Name:_________________________ Student ID:_____________________

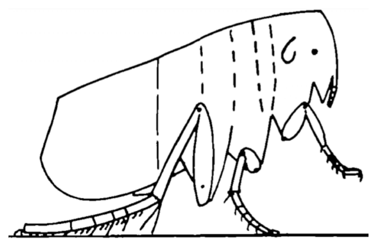
There are <u>three</u> questions. You must complete <u>all</u> of them. Ensure that you show your work (that is, equations, calculations and units). Excessive length is not encouraged.

QUESTION ONE

Give one example of a geometric shape that does not obey Galileo scaling ($V \propto A^{3/2}$). Explain with clear diagrams and graphs.

QUESTION TWO

A flea of regular size (about 1 mm tall) can jump to a height of 20 cm. To be able to jump to the height of a tree 100 m tall, how tall would the flea have to be?



QUESTION THREE

One example of a dimensionless number is the Cavitation number (C_a):

$$C_a = \frac{p - p_v}{\frac{1}{2}\rho V^2}$$

where p is the pressure, p_v is the vapour pressure, ρ is the density and V is the characteristic velocity. The cavitation number provides insight into whether a solution will cavitate or not. At room temperature and for typical values of xylem flow (10 m per hour) and pressure (–3 MPa), calculate the Cavitation number. <u>Be sure to show units</u>. If $C_a<1$, would cavitation be more likely? Or at $C_a>1$? Explain.

Table of vapour pressure (p_v) for water at the temperatures shown.

Temperature (Celsius)	Vapour Pressure (kPa)	Temperature (Celsius)	Vapour Pressure (kPa)
0	0.6	25	3.2
3	0.8	26	3.4
5	0.9	27	3.6
8	1.1	28	3.8
10	1.2	29	4
12	1.4	30	4.2
14	1.6	32	4.8
16	1.8	35	5.6
18	2.1	40	7.4
19	2.2	50	12.3
20	2.3	60	19.9
21	2.5	70	31.2
22	2.6	80	47.3
23	2.8	90	70.1
24	3	100	101.3

A rectangular area is equal to (width * length * 2) + (depth * length * 2) + width * depth * 2)
The volume is equal to (depth * width * length)

$> \ width := 2 : depth := 1 :$

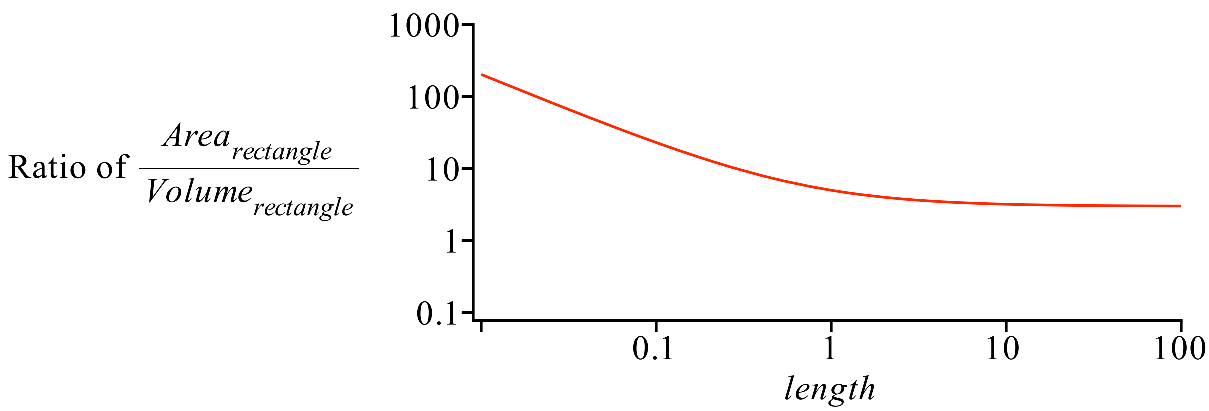
$> \ Area_{(rectangle)} = (width * length * 2) + (depth * length * 2) + (width * depth * 2)$

$$Area_{rectangle} = 6\, length + 4 \qquad (1)$$

$> \ Volume_{(rectangle)} = (depth \cdot width * length)$

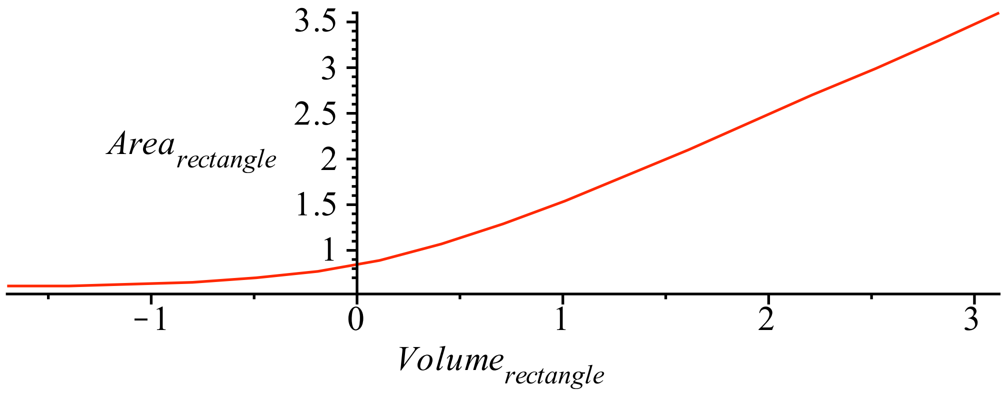
$$Volume_{rectangle} = 2.00\, length \qquad (2)$$

$> \ smartplot\left(\frac{(6\, length + 4)}{2.00\, length}\right)$



Calculating the area and volumes, then taking the logs...

$$> A := \left[\begin{bmatrix} -1.70 & 0.61 \\ -1.40 & 0.61 \\ -1.10 & 0.63 \\ -0.80 & 0.65 \\ -0.49 & 0.70 \\ -0.19 & 0.77 \\ 0.11 & 0.89 \\ 0.41 & 1.07 \\ 0.71 & 1.29 \\ 1.01 & 1.54 \\ 1.31 & 1.82 \\ 1.61 & 2.10 \\ 1.91 & 2.40 \\ 2.21 & 2.70 \\ 2.52 & 2.99 \\ 2.82 & 3.29 \\ 3.12 & 3.60 \end{bmatrix}\right] : plot(A)$$



As length increases, the area versus volume ratio initially declines (expected for Galileo scaling) but then reaches a more or less constant value

For the second question, the flea has to be 99.8 meters tall, in accordence with the principle of similitude. In other words, regardless of the flea's height, it will always jump about 20 cm (approximately true of all jumpers)

For the third question on the Cavitation number:

> $\left(C_a = \dfrac{P - P_v}{\frac{1}{2}\cdot \rho \cdot V^2}\right)$:

> $P := -3\cdot 10^6 [\![Pa]\!] : P_v := 3\cdot 10^3 [\![Pa]\!] : \rho := 1000 \dfrac{[\![kg]\!]}{[\![m]\!]^3} : V := 2.78\cdot 10^{-3} \dfrac{[\![m]\!]}{[\![s]\!]}$:

> $\dfrac{-3\cdot 10^6 - 3\cdot 10^3}{\frac{1}{2}\cdot 1000\cdot \left(2.78\cdot 10^{-3}\right)^2}$

$$-7.771336887\ 10^8 \quad \textbf{(3)}$$

>

The negative C_a indicates a strong potential to cavitate. The negative values means that $P < P_v$.
Note that this also occurs at 100 degrees Celsius, where water boils.

The area is the cylinder area plus the two ends: (2 * π * radius^2 + 2 * π * radius * height). The volume is equal to π radius^2 * height

> height := 2 :

> $\text{Area}_{(\text{cylinder})} = 2\cdot \pi \cdot radius^2 + 2\cdot height\cdot radius$

$$Area_{cylinder} = 2\,\pi\, radius^2 + 4\, radius \quad \textbf{(4)}$$

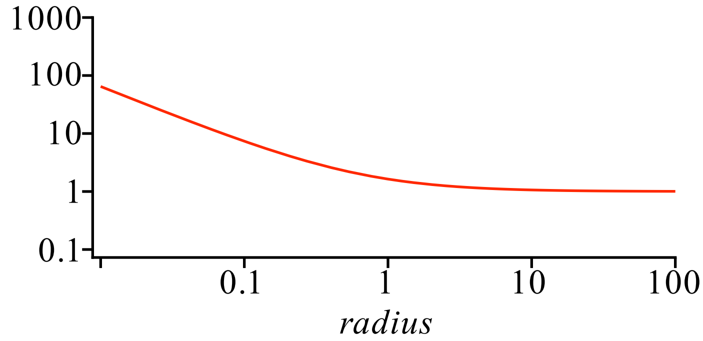
> $\text{Volume}_{(\text{cylinder})} = \pi \cdot height\cdot radius^2$

$$Volume_{cylinder} = 2\,\pi\, radius^2 \quad \textbf{(5)}$$

>

> smartplot$\left(\dfrac{\left(2\,\pi\, radius^2 + 4\, radius\right)}{2\,\pi\, radius^2}\right)$

Ratio of $\frac{Area_{cylinder}}{Volume_{cylinder}}$



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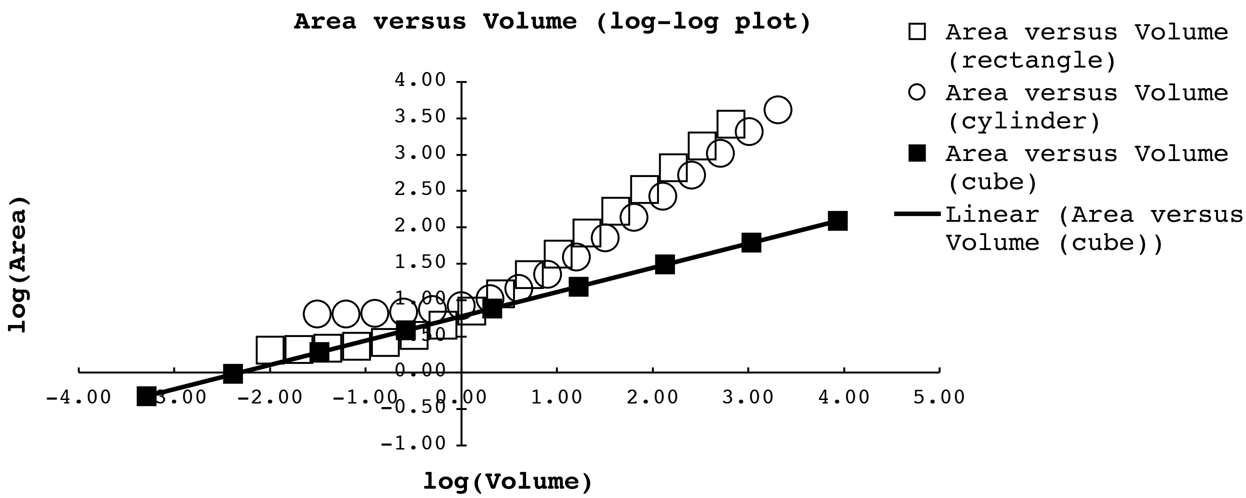
	Rectangle					Cylinder		Cube	
length	Area	log(Area)	Volume	log(Volume	log(Ratio)	log(volume)	log(area)	log(volume)	log(area)
0.01	2.04	0.31	0.01	-2.00	2.31	-1.50	0.80		
0.02	2.08	0.32	0.02	-1.70	2.02	-1.20	0.81		
0.04	2.16	0.33	0.04	-1.40	1.73	-0.90	0.81		
0.08	2.32	0.37	0.08	-1.10	1.46	-0.60	0.83	-3.29	-0.32
0.16	2.64	0.42	0.16	-0.80	1.22	-0.30	0.86	-2.39	-0.02
0.32	3.28	0.52	0.32	-0.49	1.01	0.00	0.92	-1.48	0.28
0.64	4.56	0.66	0.64	-0.19	0.85	0.30	1.01	-0.58	0.58
1.28	7.12	0.85	1.28	0.11	0.75	0.60	1.16	0.32	0.89
2.56	12.24	1.09	2.56	0.41	0.68	0.91	1.35	1.22	1.19
5.12	22.48	1.35	5.12	0.71	0.64	1.21	1.58	2.13	1.49
10.24	42.96	1.63	10.24	1.01	0.62	1.51	1.85	3.03	1.79
20.48	83.92	1.92	20.48	1.31	0.61	1.81	2.13	3.93	2.09
40.96	165.84	2.22	40.96	1.61	0.61	2.11	2.42		
81.92	329.68	2.52	81.92	1.91	0.60	2.41	2.72		
163.84	657.36	2.82	163.84	2.21	0.60	2.71	3.02		
327.68	1312.72	3.12	327.68	2.52	0.60	3.01	3.31		
655.36	2623.44	3.42	655.36	2.82	0.60	3.31	3.62		



$N_T = N_0 \bullet 2^{(T/g)}$

as time increases, t/g = 1, 2, 3 ..., thus 2^1, 2^2, 2^3, etc.

g is the generation time

N_0 is the number of cells at time T = 0

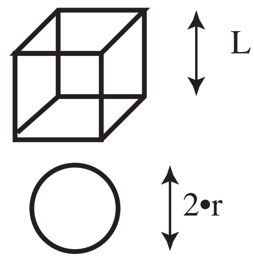
N_T is the number of cells at time T

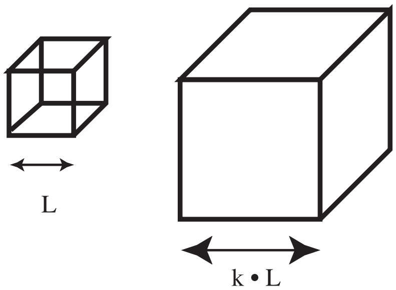
Logistic growth curve:

$$N_T = \frac{K \bullet N_0 \bullet e^{T/g}}{K + N_0(e^{T/g} - 1)}$$

K is the carrying capacity

A cube has a surface area of $6 \bullet L^2$. Its volume is L^3. As long as the shape is constant, the ratio of suraface area to volume will always be $(6 \bullet L^2) / L^3$, or 6/L.
For a sphere, the surface area is $4 \bullet \pi \bullet r^2$, and the volume is $\pi \bullet r^3$; the corresponding ratio of surface area to volume is 4/r.





(area) $A_1 = 6 \bullet L^2$ $A_k = 6 \bullet (k \bullet L)^2$ $A_k = 6 \bullet k^2 \bullet L^2$ $(= k^2 \bullet A_1)$
(volume) $V_1 = L^3$ $V_k = (k \bullet L)^3$ $V_k = k^3 \bullet L^3$ $(= k^3 \bullet V_1)$
The scaling coefficient is different for area (k^2) and for volume (k^3).

Heat conduction rates are defined by the relation: $P_{cond} = Q / t = k \bullet A \bullet [(T_a - T_b) / L]$ where P_{cond} is the rate of conduction (transferred heat, Q, divided by time, t); k is the thermal conductivity; T_a and T_b are the temperatures of the two heat reservoirs a and b; A is the area; and L is the distance. Thermal conductivities of water and air are about 0.6 and 0024 W m^{-1} K^{-1}, respectively.
Thermal radiation is defined by the relation: $P_{rad} = \sigma \bullet \varepsilon \bullet A \bullet T^4$
where P_{rad} is the rate of radiation; σ is the Stefan-Boltzmann constant ($5.6703 \bullet 10^{-8}$ W m^{-2} K^{-4}; ε is the emissivity (varies from 0 to 1, where 1 is for a blackbody radiator); A is the area; and T is the temperature (in Kelvins). The *net* radiative emission or absorption will depend upon the difference in temperature: $P_{net} = \sigma \bullet \varepsilon \bullet A \bullet (T^4_{body} - T^4_{ambient})$

$$\text{compression} = \rho \bullet h \quad F_{cr} = \frac{E \bullet I \bullet \pi^2}{L_{eff}^2} \quad \Psi_{wv} = \frac{RT}{\overline{V}_w} \ln\left(\frac{\%\text{ relative humidity}}{100}\right) + \rho_w g h$$

$$F_{cr} = \frac{E \bullet \frac{\pi \bullet r}{4} \bullet \pi^2}{(2 \bullet h)^2}, \text{ and } F_{cr} = \rho \bullet \pi \bullet r^2 \bullet h$$

velocity (meters sec^{-1})

pressure difference (Pascal = 1 kg m^{-1} sec^{-1})

tube radius

$$v = \left(\frac{\Delta p}{l}\right)\left(\frac{1}{4 \bullet \eta}\right)(R^2 - r^2)$$

distance (meters)

viscosity (water = 0.01 gm cm^{-1} sec^{-1}, or Pa sec)

distance from center of tube

density (water = 1 gm cm^{-3})

velocity (cm sec^{-1})

tube diameter (cm)

$$R_e = \frac{\rho \bullet \nu \bullet l}{\eta}$$

viscosity (water = 0.01 gm cm^{-1} sec^{-1})

$$v = \left(\frac{\Delta p}{l}\right)\left(\frac{1}{4 \bullet \eta}\right) R^2$$

$$\mathrm{J_v} = \left(\frac{\Delta \mathrm{p}}{1}\right)\left(\frac{\pi}{8 \bullet \eta}\right) \bullet R^4$$

$$J_V = -\frac{r^2}{8 \bullet \eta} \bullet \frac{\partial P}{\partial x}$$

$$J = -D\frac{\partial c}{\partial x}$$

Fick's First Law of Diffusion: The flux is proportional to the concentration gradient

$$\frac{\partial c}{\partial t} = -\frac{\partial J}{\partial x}$$

Fick's Second Law of Diffusion: Changes in concentration over time depend upon the flux gradient

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

$$J = -\frac{1}{2} \bullet \frac{\Delta^2}{\tau} \bullet \frac{dC}{dx}$$

$$\nabla \nu = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

velocity vector —the notation grad ν is sometimes used

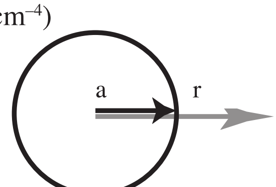
with velocity components, u, v, and w, in the three dimensions, x, y, and z.

$$D = \frac{1}{2} \bullet \frac{\Delta^2}{\tau}$$

units: moles cm^{-2} sec^{-1}

$$J_x = -D\frac{\partial c}{\partial x} + v_x \bullet c$$

(cm^2 sec^{-1})(moles cm^{-4}) (cm sec^{-1})(moles cm^{-3})



Fick's First law : $J_r = -D\dfrac{\partial C}{\partial r}$

Fick's Second Law : $\dfrac{\partial C}{\partial t} = D\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial C}{\partial r}\right) = 0$

$$C(r) = C_0\left(1 - \frac{a}{r}\right)$$

(steady state)

$$J_r(a) = -D \bullet C_0 \bullet 4 \bullet \pi \bullet a = I_D \text{ (diffusive current)}$$

(mole cm[–2] sec[–1]) (units of mole sec[–1])

$$P_e = \frac{2 \bullet a \bullet u}{D}$$

$$I_m = 4 \bullet \pi \bullet a^2 \bullet \beta \text{ (metabolic current)}$$

(cm[2]) (units of mole sec[–1])

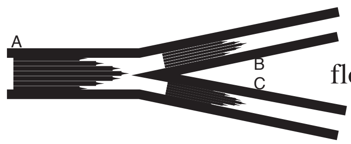
$$\frac{\partial C}{\partial t} = u \bullet \frac{\partial C}{\partial r} \bullet C + D\frac{\partial^2 C}{\partial r^2}$$

flow velocity

concentration gradient

concentration

$$Q = \frac{\Delta p}{l}\frac{\pi a^4}{8\eta}$$



$$\mu_j^{liquid} = \mu_j^* + RT\ln a_j + \overline{V}_j P + z_j FE + m_j gh$$

fluid density

velocity

$$D = \frac{1}{2} \bullet \rho \bullet V^2 \bullet A \bullet C_D$$

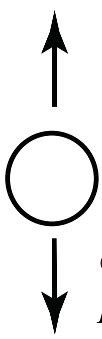
frontal area

drag coefficient
(shape-dependent)

$$m\left(-\frac{d\upsilon}{dt}\right) = 6 \bullet \pi \bullet \eta \bullet r \bullet \upsilon$$

$$\upsilon(t) = \upsilon_0 e^{\left(-\frac{6 \bullet \pi \bullet \eta \bullet r}{m} \bullet t\right)}$$

$$V_{\text{terminal}} = \sqrt{\frac{2mg}{\rho A C_D}}$$



Frictional force

$$F_f = 6\pi\eta a v$$

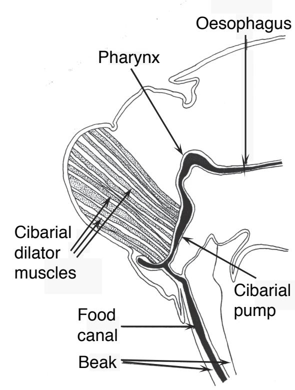
Where the frictional and gravitational forces are balenced, the velocity reaches a steady state.

Gravitational pull

$$F_g = \tfrac{4}{3}\pi a^3 \Delta\rho g$$

The energetic details of the pumping mechanism are shown below for *Rhodnius* (a blood sucking insect) and spittlebugs (*Philaenus*)[1].

	Rhodnius	*Philaenus*
Muscle tension (maximum)	600 kPa	600 kPa
Pump stroke frequency	3 Hz	1.7 Hz
Muscle contraction rate (muscle lengths per second)	1 s[–1]	0.5 s[–1]
Ratio of muscle/piston	2.5	10.0
Maximum muscle tension	–300 kPa	–2400 kPa



$$\text{Rotor}_{(n)} + mH^{+}_{\text{outside}} \longleftrightarrow \text{Rotor}_{(n+1)} + mH^{+}_{\text{inside}}$$

$$\text{ADP} + \text{P}_{\text{i}} + \text{mH}^{+}_{\text{outside}} \longleftrightarrow \text{ATP} + \text{mH}^{+}_{\text{inside}}$$

gas constant

Faraday constant

Voltage

$$\mu = \mu^{\circ} + \text{RT}\ln(a_{H^+}) + zF\Psi$$

temperature

activity of protons

$$\Delta\text{G}_{\text{ATP}} = \Delta\text{G}^{\circ}_{\text{ATP}} + \text{RT}\ln\left(\frac{[ATP]}{[ADP][P_i]}\right)$$

$$\Delta\text{G}_{\text{total}} = n \bullet \Delta\mu_{H^+} + \Delta G_{ATP} = 0$$

$$n \bullet \left(RT\ln\left(\frac{a^{inside}_{H^+}}{a^{outside}_{H^+}}\right) + F\Delta\Psi\right) + \Delta G^{o}_{ATP} + RT\ln\left(\frac{[ATP]}{[ADP][P_i]}\right) = 0$$

$$\Delta\mu_{H^+} = \frac{RT}{F}\ln\left(\frac{a^{inside}_{H^+}}{a^{outside}_{H^+}}\right) + \Delta\Psi$$

(units: mV)

RT/F is about 25 mV at 20°C.

The work exerted will depend upon the speed of the contraction, and the cross-sectional area of the muscle times its length. Muscle contraction speeds are normally in the range of 3 milliseconds. The initial velocity will equal the impulse force divided by the mass ($\nu = F_{\text{impulse}}/\text{mass}$).

The work done in the leap is proportional to mass and the height of the leap ($W \propto mH$), while the work of the muscles is proportional to the mass of the muscle (or the whole organism) ($W \propto m$). It follows then, that the total work is related solely to the height, since the organism's mass cancels out. Thus, the height of the leap is not proportional to the organisms's size, but rather is similar for any organism. D'Arcy Thompson describes this as an example of the Principle of Biological Similitude.

$$\mu^{liquid}_{j} = \mu^{*}_{j} + RT\ln a_j + \overline{V}_j P + z_j FE + m_j gh$$

$$RT\ln a_j + \overline{V}_j P + m_j gh$$

gravitational potential

$$a_j = \gamma_j c_j$$

The activity of water (a_j) is the product of the activity coefficient and the concentration of water

$$\overline{V}_j = \left(\frac{\partial V}{\partial n_j}\right)_{n_i, T, P, E, h}$$

The partial molal volume of species j is the incremental increase in volume with the addition of species j. For water, it is 18.0 X 10^{-6} $\text{m}^3\ \text{mol}^{-1}$.

$$RT\ln a_j = \overline{V}_j \Pi$$

osmotic pressure

$$\Pi_s = RT\sum_j c_j$$

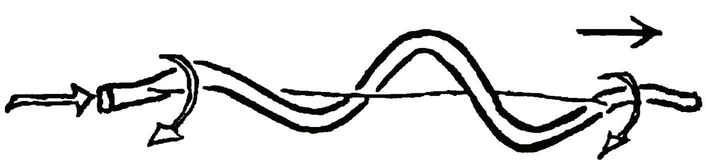
Van't Hoff relation

The terms inter-relate various properties of water: changes in its activity with the addition of solutes, and the relation to pressure.

$$F = A\upsilon + B\omega$$

$$N = C\upsilon + D\omega$$

That is, both velocity and rotation contribute to both the force and torque.



Symbol	Value	Units	Comments
GAS CONSTANT			
R	8.314	J mol^{-1} K^{-1}	R is the Boltzmann constant times Avogadro's Number ($6.023 \cdot 10^{23}$)
	1.987	cal mol^{-1} K^{-1}	
	8.314	m^{-3} Pa mol^{-1} K^{-1}	
RT	$2.437 \cdot 10^{3}$	J mol^{-1}	At 20 ºC (293 ºK)
	$5.833 \cdot 10^{2}$	cal mol^{-1}	At 20 ºC (293 ºK)
	2.437	liter MPa mol^{-1}	At 20 ºC (293 ºK)
RT/F	25.3	mV	At 20 ºC (293 ºK)
2.303 • RT	5.612	kJ mol^{-1}	At 20 ºC (293 ºK)
	1.342	kcal mol^{-1}	At 20 ºC (293 ºK)
FARADAY CONSTANT			
F	$9.649 \cdot 10^{4}$	coulombs mol^{-1}	F is the electric charge times Avogadro's Number
	$9.649 \cdot 10^{4}$	J mol^{-1} V^{-1}	
	23.06	kcal mol^{-1} V^{-1}	
CONVERSIONS			
kcal	4.187	kJ (kiloJoules)	Joules is an energy unit (equal to 1 Newton•meter)
Watt	1	J sec^{-1}	
Volt	1	J $coulomb^{-1}$	
Amperes	1	coulomb sec^{-1}	
Pascal (Pa)	1	Newton $meter^{-2}$	Pascal is a pressure unit (equal to 10^{-5} bars)
Siemens	1	Ohm^{-1}	Siemens (S) is conductance, the inverse of resistance (Ohm)
PHYSICAL PROPERTIES			
η_w	$1.004 \cdot 10^{-3}$	Pa sec	viscosity of water at 20 ºC
ν_w	$1.004 \cdot 10^{-6}$	m^2 sec^{-1}	kinematic viscosity of water at 20 ºC (viscosity/density)
V_w	$1.805 \cdot 10^{-5}$	m^3 mol^{-1}	partial molal volume of water at 20 ºC

Source: Nobel, Park S (1991) Physicochemical and Environmental Physiology