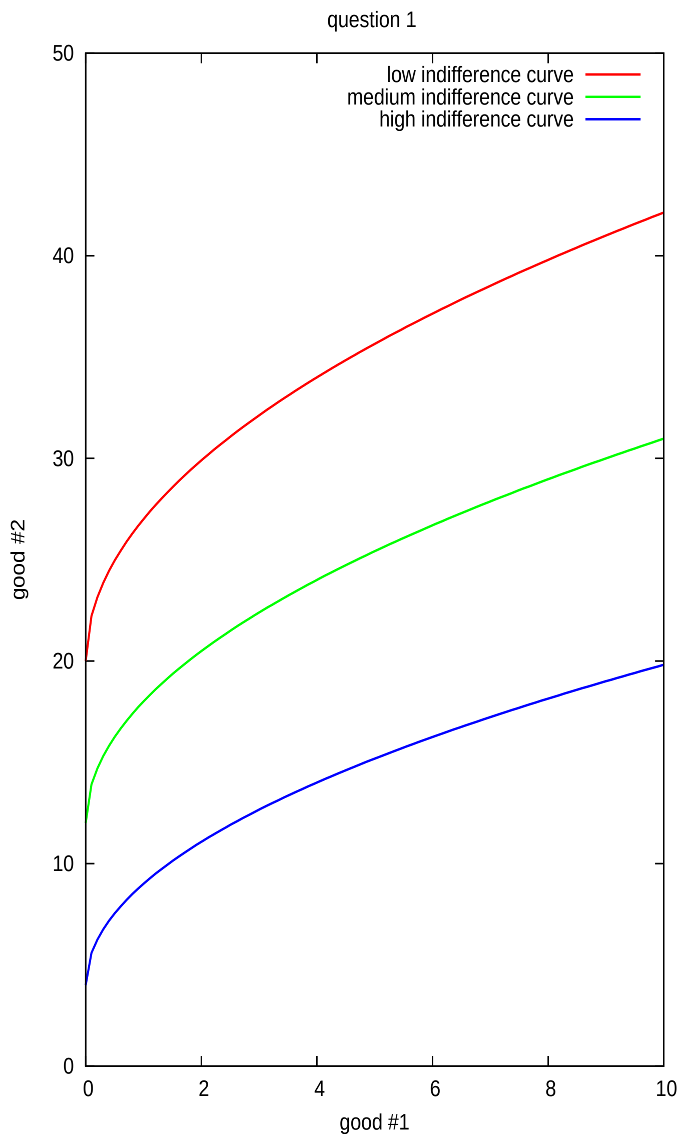
*Q*1. What would a person's indifference curves look like if her preferences had **all** of the following properties (simultaneously)? : (*i*) she regards good #1 as a good, so that a higher quantity consumed of good #1 makes her better off ; (*ii*) she regards good #2 as a bad, so that a higher quantity consumed of good #2 makes her worse off, and (*iii*) her preferences are convex.

*A*1. Since commodity 1 is a good, and commodity 2 is a bad, her indifference curves must slope **up**. Graph consumption of good 1 on the horizontal axis, and consumption of good 2 on the vertical. Now consider some consumption bundle $A$. Bundles which are above and to the left of $A$ must be **worse** than $A$ for this consumer, since bundles above and to the left have less of good 1 (which is bad) and more of good 2 (which also is bad).

The set of bundles which the person likes better than $A$, those on a higher indifference curve than the curve through $A$, are below and to the right of $A$. Property (*iii*) is that this person has convex preferences. So the set of bundles she likes better than $A$ must be a convex set. That means that the indifference curve through $A$ must get **less steep** as we move up and to the right.

Figure 1 illustrates some indifference curves for this person.


question 1
low indifference curve
medium indifference curve
high indifference curve
good #2
good #1
0
10
20
30
40
50
0
2
4
6
8
10

*Q*2. What is a person's demand function for good #1, if she regards good #1 and good #2 as *perfect complements*, so that her preferences can be represented by the utility function

$$U(X_1, X_2) = \min{(X_1, X_2)}$$

where $X_1$ and $X_2$ are the quantities consumed of goods #1 and #2?

*A*2. A person who regards two goods as perfect complements has $L$–shaped indifference curves, as depicted in figure 3.4 of the text.

So she always wants to consume the two goods in the same fixed proportions — no matter what are the relative prices.

In the example in this question, the fixed proportions are 1–to–1 : her preferred choice always has $X_1 = X_2$.

That is, she always wants to be at the kink in her indifference curve, which is located where $X_1 = X_2$.

Since these preferences are monotonic (both commodities are goods, not bads), she will also want to be on her budget line.

So the consumption bundle she chooses must satisfy the following two equations

$$X_1 = X_2 \qquad (2-1)$$

$$p_1X_1 + p_2X_2 = M \qquad (2-2)$$

The first equation says that she consumes equal quantities of the two goods ; the second says that she chooses a point on her budget line. (Figure 5.6 in the text illustrates.)

Since $X_1 = X_2$, equation $(2-2)$ can be written

$$p_1X_1 + p_2X_1 = M \qquad (2-3)$$

or

$$X_1 = \frac{M}{p_1 + p_2} \qquad (2-4)$$

Equation $(2-4)$ is this person's demand function for good 1, since it expresses the quantity $X_1$ she wants to buy of good 1 as a function of her income $M$, and the prices $p_1$ and $p_2$ of the 2 goods.

*Q*3. Write down the Slutsky equation [in a "simple" one–period model, in which a person's income is exogenous]. (You do not need to derive it.)

What does this equation say about the slope of a person's demand curve?

*A*3. The Slutsky equation splits the effect of a change in price into two terms, the substitution effect and the income effect.

So

$$\frac{\partial X_1}{\partial P_1} = \frac{\partial X_1}{\partial P_1}|_{subst} - X_1 \frac{\partial X_1}{\partial M} \tag{3-1}$$

[This is the equation at the bottom of the first page of the appendix to chapter 8 in the text.]

The first term on the right hand side of equation $(3-1)$ is the substitution effect : the effect on demand for good 1 of changing the price of good 1, and at the same time, adjusting the person's income so that her old consumption bundle is exactly on her new budget line. That means changing the person's income by $(\Delta P_1)X_1^0$, where $X_1^0$ is her original quantity demanded of good 1, and $\Delta P_1$ is the change in the good's price.

This substitution term $\frac{\partial X_1}{\partial P_1}|_{subst}$ must be **negative**.

The second term is the income effect : the effect of giving back the change in income $-(\Delta P)X_1^0$ from the first stage (and keeping prices constant). This income effect $-X_1\frac{\partial X_1}{\partial M}$ will be negative if — and only if — good 1 is **normal** (so that $\frac{\partial X_1}{\partial M} > 0$).

So the Slutsky equation says that the ordinary demand curve for a normal good must slope down, and that the ordinary demand curve for an inferior good could slope up or down, depending on whether the income effect outweighs the substitution effect.

*Q4.* How would a person's preferred level of savings vary with the interest rate $r$ if her preferences could be represented by the utility function

$$U(C_1, C_2) = \log(C_1) + C_2$$

where $C_1$ is her consumption in period 1 and $C_2$ is her consumption in period 2? The person receives exogenous income of $M_1$ in period 1 (and no exogenous income in period 2).

*A4.* The utility function $U(C_1, C_2) = \log(C_1) + C_2$ is **quasi–linear**. So demand for consumption in the first period, $C_1$ is independent of the person's exogenous income ; that must always be the case with quasi–linear preferences.

With $C_1$ independent of $M$, there is no income effect in the Slutsky equation. That means the only effect of the interest rate on the person's first–period consumption is the substitution effect, which must be negative.

Therefore, for this person, savings (which equals first–period income minus first–period consumption) must increase with the interest rate $r$.

Checking, this person chooses $C_1$ and $C_2$ so as to maximize

$$\log(C_1) + C_2 \tag{4-1}$$

subject to the constraint that

$$C_2 = (1+r)S = (1+r)(M_1 - C_1) \tag{4-2}$$

where $M_1$ is the exogenous income she gets in period 1. (The question says that she gets no exogenous income in period 2.)

Substituting from $(4-2)$ into $(4-1)$, she should pick $C_1$ so as to maximize

$$\log(C_1) + (1+r)(M_1 - C_1) \tag{4-3}$$

So she should find the value of $C_1$ which makes the derivative of expression $(4-3)$ (with respect to $C_1$) equal zero.

That means

$$\frac{1}{C_1} - (1+r) = 0 \tag{4-4}$$

or

$$C_1 = \frac{1}{1+r} \tag{4-5}$$

Since $S = M_1 - C_1$, therefore

$$S = M_1 - \frac{1}{1+r} \tag{4-6}$$

so that savings must be an increasing function of the interest rate, as implied by the Slutsky equation.