AS/ECON 2300 Answers to Mid–term Exam W2007

$Q1$. Suppose that a person consumes only food and telephone calls, and has \$200 per week to spend on the two goods. The price of food is \$1 per unit. If the person makes fewer than 20 telephone calls per week, she pays \$5 per call. If she makes 20 calls or more, then she pays \$100 for the first 20 calls, and \$1 per call for any additional calls (in excess of 20).

Sketch the person's budget set.

$A1$. If food consumption is graphed on the horizontal axis, and telephone calls on the vertical, then the slope of the budget line is the price of food divided by the price of telephone calls. The price of food is \$1. The price of telephone calls varies : it's \$5 if $T < 20$, and \$1 if $T \geq 20$, where T is the number of telephone calls.

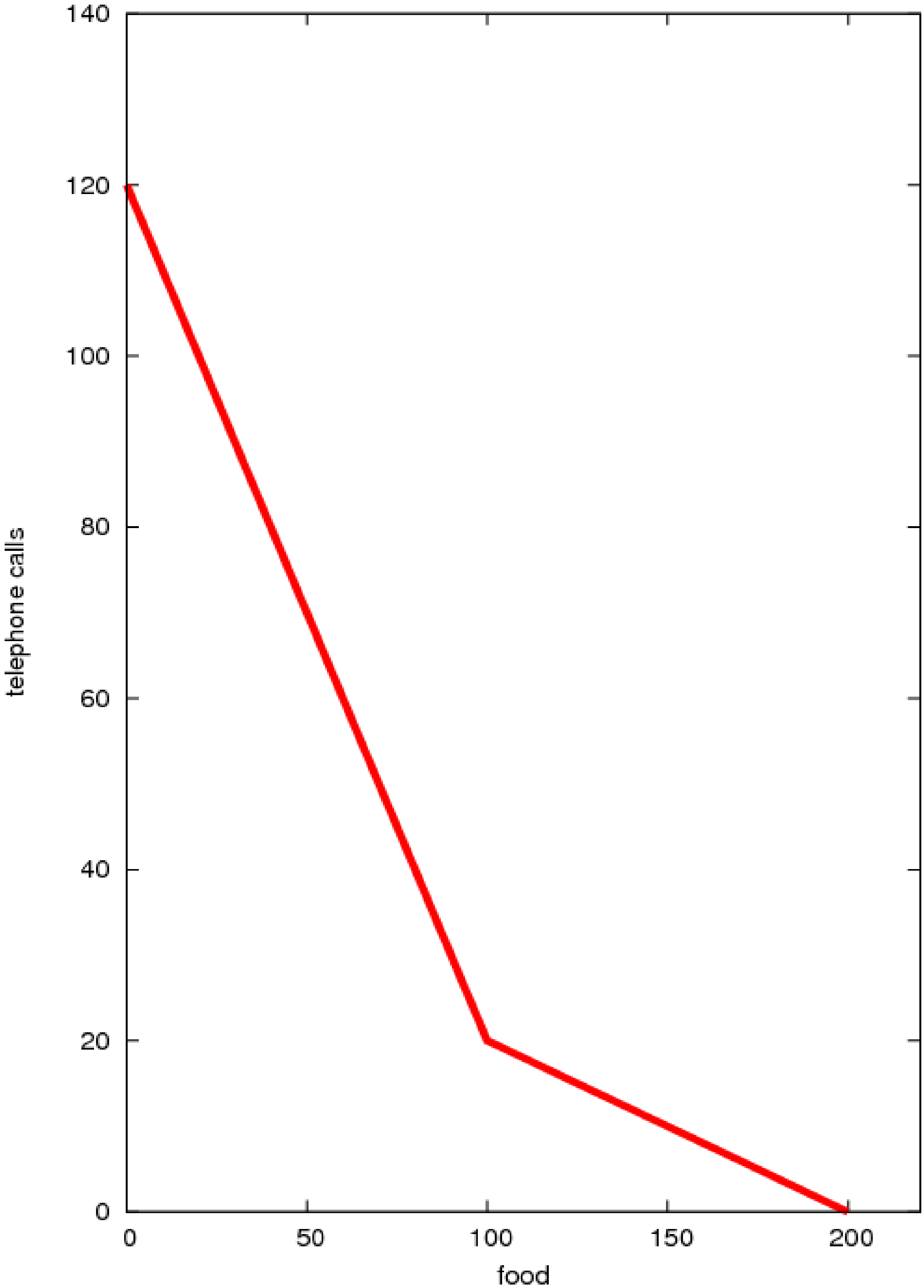
That means that the slope of the budget line is $-1/5$ if $T < 20$, and -1 if $T \geq 20$. There is a kink in the boundary of the budget set at $T = 20$.

What is food consumption F if $T = 20$? If $T = 20$, the person must spend \$100 on telephone calls, since calls cost \$5 each when $T < 20$. That leaves \$100 for food. Since food costs \$1 per unit, $F = 100$ when $T = 20$. Therefore the point $(100, 20)$ is on the boundary of her budget set : the bundle $(100, 20)$ costs exactly \$200, which is the person's available income.

The boundary of the budget set has a kink at $(100, 20)$, as in the figure. To the right of the kink, the boundary of the budget set has a slope of $-1/5$ and to the left it has a slope of -1.

One of the points on the boundary of the budget set is $(200, 0)$: she could buy 200 units of food if she bought no telephone calls. Another point on the boundary is $(0, 120)$: spending all of her income, \$200, on telephone calls gets her 120 calls — 20 with her first \$100, and then 100 more with the next \$100.

Other points on the boundary of the budget set include $(150, 10)$, $(120, 16)$, $(80, 40)$ and $(40, 80)$.



Question 1 : a budget set with a kinked boundary

$Q2$. What is a person's demand function for food, if her preferences can be represented by the utility function

$$u(F, C) = C + \ln F$$

where C is her clothing consumption, and F her food consumption?

[Note : the derivative of $\ln x$ with respect to x is $1/x$.]

$A2$. When a person chooses the most–preferred point in her budget set, she always chooses a consumption bundle for which the marginal rate of substitution between goods equals the ratio of the prices of the goods :

$$MRS = \frac{MU_F}{MU_C} = \frac{p_F}{p_C} \tag{2-1}$$

Given that her preferences are represented by the utility function $u(F, C) = C + \ln F$, here

$$MU_F = \frac{1}{F}$$

$$MU_C = 1$$

so that equation $(2-1)$ becomes

$$\frac{1}{F} = \frac{p_F}{p_C} \tag{2-2}$$

or

$$F = \frac{p_C}{p_F} \tag{2-3}$$

Equation $(2-3)$ is the person's demand function for food, since it expresses quantity demanded F of food as a function of the prices of food and clothing, and her income. (Here, because the preferences are *quasi–linear*, the quantity demanded of food does not depend on the person's income.)

(Note : If food demand is defined by equation $(2-3)$, then her expenditure on food, $p_F F$, equals $p_F(p_C/p_F) = p_C$, which leaves the person with $m - p_C$ to spend on clothing. Therefore, equation $(2-3)$ defines the person demand function for food only if $m \geq p_C$, so that she can consume a non–negative amount of clothing. If $p_C > m$, then the person chooses to spend all her money on food, and her demand function for food would be $F = m/p_F$.)

$Q3$. If the federal government made food purchases subject to the GST (that is, if they imposed a 6 percent tax on all food purchases), but also gave a cash grant to people to compensate for the damage, what would happen to a person's food consumption, if the cash grant was exactly enough so that she could exactly afford her original consumption bundle?

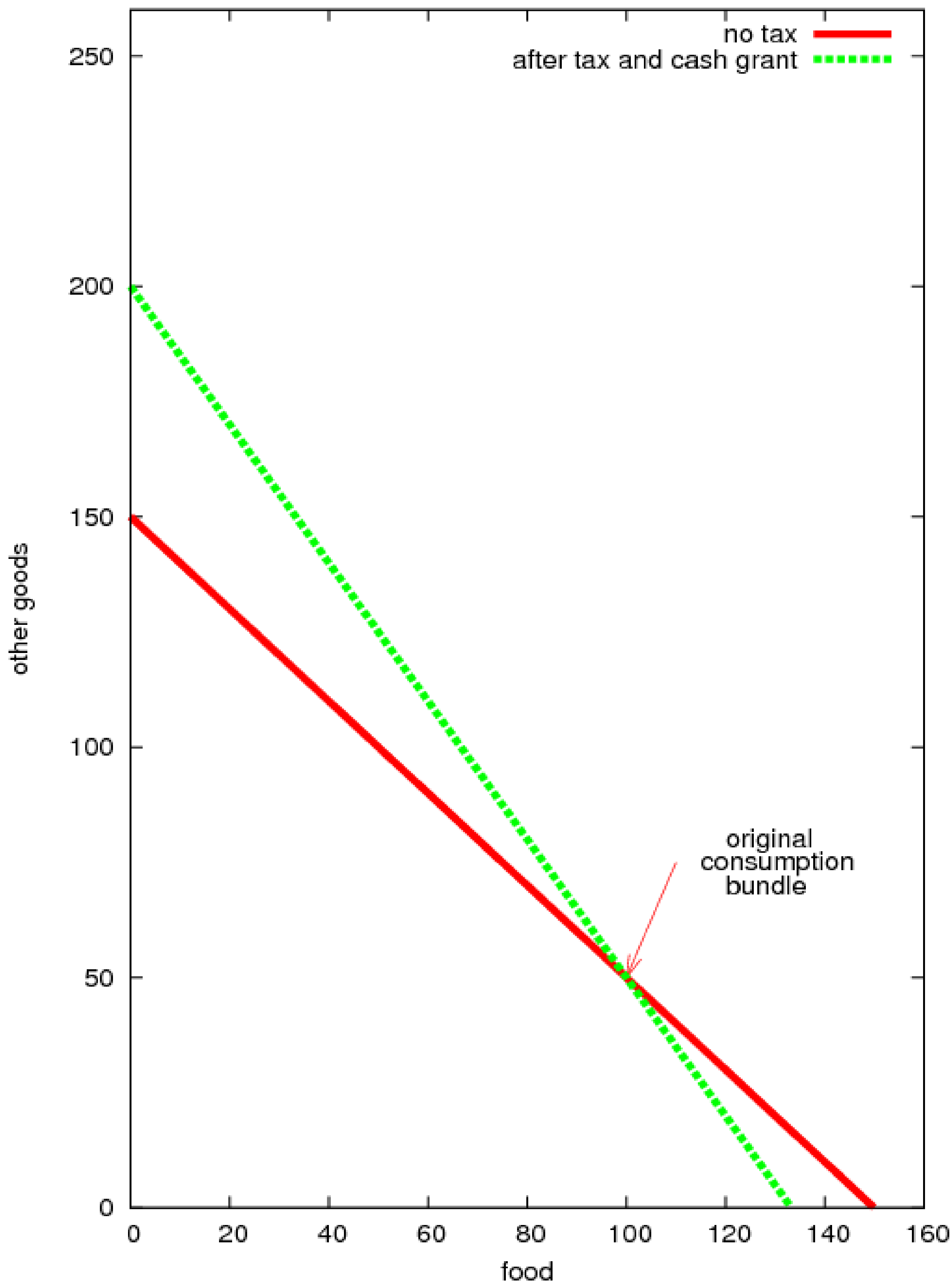
Explain briefly.

$A3$. The government policy — taxing food, but also giving a cash grant so that the person could still exactly afford her original consumption bundle — must **reduce** the person's food consumption.

(Figure 8.2, and section 8.3, in Varian's text explain why : the explanation below is basically a repeat of that argument.)

The figure below illustrates (for a higher tax rate than 6%, so that the diagram is clearer). The person's original consumption bundle, in the figure, is at $(100, 50)$ (when food is graphed along the horizontal axis). The tax on food makes the budget line steeper. The cash grant shifts the budget line out, so that it still goes through $(100, 50)$. (So the red line is the original budget line, when food is not taxed. The green line is the budget line after the government has introduced the tax on food, and paid the grant. The green line is steeper, since food is more expensive, but also goes through $(100, 50)$.

The weak axiom of revealed preference (chapter 7) shows that the person's new choice of consumption, when she faces the steeper, green budget line, must lie above and to the left of $(100, 50)$. Since she chose to consume $(100, 50)$ when food was cheap (and she faced the less–steep red budget line), she has revealed that she prefers $(100, 50)$ to everything on, or inside, the red budget line. The original consumption bundle $(100, 50)$ is still affordable after the government policy changes : it is on the green budget line. So the bundle she chooses after the policy change is something she prefers to $(100, 50)$ (since $(100, 50)$ is one of the bundles she still could have). Therefore, her new bundle cannot be inside the red budget line : if it were, then the weak axiom of revealed preference would be violated. If the new choice is on the green budget line, but outside the red budget line, then it must be above and to the left of the original consumption bundle $(100, 50)$.



Question 3 : if an indifference curve is tangent to the red line at (100,50), then the person's preferred consumption bundle on the green dotted line must be above and to the left of (100,50)

Alternatively : the person chose $(100, 50)$ when she faced the less–steep (red) budget line. That means that her indifference curve through $(100, 50)$ is tangent to the less–steep (red) budget line.

So her indifference curve through $(100, 50)$ is less steep than the new (green) budget line she faces (after the tax has been introduced, and the cash grant paid). If her preferences are convex, then her indifference curves get less steep as she moves down and to the right along a budget line. That means that all her indifference curves through points on the new (green) budget line, below and to the right of $(100, 50)$, are less steep than that budget line. The only places she can have an indifference curve which is tangent to the new (green) budget line are where her indifference curves are steeper than the old (red) budget line — above and to the left of $(100, 50)$.

*Q*4. If a person regarded leisure time as a normal good, must it be true that the person would choose to work more hours if she were paid a higher wage?

Explain briefly.

*A*4. The Slutsky equation says it is not necessarily true that the person must work more hours, if her wage increases. There is an income effect, and a substitution effect. If R denotes the person's demand for leisure, w the wage rate, and L her hours worked, then the Slutsky equation for someone choosing how many hours to work is

$$\frac{\partial R}{\partial w} = \frac{\partial R}{\partial w}|^{S} + L\frac{\partial R}{\partial m} \qquad (4-1)$$

where "$|^{S}$" denotes the substitution effect.(This is equation (9.4) in Varian's text.)

This substitution effect must be negative : a higher wage causes the person to substitute consumption of goods for leisure. But the second term on the right hand side of $(4-1)$ will be positive if leisure is a normal good.

Therefore, if the income effect were sufficiently strong, equation $(4-1)$ implies that an increase in the person's wage rate would lead to an increase in her demand for leisure — and an increase in the demand for leisure is exactly the same as a decrease in the number of hours she works.