

Logistic growth curve:

$$
N_{T}=\frac{K \bullet N_{0} \bullet e^{T / g}}{K+N_{0}\left(e^{T / g}-1\right)}
$$

K is the carrying capacity

A cube has a surface area of $6 \cdot L^{2}$. Its volume is $L^{3}$. As long as the shape is constant, the ratio of suraface area to volume will always be ( $6 \cdot \mathrm{~L}^{2}$ ) / $\mathrm{L}^{3}$, or 6/L.
For a sphere, the surface area is $4 \bullet \pi \cdot r^{2}$, and the volume is $\pi \bullet r^{3}$; the corresponding ratio of surface area to volume is $4 / \mathrm{r}$.



L

(volume) $\mathrm{V}_{1}=\mathrm{L}^{3} \quad \mathrm{~V}_{\mathrm{k}}^{\mathrm{k}}=(\mathrm{k} \cdot \mathrm{L})^{3} \quad \mathrm{~V}_{\mathrm{k}}=\mathrm{k}^{3} \bullet \mathrm{~L}^{3} \quad\left(=\mathrm{k}^{3} \cdot \mathrm{~V}_{1}\right)$ The scaling coefficient is different for area $\left(\mathrm{k}^{2}\right)$ and for volume $\left(\mathrm{k}^{3}\right)$.

Heat conduction rates are defined by the relation: $\mathrm{P}_{\text {cond }}=\mathrm{Q} / \mathrm{t}=\mathrm{k} \bullet \mathrm{A} \bullet\left[\left(\mathrm{T}_{\mathrm{a}}-\mathrm{T}_{\mathrm{b}}\right) / \mathrm{L}\right]$ where $P_{\text {cond }}$ is the rate of conduction (transferred heat, Q , divided by time, t ); k is the thermal conductivity; $\mathrm{T}_{\mathrm{a}}$ and $\mathrm{T}_{\mathrm{b}}$ are the temperatures of the two heat reservoirs a and b ; A is the area; and L is the distance. Thermal conductivities of water and air are about 0.6 and $0024 \mathrm{~W} \mathrm{~m}^{-1} \mathrm{~K}^{-1}$, respectively.
Thermal radiation is defined by the relation: $\mathrm{P}_{\mathrm{rad}}=\sigma \bullet \varepsilon \bullet \mathrm{A} \bullet \mathrm{T}^{4}$ where $P_{\text {rad }}$ is the rate of radiation; $\sigma$ is the Stefan-Boltzmann constant $\left(5.6703 \cdot 10^{-8} \mathrm{~W}\right.$ $\mathrm{m}^{-2} \mathrm{~K}^{-4} ; \varepsilon$ is the emissivity (varies from 0 to 1 , where 1 is for a blackbody radiator); A is the area; and T is the temperature (in Kelvins). The net radiative emission or absorption will depend upon the difference in temperature: $\mathrm{P}_{\text {net }}=\sigma \bullet \varepsilon \bullet \mathrm{A} \bullet\left(\mathrm{T}_{\text {body }}^{4}-\mathrm{T}_{\text {ambient }}^{4}\right)$

$$
\begin{aligned}
& \text { compression }=\rho \bullet h \quad F_{c r}=\frac{E \bullet I \bullet \pi^{2}}{L_{e f f}^{2}} \quad \Psi_{w v}=\frac{R T}{\bar{V}_{w}} \ln \left(\frac{\% \text { relative humidity }}{100}\right)+\rho_{w} g h \\
& F_{c r}=\frac{E \bullet \frac{\pi \bullet r}{4} \bullet \pi^{2}}{(2 \bullet h)^{2}}, \text { and } F_{c r}=\rho \bullet \pi \bullet r^{2} \bullet h
\end{aligned}
$$

velocity (meters sec ${ }^{-1}$ ) pressure difference

distance (meters)
center of tube
viscosity (water $=0.01 \mathrm{gm} \mathrm{cm}^{-1} \mathrm{sec}^{-1}$, or Pa sec)

$$
v=\left(\frac{\Delta p}{l}\right)\left(\frac{1}{4 \bullet \eta}\right) R^{2}
$$

$$
\mathrm{J}_{\mathrm{v}}=\left(\frac{\Delta \mathrm{p}}{1}\right)\left(\frac{\pi}{8 \bullet \eta}\right) \cdot R^{4} .
$$

$$
J=-D \frac{\partial c}{\partial x} \quad \begin{gathered}
\text { Fick's First Law of Diffusion: The flux is } \\
\text { proportional to the concentration gradient } \\
\partial c \quad \partial J
\end{gathered} \quad \text { Fick's Second Law }
$$

$$
\begin{array}{ccc}
\partial x \\
\partial c
\end{array} \quad \begin{array}{cc}
\partial^{2} c
\end{array} \quad \frac{\partial c}{\partial t}=-\frac{\partial J}{\partial x} \quad \begin{gathered}
\text { Fick's Second Law of Diftusion: } \\
\text { Changes in oconcentration over time } \\
\text { depend upon the flux gradient }
\end{gathered}
$$ in the three dimensions, $\mathrm{x}, \mathrm{y}$, and z .

units: moles $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$

$$
\text { - the notation grad } v
$$

is sometimes used

$$
J_{x}=-D \frac{\partial c}{\partial x}+v_{x} \cdot \underset{\left(\mathrm{~cm} \mathrm{sec}^{-1}\right)\left(\text { moles cm }^{-3}\right)}{c}
$$

$$
\left(\mathrm{cm}^{2} \sec ^{-1}\right)\left(\text { moles } \mathrm{cm}^{-4}\right)
$$

$$
\begin{aligned}
& J_{r}(a)=-D \bullet C_{0} \bullet 4 \bullet \pi \bullet a=I_{D} \quad \text { (diffusive current) } \\
& \text { (units of mole sec }{ }^{-1} \text { ) } \\
& P_{e}=\frac{2 \cdot a \bullet u}{D} \\
& \text { (mole } \mathrm{cm}^{-2} \mathrm{sec}^{-1} \text { ) } \\
& \mu_{j}^{\text {liquid }}=\mu_{j}^{*}+R T \ln a_{j}+\overline{V_{j}} P+z_{j} F E+m_{j} g h \\
& \begin{array}{l}
m\left(-\frac{d v}{d t}\right)=6 \cdot \pi \cdot \eta \cdot \mathrm{r} \bullet v \\
v(\mathrm{t})=\mathrm{v}_{0} e^{\left(-\frac{6 \cdot \pi \cdot \eta \cdot r}{m} \cdot t\right)}
\end{array} \\
& V_{\text {terminal }}=\sqrt{\frac{2 m g}{\rho A C_{D}}} \\
& \text { drag coefficient } \\
& \text { (shape-dependent) }
\end{aligned}
$$

Frictional force
$F_{f}=6 \pi \eta a v$

$$
\downarrow \begin{aligned}
& \text { Gravitational pull } \\
& F_{g}=\frac{4}{3} \pi a^{3} \Delta \rho g
\end{aligned}
$$

Where the frictional and gravitational forces are balenced, the velocity reaches a steady state.

| The energetic details of the pumping mechanism are shown below for Rhodnius (a blood sucking insect) and |  |  |
| :---: | :---: | :---: |
| spittlebugs (Philaenus) ${ }^{[1]}$. | Rhodnius | Philaenus |
| Muscle tension (maximum) | 600 kPa | 600 kPa |
| Pump stroke frequency | 3 Hz | 1.7 Hz |
| Muscle contraction rate (muscle lengths per second) | $1 \mathrm{~s}^{-1}$ | $0.5 \mathrm{~s}^{-1}$ |
| Ratio of muscle/piston | 2.5 | 10.0 |
| Maximum muscle tension | -300 kPa | -2400 kPa |



$$
\operatorname{Rotor}_{(\mathrm{n})}+\mathrm{mH}_{\text {outside }}^{+} \longleftrightarrow \text { Rotor }_{(\mathrm{n}+1)}+\mathrm{m} H_{\text {inside }}^{+}
$$

$\mathrm{ADP}+\mathrm{P}_{\mathrm{i}}+\mathrm{mH}_{\text {outside }}^{+} \longleftrightarrow$ ATP $+\mathrm{mH}_{\text {inside }}^{+}$

activity of protons
$\Delta \mathrm{G}_{\text {total }}=n \bullet \Delta \mu_{H^{+}}+\Delta G_{A T P}=0$
$n \bullet\left(R T \ln \left(\frac{a^{\text {inside }}}{a_{H^{+}}^{\text {ousside }}}\right)+F \Delta \Psi\right)+\Delta G_{A T P}^{o}+R T \ln \left(\frac{[A T P]}{[A D P]\left[P_{i}\right]}\right)=0$
$\Delta \mu_{H^{+}}=\frac{R T}{F} \ln \left(\frac{a_{H^{+}}^{\text {inside }}}{a_{H^{+}}^{\text {outside }}}\right)+\Delta \Psi$
(units: mV )
RT/F is about 25 mV at $20^{\circ} \mathrm{C}$.
$F=A v+B \omega$
$N=C v+D \omega$
That is, both velocity and rotation contribute to both the force and torque.

$\mu_{j}^{\text {liquid }}=\mu_{j}^{*}+R T \ln a_{j}+\overline{V_{j}} P+z_{j} F E+m_{j} g h$


The activity of water $\left(\mathrm{a}_{\mathrm{j}}\right)$ is the product of the activity coefficient and the concentration of water

$$
R T \ln a_{j}=\overline{V_{j}} \Pi
$$

The work exerted will depend upon the speed of the contraction, and the crosssectional area of the muscle times its length. Muscle contraction speeds are normally in the range of 3 milliseconds. The initial velocity will equal the impulse force divided by the mass ( $v=$ $\mathrm{F}_{\text {impulse }} /$ mass).

The work done in the leap is proportional to mass and the height of the leap ( $\mathrm{W} \propto$ mH ), while the work of the muscles is proportional to the mass of the muscle (or the whole organism) ( $\mathrm{W} \propto \mathrm{m}$ ). It follows then, that the total work is related solely to the height, since the organism's mass cancels out. Thus, the height of the leap is not proportional to the organisms's size, but rather is similar for any organism. D'Arcy Thompson describes this as an example of the Principle of Biological Similitude.

The partial molal volume of species $j$ is the incremental increase in volume with the addition of species j . For water, it is $18.0 \times 10^{-6} \mathrm{~m}^{3} \mathrm{~mol}^{-1}$.


| Symbol | Value | Units | Comments |
| :---: | :---: | :---: | :---: |
| GAS CONSTANT |  |  |  |
| R | 8.314 | $\mathrm{J} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ | R is the Boltzmann constant times Avogadro's Number ( $6.023 \bullet 10^{23}$ ) |
|  | 1.987 | $\mathrm{cal} \mathrm{mol}{ }^{-1} \mathrm{~K}^{-1}$ |  |
|  | 8.314 | $\mathrm{m}^{3} \mathrm{~Pa} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$ |  |
| RT | $2.437 \cdot 10^{3}$ | $\mathrm{J} \mathrm{mol}^{-1}$ | At $20^{\circ} \mathrm{C}\left(293{ }^{\circ} \mathrm{K}\right)$ |
|  | $5.822 \cdot 10^{2}$ | cal mol ${ }^{-1}$ | At $20^{\circ} \mathrm{C}\left(293{ }^{\circ} \mathrm{K}\right)$ |
|  | 2.437 | liter MPa mol ${ }^{-1}$ | At $20^{\circ} \mathrm{C}\left(293{ }^{\circ} \mathrm{K}\right)$ |
| RT/F | 25.3 | mV | At $20^{\circ} \mathrm{C}\left(293{ }^{\circ} \mathrm{K}\right)$ |
| $2.303 \cdot \mathrm{RT}$ | 5.612 | $\mathrm{kJ} \mathrm{mol}^{-1}$ | At $20^{\circ} \mathrm{C}\left(293{ }^{\circ} \mathrm{K}\right)$ |
|  | 1.342 | kcal mol ${ }^{-1}$ | At $20^{\circ} \mathrm{C}\left(293{ }^{\circ} \mathrm{K}\right)$ |
| $\mathrm{k}_{\mathrm{B}}$ | $1.381 \cdot 10^{-23}$ | J molecule ${ }^{-1} \mathrm{~K}^{-1}$ | Boltzmann's constant |
|  |  |  |  |
| FARADAY CONSTANT |  |  |  |
| F | $9.649 \cdot 10^{4}$ | coulombs $\mathrm{mol}^{-1}$ | F is the electric charge times Avogadro's Number |
|  | $9.649 \cdot 10^{4}$ | $\mathrm{J} \mathrm{mol}^{-1} \mathrm{~V}^{-1}$ |  |
|  | 23.06 | kcal mol ${ }^{-1} \mathrm{~V}^{-1}$ |  |
| CONVERSIONS |  |  |  |
| kcal | 4.187 | kJ (kiloJoules) | Joules is an energy unit (equal to 1 Newton•meter) |
| Watt | 1 | $\mathrm{J} \mathrm{sec}^{-1}$ |  |
| Volt | 1 | $\mathrm{J}^{\text {coulomb }}{ }^{-1}$ |  |
| Amperes | 1 | coulomb sec ${ }^{-1}$ |  |
| Pascal (Pa) | 1 | Newton meter ${ }^{-2}$ | Pascal is a pressure unit (equal to $10^{-5}$ bars) |
| Radians | Radians•(180\% $/$ ) | degrees | Conversion of radians to degrees |
| PHYSICAL PROPERTIES |  |  |  |
| $\eta_{\text {w }}$ | $1.002 \cdot 10^{-3}$ | Pa sec | viscosity of water at $20^{\circ} \mathrm{C}$ |
| $\nu_{\text {w }}$ | $1.002 \cdot 10^{-6}$ | $\mathrm{m}^{2} \mathrm{sec}^{-1}$ | kinematic viscosity of water at 20 ${ }^{\circ} \mathrm{C}$ (viscosity/density) |
| $\mathrm{V}_{\mathrm{w}}$ | $1.805 \cdot 10^{-5}$ | $\mathrm{m}^{3} \mathrm{~mol}^{-1}$ | partial molal volume of water at $20^{\circ} \mathrm{C}$ |

Source: Nobel, Park S (1991) Physicochemical and Environmental Physiology

