

## ASSIGNMENT ONE (deadline: 28 September 4:30 pm at 229 Farquharson)

A biophysicist colleague viewed the diagram of the physical approaches that impact on the height of a tree (right) and asked the following:

"One key word that seemed missing here to me was \*osmosis\*. Wouldn't that be playing a significant role here?"

From my point of view, this is a popular misconception, but I haven't explored it formally. So, the assignment is to ask you to use the analytical tools of physics to test the idea that osmosis could be the force causing water transport from the soil to the top of a 100-meter tree. There are two aspects of this:

### How High Can a Tree Grow?

#### Biological Problem

Evolution and adaptation  
 Colonization of land  
 Competition for light  
 Physical aspects  
 Pumping water  
 Water piping  
 Structural support

#### Biological Structure

Leaves  
 Photosynthesis  
 Woody stem  
 Xylem vessels  
 Columnar structure  
 Woody roots  
 Water uptake  
 Structure foundation



#### Physical Approach

Evaporative pump  
 (thermodynamics)  

$$\Psi_{wv} = 135 \times \ln \frac{\%RH}{100}$$

Poiseuille flow  
 (fluid dynamics)  

$$J_v = \left( \frac{\Delta P}{l} \right) \left( \frac{\pi}{8\eta} \right) R^4$$

Euler's column  
 (mechanics)  

$$F_{critical} = \frac{EI\pi^2}{L_{eff}^2}$$

Tensile strength of water  
 (condensed matter)

- One is whether osmosis could be used to create a high enough hydrostatic pressure. You can compare the required osmotic 'strength' with known values for cell osmolarity from the biological literature. The potential ( $\Psi$ ) of soil water is extraordinarily complex. For the purposes of this assignment, consider only the range of osmotic pressures, which vary from 0.01 to 0.2 MPa<sup>1</sup>.
- The other aspect is to account for water loss from the tree canopy. In other words, could osmosis be used to provide a flow rate similar to those known to occur in trees (say, 10 meters hour<sup>-1</sup>)?

In support of this assignment, you can refer to the course notes (and elsewhere) for pertinent equations. A helpful overview is available from *A Companion to Plant Physiology, Fifth Edition* by Lincoln Taiz and Eduardo Zeiger at the following website:

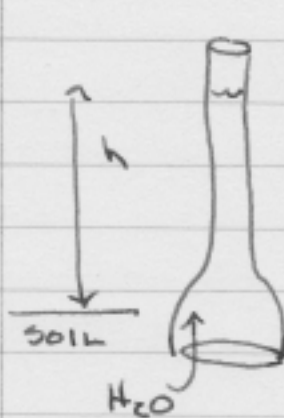
<http://5e.plantphys.net/categories.php?t=t>    Topic 3 — *Water and Plant Cells* and  
 Topic 4 — *Water Balance of Plants*

#### Guidelines

I expect that students may wish to work together on the assignment, that is fine, but be sure that your assignment is in your own words. Remember that you have to explain your answers with sufficient clarity, so that a non-physicist like Dr. Lew will understand them. He often finds diagrams helpful and is obsessed with ensuring that the units work, so showing the units is obligatory. Excessive length is not encouraged.

<sup>1</sup> Nobel PS (1991) *Physicochemical and Environmental Plant Physiology*. Academic Press. Page 497.

Could osmotic pressure be high enough to "push" water to the top of a tree



H<sub>2</sub>O density (1000 kg m<sup>-3</sup>)

$$P_{tree} = \rho g h$$

gravity (N/kg)

height (100 m)

The osmotic pressure is

$$P_{soil} = 0.01 \text{ to } 0.2 \text{ MPa}$$

$$RTc$$

concentration

gas constant  
(8.314 J mol<sup>-1</sup> K<sup>-1</sup>)

temperature  
(298 K)

Equating

$$\rho g h = RTc - P_{soil}$$

Solve for c

$$c = \frac{\rho g h + P_{soil}}{RT}$$

$$\frac{(1000 \text{ kg/m}^3)(9.8 \text{ N/kg})(100 \text{ m}) + 0.2 \times 10^6 \text{ Pa}}{(8.314 \text{ J mol}^{-1} \text{ K}^{-1})(298 \text{ K})}$$

$$c = 476 \text{ mol/m}^3$$

$$\frac{\text{mol}}{\text{m}^3} = c = 476 \text{ mol/m}^3 \text{ (or 476 mM)}$$

(equivalent to 1.18 MPa)

Measured cell pressures are about 0.65 MPa [1], so similar to required osmolarity

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[1] Shabala and Hew 2002. Plant Physiology 129:290-299

Could the osmotic pressure cause water to flow at a velocity of 10 m/h?

$$\text{velocity} = v = \left( \frac{\Delta P}{\eta} \right) \left( \frac{1}{4\pi} \right) (R^2) \quad \text{(xylem radius)}$$

$\xrightarrow{1.18 \times 10^6 \text{ Pa}}$        $\xrightarrow{100 \times 10^{-6} \text{ m}}$   
 $\xrightarrow{100 \text{ m}}$        $\xrightarrow{\text{viscosity } 0.001 \text{ Pa}\cdot\text{sec}}$

$$\frac{(1.18 \times 10^6 \text{ Pa})}{(100 \text{ m})} \frac{1}{4 (0.001 \text{ Pa}\cdot\text{sec})} (100 \times 10^{-6} \text{ m})^2$$

$$\frac{\text{m}}{\text{sec}} = v = 0.0295 \text{ m/sec}$$

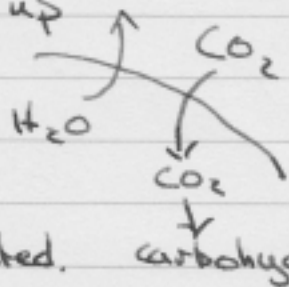
$$= 106 \text{ m/h} \quad \begin{matrix} 60 \text{ sec} & 60 \text{ min} \\ \text{min} & \text{hour} \end{matrix}$$

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The pressure would be more than sufficient to cause water to flow at a velocity of 10 m/h

So, what's wrong with this picture?

The problem is evaporation of water from the tree canopy. For photosynthesis, CO<sub>2</sub> must be taken up but water will leave.



When water leaves, osmotic pressure will increase as the solutes become more concentrated. But they will eventually saturate, and the system will plug.

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In the "real" biological world, xylem sap is very dilute to avoid saturating levels of solutes at the tree canopy.