Q1. Suppose that a person has a most preferred "ideal" combination of hours T watching television and hours S spent playing soccer (T^*, S^*). Her preferences over other combinations (T, S) of hours spent watching television and playing soccer are determined entirely by how close a combination is to her ideal combination. That is, she prefers the combination (T_1, S_1) to the combination (T_2, S_2) if and only if (T_1, S_1) is closer to (T^*, S^*) than (T_2, T_2) is, when combinations are graphed in a diagram (with T on the horizontal axis, and S on the vertical).

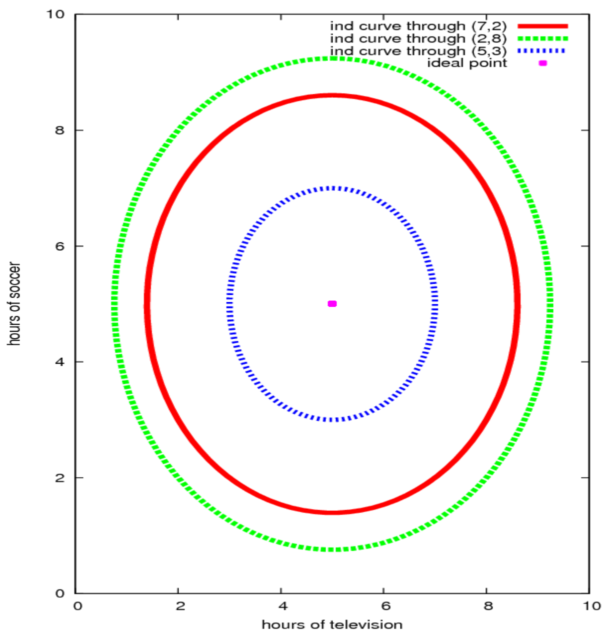
Are these preferences monotonic? Convex?

Explain briefly.

A1. The indifference curves for this example are illustrated in figure 1 below (and are similar to those in figure 3.7 in Varian.)

The preferences are **not** monotonic : if $T > T^*$, then the person already is spending more time watching television than she considers ideal. So further increases in T — moving right in the figure — move her to a lower indifference curve. Similarly, if $S > S^*$, then increasing S further (moving up in the diagram) moves her to a lower indifference curve.

The preferences **are** convex, since the combinations of (T, S) which she likes at least as much as some given combination (T_1, S_1) are a convex set. For example, the combinations which she likes at least as much as the combination $(7, 2)$ in the figure are all the combinations on or inside the red circle in the diagram. The points on and inside a circle form a convex set.



Q2. What is a person's demand function for food, if her preferences can be represented by the utility function

$$u(F,C) = C - \frac{1}{F}$$

where C is her clothing consumption, and F her food consumption?

A2. If a person chooses a bundle (F,C) so as to get to the highest indifference curve on her budget line $p_F F + pCC = m$, then it must be true that her marginal rate of transformation equals the price ratio.

So the point she chooses on the budget line, where her indifference curve is tangent to the budget line, must be a consumption bundle at which

$$MRS = \frac{p_F}{p_C} \qquad (2-1)$$

As well, her chosen consumption bundle must be on the budget line, and so must obey the equation

$$p_F F + pCC = m \qquad (2-2)$$

In this question, the preferences are represented by the utility function $u(F,C) = C - \frac{1}{F}$, so that the marginal tutilities of food and clothing consumption are

$$MU_F \equiv \frac{\partial u}{\partial F} = \frac{1}{F^2} \qquad (2-3)$$

$$MU_C \equiv \frac{\partial u}{\partial C} = 1 \qquad (2-4)$$

So that her marginal rate of substitution is

$$MRS \equiv \frac{MU_F}{MU_C} = \frac{1}{F^2} \qquad (2-5)$$

Using $(2-5)$, the optimality condition for the consumer's choice of consumption bundle $(2-1)$ becomes

$$\frac{1}{F^2} = \frac{p_F}{p_C} \qquad (2-6)$$

Multiplying both sides of $(2-6)$ by F^2, it becomes

$$1 = F^2 \frac{p_F}{p_C} \qquad (2-7)$$

or

$$F^2 = \frac{p_C}{p_F} \qquad (2-8)$$

Taking the square root of both sides of $(2-7)$ yields the person's demand function for food

$$F = \sqrt{\frac{p_C}{p_F}} \tag{2-9}$$

[Since preferences here are *quasi–linear*, the person's demand for food is independent of income.

The demand function for clothing can then be calculated from the budget line equation. Since

$$C = \frac{m - p_F F}{p_C} \tag{2-10}$$

equation $(2-9)$ implies that

$$C = \frac{m}{p_C} - \sqrt{\frac{p_F}{p_C}} \tag{2-11}$$

is the demand function for clothing.

Actually, $(2-9)$ and $(2-11)$ define the demands for food and clothing only if they define non–negative levels of consumption. So, from $(2-11)$, equations $(2-9)$ and $(2-11)$ define the demands for food and clothing only if the person's income is high enough so that

$$m \geq \sqrt{p_F p_C} \tag{2-12}$$

If condition $(2-12)$ did not hold, the person would instead be at a corner solution, spending all her money on food, with $F = m/p_F$.]

Q3. Give an example (in numbers, or in a graph) of behavior which violates the *Weak Axiom of Revealed Preference.*

A4. The weak axiom of revelaled preference says that if the person chose the bundle (F_1, C_1) in period 1, and (F_2, C_2) in period 2, and if she could have afforded bundle (F_2, C_2) in period 1, **then** she could **not** have afforded bundle $((F_1, C_1)$ in period 2.

In equation form, *WARP* ays that (if the person chose the bundle (F_1, C_1) in period 1, and (F_2, C_2) in period 2),

$$\text{if} \quad p_F^1 F_1 + p_C^1 C_1 \geq p_F^1 F_2 + p_C^1 C_2 \quad \text{then} \quad p_F^2 F_1 + p_C^2 C_1 > p_F^2 F_2 + p_C^2 C_2 \tag{3-1}$$

if (p_F^1, p_C^1) and (p_F^2, p_C^2) are the prices of food and clothing in periods 1 and 2.

Graphically, a pircture like figure 7.4 of the text shows a violation of $WARP$: period 2's choice is inside period 1's budget line, but period 1's choice is inside period 2's budget line.

Numbers which would give rise to this sort of graph might include initial period prices of $(p_F^1, p_C^1) = (2, 1)$, second–period prices of $(p_F^2, p_C^2) = (1, 2)$ and choices by the consumer of $(F_1, C_1) = (10, 5)$ in the first period and $(F_2, C_2) = (5, 10)$ in the second period. [Here period 2's bundle costs \$20 in period 1, less than the cost $(2)(10) + (1)(5) = 25$ of the bundle she actually chose, but period 1's bundle costs only \$20 in period 2, less than the cost $(1)(5) + (2)(10) = 25$ of the bundle actually chosen in period 2.]

Q4. Use the Slutsky equation to explain why a person's supply of labour might decrease with the net hourly wage which she can earn.

A4. If the person has an "endowment" $\bar{R}$ of hours per week, which she can use either for paid work, or for leisure, then the Slutsky equation can be written (as in (9.4) of the text as

$$\frac{\partial R}{\partial w} = \frac{\partial R}{\partial w}|^{subst} + (\bar{R} - R)\frac{\partial R}{\partial m} \tag{4-1}$$

where R is the total time per week spent at lesisure (i.e. not at work), w is the person's hourly wage, and $\frac{\partial R}{\partial w}|^{subst}$ is the substitution effect, the compensated derivative of leisure demand with respect to the wage. Since the wage is the price of an hour of leisure, this substitution effect $\frac{\partial R}{\partial w}|^{subst}$ must be negative.

So the first term on the right hand side of the Slutsky equation $(5-1)$ must be negative. The second term, however, will be positive if leisure is a normal good. By definition, leisure is a normal good if and only demand for leisure increases with a person's outside income, or $\frac{\partial R}{\partial m} > 0$. Since leisure consumption R must be less than the total available hours $\bar{R}$ of time per week if the person chooses to do any paid work, the second term on the right side of $(5-1)$ will be positive if leisure is a normal good.

Since hours available $\bar{R}$ can be spent either at work or at leisure

$$R + L = \bar{R} \tag{5-2}$$

where L is the amount of work the person does per week, so that

$$\frac{\partial L}{\partial w} = -\frac{\partial R}{\partial w} \tag{5-3}$$

and the Slutsky equation $(5-1)$ can be written

$$\frac{\partial L}{\partial w} = -\frac{\partial R}{\partial w}|^{subst} - L\frac{\partial R}{\partial m} \tag{5-4}$$

The first term on the right side of $(5-4)$ is positive ; the second is negative if leisure is a normal good. So if leisure is a normal good, and if the substitution effect is relatively small, the left side of $(5-4)$ will be negative, and the person's choice of labour supply L will decline with her wage w.