AS/ECON 2300 3.0MW February 2007

Answers to Assignment 1

$Q1$. Draw the budget set for the following person.

The person consumes milk and cookies. She owns a farm, which produces 100 litres of milk per week. She is allowed to sell up to 40 litres of milk each week to the milk marketing board, at a price of \$2 per litre. If she wants to sell more than 40 litres of milk per week, she must sell it at the farmers' market, where the price of milk is \$1 per litre.

Any money she earns from selling milk, she can use to buy cookies. Cookies cost \$1 each. Any milk which she chooses not to sell, she can consume herself.

$A1$. One option the person has is not to sell any milk. That is, she could consume all 100 litres per week of the milk produced on her farm. If she did consume 100 litres of milk, then she could not consume any cookies, since she would not have any money to spend on cookies. So the point $(100, 0)$ is on the edge of her budget set, where the first number is the quantity (in litres per week) of milk, and the second is the quantity of cookies per week.

If she sold x litres of milk to the marketing board (where $x \leq 40$), she would get $2x$ dollars to spend on cookies, which would enable her to buy $2x$ cookies, since cookies cost \$1 each. So combinations $(100-x, 2x)$ are on the border of her budget set, if $0 \leq x \leq 40$. For example, $(95, 10)$ and $(70, 60)$ are on the border of her budget set. This border has a slope of -2 : if she reduces milk consumption by one litre, she can consume 2 more cookies.

The most milk that she can sell to the milk marketing board is 40 litres. So the point $(60, 80)$ is on the border of her budget set : selling exactly 40 litres of milk to the marketing board, at \$2 per litre, enables her to buy 80 cookies, and leaves her with 60 litres of milk to consume.

If she sells more than 40 litres of milk, then she only gets \$1 per litre, and can only consume 1 cookie for each additional litre of milk sold. So points of the form $(60 - y, 80 + y)$ are on the boundary of her budget set, with $0 \leq y \leq 60$ For example, $(30, 110)$ is on the boundary of her budget set : if she sold 70 litres of milk, she would have 30 litres left to consume ; she would get \$110 from selling the milk (\$80 for the first 40 litres, and \$30 for the next 30 litres), enabling her to consume 110 cookies.

This, to the left and above the point $(60, 80)$, the boundary of the budget set has a slope of -1 : each litre of milk sold means one more cookie can be bought.

The most milk she can sell is all 100 litres : that would leave her with no milk to consume, and with $40(2) + 60(1) = 140$ dollars to spend on cookies. Therefore, the point $(0, 140)$ is at the top end of her budget set.

The accompanying figure shows the boundary of the budget set. Every combination (M, C) of milk and cookies which is on or below the red curve in the figure (and above the axes) is in her budget set.

*Q*2. Are the following preferences well–behaved? Explain briefly.

The person measures the value of a bundle by its distance from the origin, in a diagram. That is, he prefers the bundle (F, C) to the bundle $(\tilde{F}, \tilde{C})$ if (and only if) the point (F, C) is farther from the point $(0, 0)$ than is the point $(\tilde{F}, \tilde{C})$.

*A*2. The preferences are monotonic : increasing either the person's food consumption, or his clothing consumption, moves the consumption bundle further away from the bundle $(0, 0)$, which means that the person values the bundle more.

But the preferences are not convex. One way of seeing that is by noting that the "better than" sets are not convex. For example, the bundles $(6, 0)$ and $(0, 6)$ are both on the same indifference curve ; they are each a distance 6 from the origin. The bundle $(3, 3)$ is halfway between them, but it's closer to the origin than either $(6, 0)$ or $(0, 6)$. So that example shows that preferences are not convex : two bundles are on the same indifference curve, but the bundle which is halfway between them is on a lower indifference curve.

Since the length of a line connecting the origin $(0, 0)$ with the point (x, y) is $\sqrt{x^2 + y^2}$, the preferences in this question can be represented by the utility function

$$u(F, C) = \sqrt{F^2 + C^2}$$

or (taking a monotonic transformation of $u(F, C)$ by squaring it)

$$U(F, C) = F^2 + C^2$$

Then the MRS is

$$MRS = \frac{U_F}{U_C} = \frac{F}{C}$$

which gets **bigger**, not smaller, in absolute value as F increases and C decreases.

*Q*3. What do the indifference curves look like for a person whose preferences can be represented by the utility function below?

$$u(x_1, x_2) = \min(x_1 + 2x_2, 2x_1 + x_2)$$

Are the preferences well–behaved?

*A*3. The score the person assigns to a bundle (x_1, x_2) is the smaller of $x_1 + 2x_2$ and $2x_1 + x_2$. Which of those numbers is smaller? $x_1 + 2x_2 > 2x_1 + x_2$ is the same thing as $x_2 > x_1$. (Just subtract $x_1 + x_2$ from both sides.) So the person's preferences can be represented by the utility function

$$u(x) = \begin{cases} = 2x_1 + x_2 & \text{if} \quad x_1 < x_2 \\ = x_1 + 2x_2 & \text{if} \quad x_1 > x_2 \end{cases}$$

Those preferences are monotonic : the higher x_1 or x_2 is, the higher is $u(x_1, x_2)$. Alternatively, the marginal utility of good 1 is either 1 or 2, depending on whether or not $x_1 > x_2$; so regardless of whether or not $x_1 > x_2$, $MU_1 > 0$. (Similarly, MU_2 is either 2 or 1.)

The slope of the indifference curve is the ratio of the marginal utilities. So if $x_1 < x_2$, then

$$MRS = \frac{MU_1}{MU_2} = 2$$

and if $x_2 < x_1$, then

$$MRS = \frac{MU_1}{MU_2} = \frac{1}{2}$$

So the indifference curves are like the ones in the accompanying figure. Above the (dotted) line $x_1 = x_2$ they are straight lines with slope -2 and below the dotted line they are straight lines with slope $-1/2$. Since the indifference curves get less steep as we move from above the 45–degree line to below the 45–degree line, the preferences are convex.

*Q*4. If a person's preferences could be represented by the utility function

$$u(x_1, x_2) = x_1 + x_2 + 2\sqrt{x_1}$$

are the preferences well–behaved? What is the person's marginal rate of substitution between the two goods if she has these preferences.

*A*4. Since

$$\frac{\partial u}{\partial x_1} = 1 + \frac{1}{\sqrt{x_1}}$$

$$\frac{\partial u}{\partial x_2} = 1$$

both marginal utilities are strictly positive : the preferences are monotonic.

Here

$$MRS = \frac{MU_1}{MU_2} = 1 + \frac{1}{\sqrt{x_1}}$$

As we move down and to the right along an indifference curve, x_1 increases, so that the MRS falls in absolute value : the preferences are convex.

*Q*5. What would a person's demand function be for good 1, if her preferences could be represented by the utility function defined in question #4 above?

*A*5. If the person's indifference curve is tangent to her budget line, then

$$MRS = \frac{p_1}{p_2}$$

From the answer to question 4, the MRS will equal the price ratio if (and only if)

$$1 + \frac{1}{\sqrt{x_1}} = \frac{p_1}{p_2} \tag{5 − 1}$$

But notice that the left side of equation $(5 - 1)$ is always greater than 1 in value : $1 + \sqrt{x_1} > 1$. So if $p_1/p_2 < 1$, then equation $(5 - 1)$ cannot hold : the right side is less than 1, and the left side is greater than 1.

That is, if $p_1 < p_2$, then this person's indifference curves are always steeper than her budget line, no matter how far down and to the right we go. So if $p_1 < p_2$, the person will want to spend all her money on good 1. Instead of having an indifference curve tangent to the budget line, she will choose a consumption bundle at which $x_2 = 0$, choosing to spend all her money on good 1.

In fact, this corner solution may arise even when $p_1 > p_2$. Equation $(5-1)$ can also be written

$$\frac{1}{\sqrt{x_1}} = \frac{p_1}{p_2} - 1 \tag{5 − 2}$$

or

$$\sqrt{x_1} = \frac{p_2}{p_1 - p_2} \tag{5 − 3}$$

so that

$$x_1 = (\frac{p_2}{p_1 - p_2})^2 \tag{5 − 4}$$

The demand function defined by equation $(5 - 3)$ does not make sense if $p_1 < p_2$, confirming what was mentioned earlier : if $p_1 < p_2$, the person spends all her money on good 1.

But even if $p_1 > p_2$, there may be a problem with $(5 - 4)$. How much money will the person spend on good 1, if her demand obeys equation $(5 - 4)$? She spends p_1 per unit of good 1, so that her expenditure on good 1 is

$$p_1(\frac{p_2}{p_1 - p_2})^2$$

She cannot spend a negative amount of money on good 2. So if her demands make sense, then it must be true that

$$p_1(\frac{p_2}{p_1 - p_2})^2 \leq m \tag{5 − 5}$$

where m is her income.

If good 1 is cheap enough, and her income is high enough, then her demand for good 1 is defined by equation $(5 - 4)$. But if $(5 - 5)$ does not hold, then her indifference curves are steeper than her budget line — even if she spends all her money on good 1 — so that she will choose to spend all her money on good 1, and her demand function is

$$x_1 = \frac{m}{p_1}$$