AS/ECON 2300G   Answers to Assignment 2   March 2004

1. A bundle (x_1, x_2) is directly revealed preferred to some other bundle (x_1', x_2') if the person chose (x_1, x_2) when she could have afforded (x_1', x_2') : that is when (x_1, x_2) was actually chosen at prices (p_1, p_2), but when $p_1 x_1' + p_2 x_2' \leq p_1 x_1 + p_2 x_2$.

The table below lists the costs of each of the three bundles in the question, in each of the three years. (For example, in year 1, $p_1 = 10$ and $p_2 = 1$, so that bundle 2, $(15, 12)$, cost $10(15) + 1(12) = 162$.)

bundle	1	2	3
year			
1	134	162	310
2	78*	81	120
3	82	75*	80

As in the text, asterisks indicate that the bundle chosen in that year has been revealed preferred to the asterisked bundle. So bundle #1 cost 78 in year 2 ; since bundle 2 was chosen, at a cost of 80, when bundle 1 was affordable, then bundle 2 is revealed preferred to bundle 1.

In the table, there are two instances of a bundle being directly revealed preferred to another : bundle 2 is revealed preferred to bundle 1, and bundle 3 is revealed preferred to bundle 2. That means that bundle 3 is *indirectly* revealed preferred to bundle 1 : it is directly preferred to bundle 2, which is directly preferred to 1.

But there are no violations of either $SARP$ or $WARP$ here : everything is consistent with the consumer ranking bundle #3 as best, followed by bundle #2, followed by bundle #1. She does not directly reveal bundle #1 preferred to anything, since neither of the other bundles is affordable in year #1.

(This could also be done graphically : for example, bundle #2 is directly revealed preferred to bundle #1 because bundle #1 is inside the budget set for year #2, which means that it is affordable.)

2. By definition,

$$\mathcal{L}_q \equiv \frac{p_1^b x_1^c + p_2^b x_2^c + p_3^b x_3^c}{p_1^b x_1^b + p_2^b x_2^b + p_3^b x_3^b}$$

$$\mathcal{P}_q \equiv \frac{p_1^c x_1^c + p_2^c x_2^c + p_3^c x_3^c}{p_1^c x_1^b + p_2^c x_2^b + p_3^c x_3^b}$$

Plugging in the values here

$$\mathcal{L}_q = \frac{10(15) + 1(2) + 1(5)}{10(5) + 1(5) + 1(10)} = \frac{157}{65} \approx 2.41$$

$$\mathcal{P}_q = \frac{5(15) + 3(2) + 3(5)}{5(5) + 3(5) + 3(10)} = \frac{96}{70} \approx 1.37$$

Since $\mathcal{P}_q > 1$, the current bundle $(15, 2, 5)$ is revealed preferred to the base bundle $(5, 5, 10)$, the consumer is better off in the current period : she could afford the base bundle in the current period if she wanted it, but instead chose the current bundle.

3. First, the consumer's demand function for good 1 must be calculated. If

$$U(x_1, x_2) = x_1 + 2\sqrt{x_2}$$

for the consumer, then

$$MU_1 = 1$$

and

$$MU_2 = \frac{1}{\sqrt{x_2}}$$

so that

$$MRS \equiv MU_1/MU_2 = \sqrt{x_2}$$

The consumer's optimization results in a tangency of her indifference curve with her budget line, or $MRS = p_1/p_2$, so that here she sets

$$MRS = \sqrt{x_2} = \frac{p_1}{p_2}$$

implying a demand function for good 2 of

$$x_2 = [\frac{p_1}{p_2}]^2$$

(Note that quantity demanded of good 2 is independent of the person's income, due to the quasi–linearity of her preferences.) But we need the demand function here for good 1. To find how much she buys of good 1, we use the consumer's budget constraint

$$p_1 x_1 + p_2 x_2 = m$$

so that

$$x_1 = \frac{m - p_2 x_2}{p_1}$$

Plugging in the demand function for good 2,

$$x_1(p_1, p_2, m) = \frac{m}{p_1} - \frac{p_2}{p_1}[\frac{p_1}{p_2}]^2 = \frac{m}{p_1} - \frac{p_1}{p_2}$$

is the consumer's demand function for good 1.

(Note :this is only valid if $m > (p_1)^2/p_2$; otherwise she would choose to buy $x_1 = 0$ and $x_2 = m/p_2$.)

Now that the demand function has been calculated, I can find how quantity demanded of good 1 varies with its own price, and with income :

$$\frac{\partial x_1}{\partial p_1} = -\frac{m}{(p_1)^2} - \frac{1}{p_2}$$

$$\frac{\partial x_1}{\partial m} = \frac{1}{p_1}$$

The Slutsky equation can now be used to calculate the *compensated* derivative of x_1 with respect to p_1. The Slutsky equation says that the compensated derivative, ($\partial x^s/\partial p_1$ in the notation of the text) equals

$$\frac{\partial x_1}{\partial p_1} + x_1(p_1, p_2, m)\frac{\partial x_1}{m}$$

So here

$$\frac{\partial x_1^s}{\partial p_1} = -\frac{m}{(p_1)^2} - \frac{1}{p_2} + (\frac{m}{p_1} - \frac{p_1}{p_2})\frac{1}{p_1} = -\frac{2}{p_2}$$

Here the compensated own–price derivative is negative, as it must be. Also, since good 1 is a normal good here, the substitution effect and income effect work in the same direction, so that the overall ("'uncompensated") demand derivative $\partial x_1/\partial p_1$ is negative.

4. In this case the person is (originally) a net buyer of clothing, and a net seller of food : when the price vector is $(4, 5)$, she sells 2 units of food in order to buy 1.6 units of clothing.

If a person is a net seller of food, then an increase in the price of food *must* make the person better off. To see this, note that if the price of food increases from 4 to 5, her original choice of consumption bundle $(8, 13.6)$ is still affordable : the cost of this bundle, at prices $(5, 5)$, is 108, and the value of her endowment is $(5)(10) + (5)(12) = 110$. So her original choice, $(8, 13.6)$ is inside her budget line when the price of food increases, and when the budget line pivots about her endowment point $(10, 12)$, as in figure 1.

Now if the price of clothing were to increase from 5 to 10 (and the price of food stayed at 4), the person's budget line pivots the other way : now her original choice $(8, 13.6)$ is no longer affordable. The value of her endowment is now 160, but the cost of the bundle $(8, 13.6)$ is 168, so the original choice is outside her budget set.

So it is not necessarily the case that an increase in the price of clothing will make her better off. It might make her worse off. In fact, section 9.4 of the text shows that she must be worse off, if she continues to buy clothing and sell food after the price change. So if she chooses a bundle such as $(9, 12.4)$ after the price increase, then she must be worse off. In figure 2, this new choice is a bundle which she could have chosen before, when the price of clothing was 5. Her new choice is inside her old budget set, so that — if she still chooses to sell food and buy clothing — her original choice $(8, 13.6)$ has been revealed preferred to her new one.

On the other hand, an increase in the price of clothing might induce her to switch, and to sell clothing and buy food. She might choose a bundle such as $(15, 10)$ when the price vector is $(4, 10)$.

In this case, the new choice might be better than the old one. In figure 3, revealed preference cannot tell us which bundle is better : her old choice $(8, 13.6)$ is outside her budget set at the new prices, but her new choice $(15, 10)$ is outside her budget set at the old prices.

5. This is the Slutsky equation again, now applied to a situation in which the person has an endowment of leisure which can be used either for consumption, or to be sold to earn income. In this case, if R is the number of hours a week she consumes of leisure, and L is the number of hours a week she works, then the Slutsky equation is

$$\frac{\partial R}{\partial w} = \frac{\partial R^s}{\partial w} + L\frac{\partial R}{\partial m}$$

where w is the hourly wage rate, m her exogenous income, and $\partial R^s/\partial w$ is her compensated demand derivative of leisure with respect to its own price, which is the wage rate w.

The question states that she would reduce her hours worked per week by 2 hours, for every extra \$100 a week that she received. That implies that

$$\frac{\partial R}{\partial m} = \frac{2}{100} = 0.02$$

so that the Slutsky equation says that

$$\frac{\partial R}{\partial w} = -1 + 40(0.02) = -0.20$$

in this case. Here the substitution effect outweighs the income effect. Every dollar increase in her hourly wage rate will cause her to demand 0.2 fewer hours per week of leisure. Reducing hours of leisure is another way of saying increasing hours of work. This person would work more hours if her wage rate were to rise : every \$1 per hours increase in the wage rate would induce her to want to work 0.2 more hours per week.