your own words. Remember that you have to explain your answers with sufficient clarity, so that a non-physicist like Dr. Lew will understand them. He often finds diagrams helpful and is obsessed with ensuring that the units work, so showing the units is obligatory. Excessive length is not encouraged.

Question Two. Tensile strength of water.

One technique used to measure the tensile strength of water was a Z-shaped tube mounted on a rotating plate.





The force required to 'break' the water column at the center can be calculated from the angular velocity, radius, water density, and the area of the tube lumen. The data are shown in the figure. Maximal tensile strength is about 280 bars, equivalent to 28 MPa. Source: Briggs, LJ (1950) Limiting negative pressure of water. Journal of Applied Physics 21:721–722.

- Provide a formal derivation of the equation used to calculate the tensile strength.
- Predict the effect of doubling the lumen (the inside of the tube) diameter.
- Explain why the tube is Z-shaped (Briggs used an angle of 140º).
- Provide an explanation(s) for the peculiar temperature dependence of the tensile strength of water.
- Predict the tensile strength of another liquid (the molecular weight should be similar to water) — *with a rationale*.

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> $\sigma := 5.670\cdot 10^{-8}$: $\epsilon := 1$:

> $P_{radiant(cube)} = \sigma\cdot\epsilon\cdot(6\cdot a^2)\cdot(373^4 - 300^4)$

$$P_{radiant(cube)} = 3829.590114\, a^2 \quad (1)$$

> $P_{radiant(rectangle)} = \sigma\cdot\epsilon\cdot(2\cdot 1.6^2 + 1\cdot 0.15^2 + 2\cdot a^2)\cdot(373^4 - 300^4)$

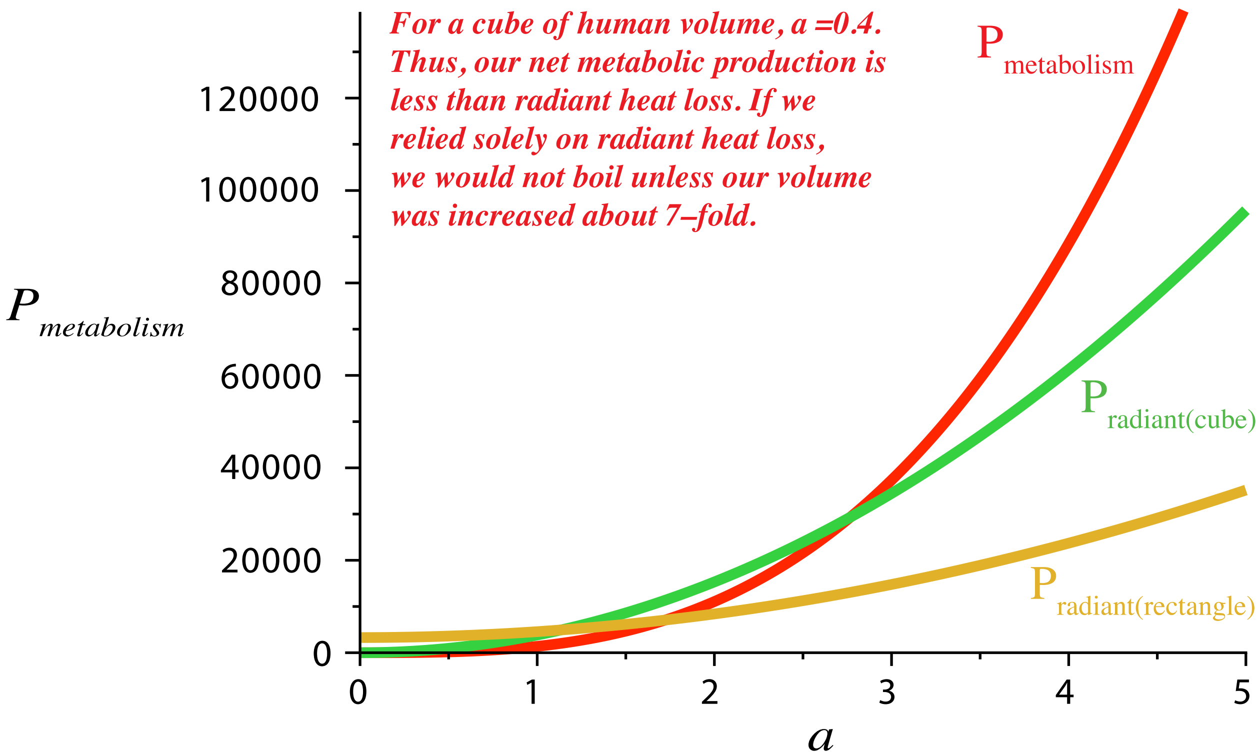
$$P_{radiant(rectangle)} = 3282.277860 + 1276.530038\, a^2 \quad (2)$$

A rectangle is reasonably close to a match for human radiative area (size and shape).

> $P_{metabolism} = \left(\frac{2000\cdot 1000\cdot 4.184}{(60\cdot 60\cdot 24)(70)}\right)\cdot(1000\cdot a^3)$

$$P_{metabolism} = 1383.6\; a^{3.00} \quad (3)$$

> smartplot$\left(P_{metabolism} = 1383.597884\, a^3, P_{radiant(cube)} = 3829.590114\, a^2, P_{radiant(rectangle)} = 3282.277860 + 1276.530038\, a^2\right)$



> $area = 2\cdot 1.6^2 + 2\cdot 0.15^2 + 2\cdot 0.3^2$

$$area = 5.3450 \quad (4)$$

> $volume = 1.6\cdot 0.15\cdot 0.30$

$$volume = 0.07200 \quad (5)$$

> $a = \sqrt[3]{0.072}$

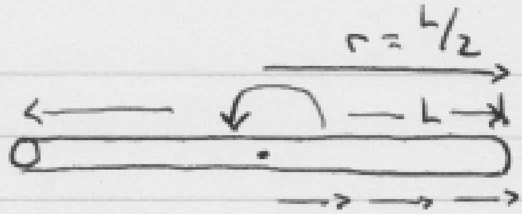
$$a = 0.4160167646 \quad (6)$$

> $volume(human) = 72\cdot 0.001$

$$volume(human) = 0.072 \quad (7)$$

>

(10/10)



The force varies with distance from the center: we have to add them up.

The force:

$$F = a \rho \omega^2 x$$

- ρ → density
- a → area
- x → distance
- ω → angular velocity

note that $a \rho x$ = <u>mass</u>

we need to consider increments of distance, dx : $dF = a\rho\omega^2 x\,dx$

Now, we can 'sum' by integration

$$\int_{x=0}^{x=r\ (=L/2)} a\rho\omega^2 x\,dx \qquad a\rho\omega^2 \underbrace{\int_{x=0}^{x=r} x\,dx}_{\frac{1}{2}x^2}$$

$$a\rho\omega^2 \Big[\tfrac{1}{2}r^2 - \tfrac{1}{2}0^2 \Big]_{x=0}^{x=r} = a\rho\omega^2 \tfrac{1}{2} r^2$$

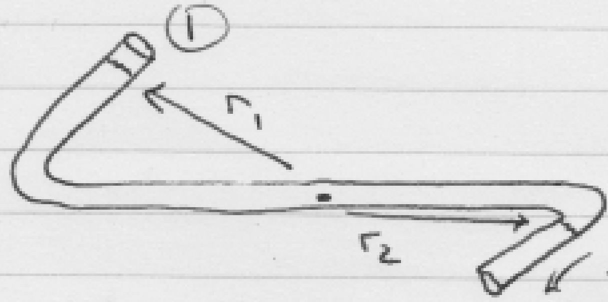
for force/area divide by a

$$= \tfrac{1}{2}\rho\omega^2 r^2 \qquad \text{dynes cm}^{-2} \text{ or Newtons m}^{-2} \qquad (2/10)$$

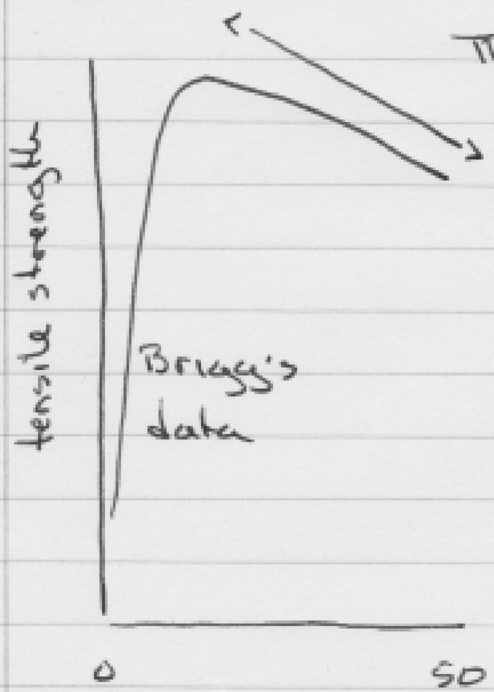
If we double the lumen, mass increases more than the surface area, the water column will break at a slower ω. (2/10)

With a straight tube, a slight difference in distance from the center will create a force imbalance and the water will "fly" out

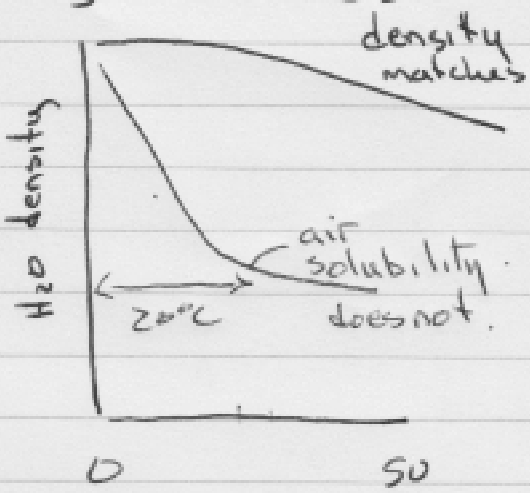
With bent ends



The shorter distance (r_1) will exert less force. Thus the water column in ② will move towards the end of the tube until $r_1' = r_2$ & the forces balance. (2/10)



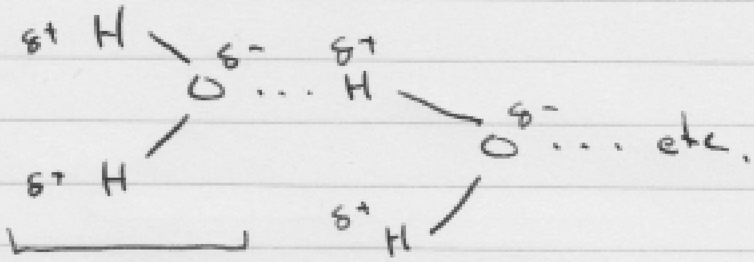
The decline in tensile strength matches the T-dependence of density surprisingly well



The strange behaviour from 0-4 C can't be a "mysterious" change in hydrogen bonding - a decline in H-bonding only occurs upon ice nucleation at 0 °C. (2/10)

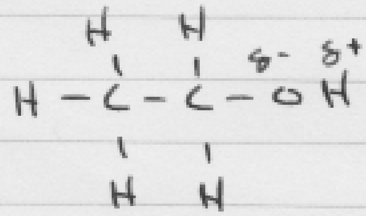
Some think Briggs's data is wrong.

Probably the major factor contributing to the 'cohesive' strength of water in hydrogen bonding.



note how the dipole spans the molecule

Some molecules (like methanol & ethanol) have a dipole that does not span the molecule



and thus will not contribute effectively to inter-molecular H-bonding

Others could:

For example, $\overset{\delta+}{H} \cdot \overset{\delta-}{F}$, but HF will dissociate into H^+ & F^-

(2/10)