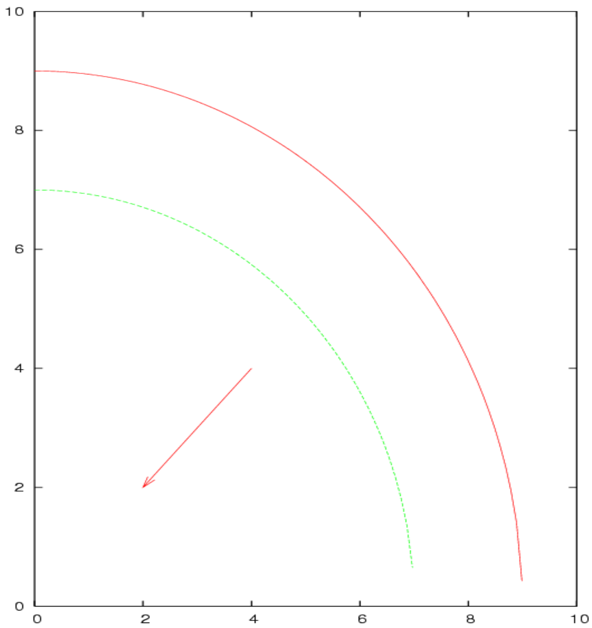
AS/ECON 2300 Answers to Midterm February 2004

1. The picture below illustrates two indifference curves for a person who has strictly convex preferences, and who regards both good 1 and good 2 as "bads". The indifference curves slope down, since more of good 1 makes the person worse off, so that she would need less of good 2 to stay on the same indifference curve. The preference direction is down and to the left : less is better. The indifference curves also get more steep as we move down and to the right : as drawn, the "better–than" sets are the sets *below* the indifference curves. These better–than sets will be convex only if the curves get steeper as we move down and to the right.



indifference curves when both commodities are bads, but preferences are convex

2. The answer is "no, they can't cross". The reason why is outlined on pages 36–38 of the text.

In short, suppose that two distinct indifference curves cross at some point (x_1, x_2). Since they are two distinct indifference curves, they represent different levels of utility. One of them represents a higher level of utility. So suppose that (x_1', x_2') is on the indifference curve representing the more–preferred bundles, and $(x_1", x_2")$ is on the curve representing less–preferred bundles. Since they are on distinct indifference curves, $(x_1', x_2') \succ (x_1", x_2")$. But (x_1, x_2) is the point where the indifference curves cross. That means that (x_1, x_2) is on the same indifference curve as (x_1', x_2'), and on the same indifference curve as $(x_1", x_2")$. So $(x_1, x_2) \sim (x_1', x_2')$ and $(x_1, x_2) \sim (x_1", x_2")$ which (by transitivity of preferences) means that $(x_1', x_2') \sim (x_1", x_2")$, contradicting the fact that (x_1', x_2') is on a higher indifference curve than $(x_1", x_2")$.

3. If

$$U(x_1, x_2) = x_1 + 10\sqrt{x_2}$$

then

$$MU_1 = 1$$
$$MU_2 = \frac{5}{\sqrt{x_2}}$$

so that

$$MRS = MU_1/MU_2 = \frac{\sqrt{x_2}}{5}$$

The person chooses a bundle such that $MRS = p_1/p_2$. Since here $p_1 = p_2 = 1$, therefore, her optimal choice implies that

$$\frac{\sqrt{x_2}}{5} = 1$$

or

$$x_2 = 25$$

Her budget line has the equation

$$x_1 + x_2 = 125$$

when $p_1 = p_2 = 5$ and $m = 125$, so that $x_1 = 100$ when $x_2 = 25$.

4. If

$$U(x_1, x_2) = \min(ax_1, bx_2)$$

then the person's indifference curves are "backwards L" shaped, with kinks at points for which $ax_1 = bx_2$.

The person will choose a bundle where such an indifference curve is "tangent" to the budget line, that is where $ax_1 = bx_2$ (as in figure 5.6 of the text). So if p_1 and p_2 are the prices of the two goods, and m is the person's income, her demands satisfy the equations

$$ax_1 = bx_2$$

$$p_1x_1 + p_2x_2 = m$$

Which can be solved

$$x_1 = \frac{b}{bp_1 + ap_2}m$$
$$x_2 = \frac{a}{bp_1 + ap_2}m$$

(The text solves the demands for the case $a = b = 1$ on pages 79–81.) So demands for each good are proportional to income. Increasing income will increase x_1 and x_2 by the same proportion. That is, the income expansion path is a straight line through the origin, with slope a/b.

More directly, the person will always be at the kink of the backwards–L shaped indifference curves, so as her income increases, she moves up the curve $ax_1 = bx_2$ which is a straight line through the origin with slope a/b.