

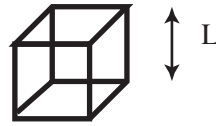
$$N_T = N_0 \cdot 2^{(T/g)}$$

as time increases,  $T/g = 1, 2, 3 \dots$ , thus  $2^1, 2^2, 2^3$ , etc.  
 $g$  is the generation time  
 $N_0$  is the number of cells at time  $T = 0$   
 $N_T$  is the number of cells at time  $T$

Logistic growth curve:

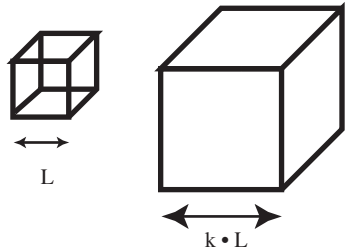
$$N_T = \frac{K \cdot N_0 \cdot e^{T/g}}{K + N_0(e^{T/g} - 1)}$$

$K$  is the carrying capacity



A cube has a surface area of  $6 \cdot L^2$ . Its volume is  $L^3$ . As long as the shape is constant, the ratio of surface area to volume will always be  $(6 \cdot L^2) / L^3$ , or  $6/L$ .

For a sphere, the surface area is  $4 \cdot \pi \cdot r^2$ , and the volume is  $\pi \cdot r^3$ ; the corresponding ratio of surface area to volume is  $4/r$ .



(area)  $A_1 = 6 \cdot L^2$       $A_k = 6 \cdot (k \cdot L)^2$       $A_k = 6 \cdot k^2 \cdot L^2$       $(= k^2 \cdot A_1)$   
(volume)  $V_1 = L^3$       $V_k = (k \cdot L)^3$       $V_k = k^3 \cdot L^3$       $(= k^3 \cdot V_1)$   
The scaling coefficient is different for area ( $k^2$ ) and for volume ( $k^3$ ).

Heat conduction rates are defined by the relation:

$$P_{\text{cond}} = Q / t = k \cdot A \cdot [(T_a - T_b) / L]$$

where  $P_{\text{cond}}$  is the rate of conduction (transferred heat,  $Q$ , divided by time,  $t$ );  $k$  is the thermal conductivity;  $T_a$  and  $T_b$  are the temperatures of the two heat reservoirs  $a$  and  $b$ ;  $A$  is the area; and  $L$  is the distance. Thermal conductivities of water and air are about  $0.6$  and  $0.024 \text{ W m}^{-1} \text{ K}^{-1}$ , respectively.

Thermal radiation is defined by the relation:

$$P_{\text{rad}} = \sigma \cdot \epsilon \cdot A \cdot T^4$$

where  $P_{\text{rad}}$  is the rate of radiation;  $\sigma$  is the Stefan-Boltzmann constant ( $5.6703 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ );  $\epsilon$  is the emissivity (varies from  $0$  to  $1$ , where  $1$  is for a blackbody radiator);  $A$  is the area; and  $T$  is the temperature (in Kelvins). The *net* radiative emission or absorption will depend upon the difference in temperature:

$$P_{\text{net}} = \sigma \cdot \epsilon \cdot A \cdot (T_{\text{body}}^4 - T_{\text{ambient}}^4)$$

$$\text{compression} = \rho \cdot h \quad h_{\text{critical}} = \frac{42.4 \cdot 10^6 (\text{N} \cdot \text{m}^{-2}) \cdot \frac{1(\text{kg}(\text{f}))}{9.80665(\text{N})}}{436(\text{kg} \cdot \text{m}^3)}$$

$$F_{cr} = \frac{E \cdot I \cdot \pi^2}{L_{\text{eff}}^2} \quad F_{cr} = \frac{E \cdot \frac{\pi \cdot r}{4} \cdot \pi^2}{(2 \cdot h)^2}, \text{ and } F_{cr} = \rho \cdot \pi \cdot r^2 \cdot h$$

$$\Psi_{\text{wv}} = \frac{RT}{V_w} \ln\left(\frac{\% \text{ relative humidity}}{100}\right) + \rho_w g h$$

$$v = \left(\frac{\Delta p}{l}\right) \left(\frac{1}{4 \cdot \eta}\right) (R^2 - r^2)$$

velocity (meters sec<sup>-1</sup>)     pressure difference (Pascal = 1 kg m<sup>-1</sup> sec<sup>-2</sup>)  
tube radius  
distance (meters)     distance from center of tube  
viscosity (water = 0.01 gm cm<sup>-1</sup> sec<sup>-1</sup>, or Pa sec)

$$v = \left(\frac{\Delta p}{l}\right) \left(\frac{1}{4 \cdot \eta}\right) R^2 \quad J_v = \left(\frac{\Delta p}{l}\right) \left(\frac{\pi}{8 \cdot \eta}\right) \cdot R^4$$

$$R_e = \frac{\rho \cdot v \cdot l}{\eta}$$

density (water = 1 gm cm<sup>-3</sup>)     velocity (cm sec<sup>-1</sup>)  
tube diameter (cm)  
viscosity (water = 0.01 gm cm<sup>-1</sup> sec<sup>-1</sup>)

$$J = -D \frac{\partial c}{\partial x}$$

Fick's First Law of Diffusion: The flux is proportional to the concentration gradient

$$\frac{\partial c}{\partial t} = - \frac{\partial J}{\partial x}$$

Fick's Second Law of Diffusion: Changes in concentration over time depend upon the flux gradient

$$J = -\frac{1}{2} \cdot \frac{\Delta^2}{\tau} \cdot \frac{dC}{dx}$$

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

$$D = \frac{1}{2} \cdot \frac{\Delta^2}{\tau}$$

$$\nabla v = u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

velocity vector  
—the notation grad v is sometimes used

with velocity components, u, v, and w, in the three dimensions, x, y, and z.

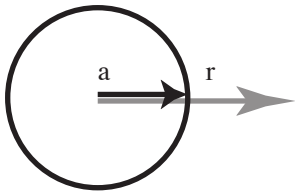
$$J_x = -D \frac{\partial c}{\partial x} + v_x \cdot c$$

units: moles cm<sup>-2</sup> sec<sup>-1</sup>

(cm<sup>2</sup> sec<sup>-1</sup>)(moles cm<sup>-4</sup>)

(cm sec<sup>-1</sup>)(moles cm<sup>-3</sup>)

$$J_V = -\frac{r^2}{8 \cdot \eta} \cdot \frac{\partial P}{\partial x}$$



Fick's First law :  $J_r = -D \frac{\partial C}{\partial r}$

Fick's Second Law : (steady state)

$$C(r) = C_0 \left(1 - \frac{a}{r}\right)$$

$$\frac{\partial C}{\partial t} = D \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C}{\partial r} \right) = 0$$

$$J_r(a) = -D \cdot C_0 \cdot 4 \cdot \pi \cdot a = I_D \text{ (diffusive current)}$$

(units of mole sec<sup>-1</sup>)

(mole cm<sup>-2</sup> sec<sup>-1</sup>)

$$I_m = 4 \cdot \pi \cdot a^2 \cdot \beta \text{ (metabolic current)}$$

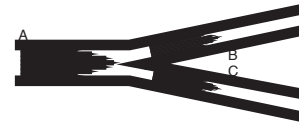
(cm<sup>2</sup>) (units of mole sec<sup>-1</sup>)

$$\frac{\partial C}{\partial t} = u \cdot \frac{\partial C}{\partial r} \cdot C + D \frac{\partial^2 C}{\partial r^2}$$

flow velocity  
concentration gradient  
concentration

$$P_e = \frac{2 \cdot a \cdot u}{D}$$

$$Q = \frac{\Delta p \pi a^4}{l 8 \eta}$$



$$\mu_j^{liquid} = \mu_j^* + RT \ln a_j + \bar{V}_j P + z_j F E + m_j g h$$

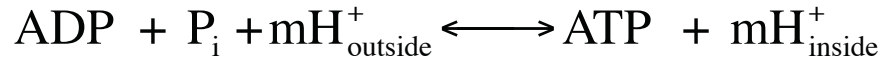
$$D = \frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D$$

fluid density  
velocity  
frontal area  
drag coefficient (shape-dependent)

$$m \left( -\frac{dv}{dt} \right) = 6 \cdot \pi \cdot \eta \cdot r \cdot v$$

$$v(t) = v_0 e^{\left( -\frac{6 \cdot \pi \cdot \eta \cdot r \cdot t}{m} \right)}$$

$$p_o(t) = e^{-\lambda t} \quad p_n = e^{-\lambda T} \frac{(\lambda T)^n}{n!} = \frac{e^{-\mu} \mu^k}{k!}$$



$$\mu = \mu^\circ + RT \ln(a_{H^+}) + zF\Psi$$

gas constant  $R$ , Faraday constant  $F$ , Voltage  $\Psi$ , temperature, activity of protons  $a_{H^+}$

$$\Delta G_{ATP} = \Delta G_{ATP}^\circ + RT \ln\left(\frac{[ATP]}{[ADP][P_i]}\right)$$

$$\Delta G_{\text{total}} = n \cdot \Delta\mu_{H^+} + \Delta G_{ATP} = 0$$

$$n \cdot \left( RT \ln\left(\frac{a_{H^+}^{\text{inside}}}{a_{H^+}^{\text{outside}}}\right) + F\Delta\Psi \right) + \Delta G_{ATP}^\circ + RT \ln\left(\frac{[ATP]}{[ADP][P_i]}\right) = 0$$

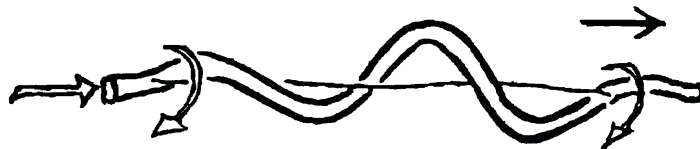
$$\Delta\mu_{H^+} = \frac{RT}{F} \ln\left(\frac{a_{H^+}^{\text{inside}}}{a_{H^+}^{\text{outside}}}\right) + \Delta\Psi \quad (\text{units: mV})$$

$RT/F$  is about 25 mV at 20°C.

$$F = Av + B\omega$$

$$N = Cv + D\omega$$

That is, both velocity and rotation contribute to both the force and torque.



The work exerted will depend upon the speed of the contraction, and the cross-sectional area of the muscle times its length. Muscle contraction speeds are normally in the range of 3 milliseconds. The initial velocity will equal the impulse force divided by the mass ( $v = F_{\text{impulse}}/\text{mass}$ ).

The work done in the leap is proportional to mass and the height of the leap ( $W \propto mH$ ), while the work of the muscles is proportional to the mass of the muscle (or the whole organism) ( $W \propto m$ ). It follows then, that the total work is related solely to the height, since the organism's mass cancels out. Thus, the height of the leap is not proportional to the organisms's size, but rather is similar for any organism. D'Arcy Thompson describes this as an example of the Principle of Biological Similitude.

$$\mu_j^{\text{liquid}} = \mu_j^* + RT \ln a_j + \bar{V}_j P + z_j FE + m_j gh$$

$$RT \ln a_j + \bar{V}_j P + m_j gh$$

gravitational potential

$$a_j = \gamma_j c_j$$

$$\bar{V}_j = \left( \frac{\partial V}{\partial n_j} \right)_{n, T, P, E, h}$$

The activity of water ( $a_j$ ) is the product of the activity coefficient and the concentration of water

The partial molal volume of species  $j$  is the incremental increase in volume with the addition of species  $j$ . For water, it is  $18.0 \times 10^{-6} \text{ m}^3 \text{ mol}^{-1}$ .

$$RT \ln a_j = \bar{V}_j \Pi$$

osmotic pressure

$$\Pi_s = RT \sum_j c_j \quad \text{Van't Hoff relation}$$

The terms inter-relate various properties of water: changes in its activity with the addition of solutes, and the relation to pressure.