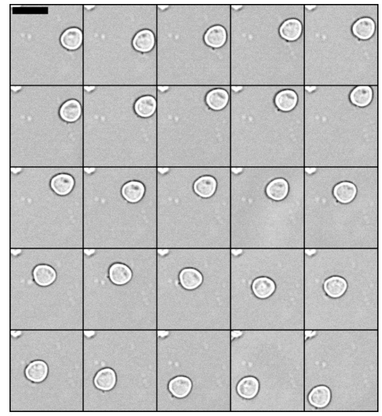
Name:____________________________ SC/BPHS2090 Final Exam (Lew)

Student ID:________________________

Be sure to write your name and student ID above. Read the four sub-questions carefully, think, then write your answers in the lined space (front and back of this page). When finished, please hand your answer in, separate from your exam booklet.



**QUESTION:** Einstein[1] noted that particles undergoing Brownian Motion have both translational and rotational Diffusion coefficients. That is, not only does the particle move within the media, but it also rotates. For both forms of diffusion, the time dependence is described similarly[2]. The root mean square distance ($\langle d^2 \rangle$) is a function of the square root of time:

$$\langle d^2 \rangle = 2Dt$$

As is the root mean square of angular displacement (that is rotation, in radians per second, $\langle \theta^2 \rangle$):

$$\langle \theta^2 \rangle = 2D_r t$$

where the translational diffusion coefficient is:

$$D = \frac{kT}{6\pi \eta a}$$

and the rotational diffusion coefficient is:

$$D_r = \frac{kT}{8\pi \eta a^3}$$

(in radians$^2$ sec$^{-1}$)

• Calculate D and $D_r$ for a spherical bacterium of radius 0.5 micron. Show units.

Translational diffusion coefficient (D)

**(Continued on Back of Page)**

[1] Einstein A (1906) On the Theory of the Brownian Motion. Ann. Phys 17:549.

[2] Berg HC (1993) Random Walks in Biology. Princeton University Press. pp. 77

• Rotational diffusion coefficient ($D_r$)

• Bacterial motility is often characterized as run and tumble: The bacterium swims in a straight line (the run), then stops (tumbles) and swims in a different direction. If the bacterium stops swimming for 1 second, what would you predict the change in direction would be (measured in radians)[3]?

• Why is the rotational diffusion coefficient ($D_r$) dependent on $1/a^3$, even though the translational diffusion coefficient (D) is dependent on $1/a$?

**(End)**

[3] Saragosti J, Silberzan P and Buguin A (2012) Modeling *E. coli* tumbles by rotational diffusion, implications for chemotaxis. PLoS One 7(4):e35412.

Here are the constants and other values we require. Note that Pa•s is equal to kg/(m•s^2), Joules are equal to (kg•m^2)/s^2, and kB is joules/K.

> $k_B := 1.381 \cdot 10^{-23} \frac{[\![kg]\!][\![m]\!]^2}{[\![s]\!]^2[\![K]\!]} : T := 293[\![K]\!] : \eta := 1.004 \cdot 10^{-3} \frac{[\![kg]\!]}{[\![m]\!][\![s]\!]} : a := 0.5 \cdot 10^{-6}[\![m]\!] :$

First, we solve for translational diffusion coefficient

> $solve\left(\mathrm{D} = \frac{k_B \cdot T}{6 \cdot \pi \cdot \eta \cdot a}, \mathrm{D}\right)$

$$\frac{4.28 \times 10^{-13} [\![m]\!]^{2.00 \times 10^0}}{[\![s]\!]} \quad \textbf{(1)}$$

Then, for the rotational diffusion coefficient

> $solve\left(\mathrm{D}_r = \frac{k_B \cdot T}{6 \cdot \pi \cdot \eta \cdot a^3}, \mathrm{D}_r\right)$

$$\frac{1.71 \times 10^0}{[\![s]\!]} \quad \textbf{(2)}$$

in units of radians$^2$ per second.

Finally, the number of radians the bacterium will rotate (on average) in 1 second.

> $solve\left(\mathrm{radians} = \sqrt{2 \cdot 1.71 \cdot 1}, radians\right)$

$$1.85 \times 10^0 \quad \textbf{(3)}$$

Or in degrees

> $solve\left(\mathrm{degrees} = \frac{180}{\pi} \cdot 1.85, degrees\right)$

$$1.06 \times 10^2 \quad \textbf{(4)}$$

106 degrees (it will go backwards!)

>

For the final sub-question, there is no simple answer. Inertial forces are insignificant at low Reynolds number, so assuming that mass (density times volume) is the key factor (and would scale as $a^3$) is unlikely. The problem is well-known from Landau and Lifshitz (1987) Fluid Mechanics (2d edition)(Pergamon Press). pages 235-237: "Determine the order of magnitude of the time τ during which a particle suspended in a fluid turns through a large angle of its axis.".

Solution: The required time τ is that during which a particle in Brownian motion moves over a distance of the order of its linear dimension a.

> $From < r^2 > = 6\ \mathrm{D} \cdot \tau,\ \tau \sim \frac{a^2}{\mathrm{D}}$

> And $since\ \mathrm{D} = \frac{R \cdot T}{6 \cdot a \cdot \pi \cdot \eta \cdot N},\ \ \mathrm{D} \sim \frac{T}{a \cdot \eta}$

> $Combining,\ \tau \sim \frac{a^2}{\frac{T}{\eta \cdot a}} \sim \frac{\eta \cdot a^3}{T}$

Hence, the $a^3$ dependence.

| Symbol | Value | Units | Comments |
|---|---|---|---|
| GAS CONSTANT | | | |
| R | 8.314 | J mol$^{-1}$ K$^{-1}$ | R is the Boltzmann constant times Avogadro's Number (6.023•10$^{23}$) |
| | 1.987 | cal mol$^{-1}$ K$^{-1}$ | |
| | 8.314 | m$^{-3}$ Pa mol$^{-1}$ K$^{-1}$ | |
| RT | 2.437 • 10$^{3}$ | J mol$^{-1}$ | At 20 ºC (293 ºK) |
| | 5.833 • 10$^{2}$ | cal mol$^{-1}$ | At 20 ºC (293 ºK) |
| | 2.437 | liter MPa mol$^{-1}$ | At 20 ºC (293 ºK) |
| RT/F | 25.3 | mV | At 20 ºC (293 ºK) |
| 2.303 • RT | 5.612 | kJ mol$^{-1}$ | At 20 ºC (293 ºK) |
| | 1.342 | kcal mol$^{-1}$ | At 20 ºC (293 ºK) |
| $k_B$ | 1.381 • 10$^{-23}$ | J K$^{-1}$ | Boltzmann constant |
| | | | |
| FARADAY CONSTANT | | | |
| F | 9.649 • 10$^{4}$ | coulombs mol$^{-1}$ | F is the electric charge times Avogadro's Number |
| | 9.649 • 10$^{4}$ | J mol$^{-1}$ V$^{-1}$ | |
| | 23.06 | kcal mol$^{-1}$ V$^{-1}$ | |
| CONVERSIONS | | | |
| kcal | 4.187 | kJ (kiloJoules) | Joules is an energy unit (equal to 1 Newton•meter) |
| Watt | 1 | J sec$^{-1}$ | |
| Volt | 1 | J coulomb$^{-1}$ | |
| Amperes | 1 | coulomb sec$^{-1}$ | |
| Pascal (Pa) | 1 | Newton meter$^{-2}$ | Pascal is a pressure unit (equal to 10$^{-5}$ bars) |
| Radians | Radians•(180º/π) | degrees | Conversion of radians to degrees |
| PHYSICAL PROPERTIES | | | |
| $\eta_w$ | 1.004 • 10$^{-3}$ | Pa sec | viscosity of water at 20 ºC |
| $\nu_w$ | 1.004 • 10$^{-6}$ | m$^{2}$ sec$^{-1}$ | kinematic viscosity of water at 20 ºC (viscosity/density) |
| $V_w$ | 1.805 • 10$^{-5}$ | m$^{3}$ mol$^{-1}$ | partial molal volume of water at 20 ºC |

Source: Nobel, Park S (1991) Physicochemical and Environmental Physiology