1. In this question, the service sector uses only labour and not capital. So if L_S denotes the allocation of labour to the service sector, and K_S the allocation of capital to the service sector, then

$$K_S = 0$$

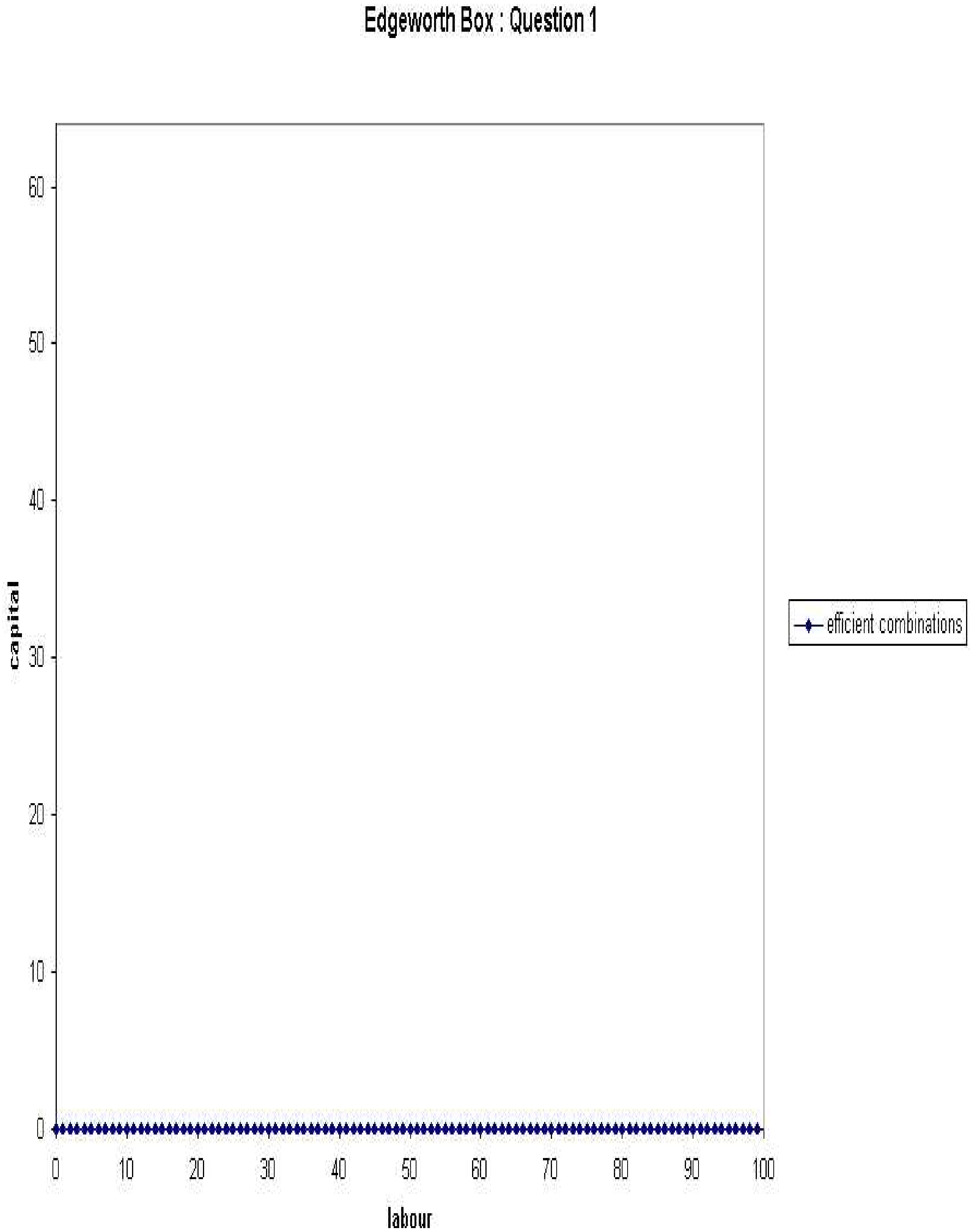
$$L_S = S$$

where S is the output of the service sector (since the question stated that 1 hour of labour produces 1 unit of services).

That means that all of the available capital, and the remaining $100 - S$ hours of labour, will be used in the housing sector, in which both of these inputs are productive.

So the efficient allocation of inputs to the two sectors are any combination : (L_S, K_S, L_H, K_H) of the form $(S, 0, 100 - S, 64)$, with $0 \leq S \leq 100$.

In the Edgeworth box diagram (with inputs to the service sector measured from the bottom left, and labour measured along the horizontal), the efficient allocations are those on the left edge of the box.


Edgeworth Box : Question 1
capital
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
labour
efficient combinations

2. From the answer to question #1,

$$L_H = 100 - S$$

$$K_H = 64$$

are the inputs of labour and capital to the housing industry. Given the production function for housing, then,

$$H = \sqrt{L_H K_H} = \sqrt{64(100 - S)}$$

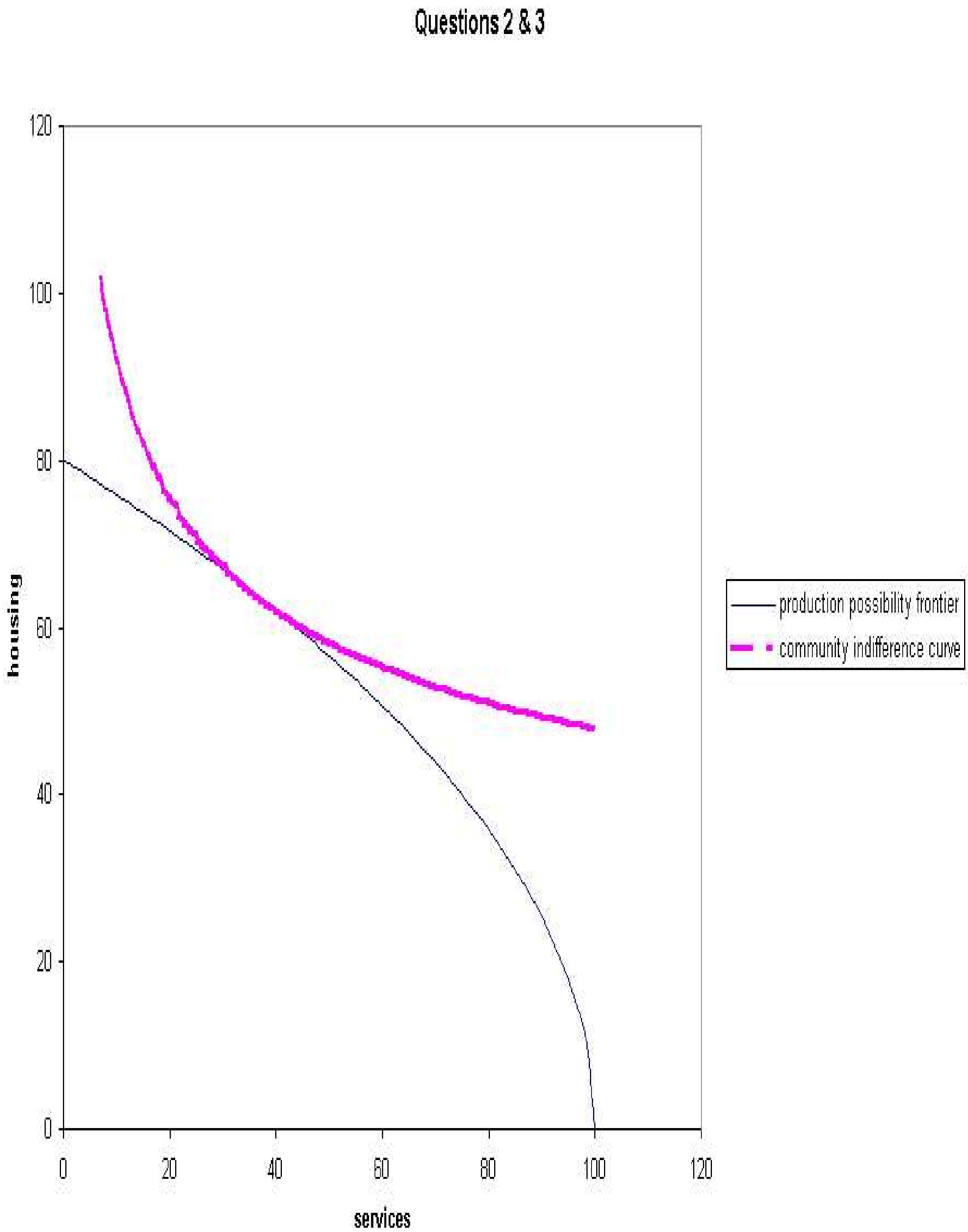
or

$$H = 8\sqrt{100 - S}$$

which defines a downward–sloping possibility frontier, with slope

$$\frac{dH}{dS}_{ppf} = -\frac{4}{\sqrt{100 - S}} = -\frac{32}{H}$$

This curve gets steeper as we move down and to the right ; with S on the horizontal axis, $-(dH/dS)_{ppf}$ gets bigger as S increases and H decreases.



3. The optimal production plan is the (S, H) combination at which the slope of the community's indifference curve equals the slope of the production possibility frontier.

("Community indifference curves" may not exist ; but when every person has the same preferences, and those preferences are *homothetic*, then these community interference curves do exist. The preferences described in the question are homothetic, and identical for all people. In this case, then every person's ratio s/h of service consumption to housing consumption is the same at an efficient solution, and this ratio equals the economy–wide ratio S/H of service production to housing production.)

The slope of an indifference curve, the marginal rate of substitution, is equal to the ratio of marginal utilities.

The marginal utilities are the derivatives of the utility function with respect to the consumption of the commodities. Here

$$MU_S = \frac{9}{s}$$

which implies that

$$H = 8\sqrt{100 - 36} = 64$$

4. Efficiency in production requires that the marginal rate of technical substitution (RTS) be equal in the two industries. The RTS is the ratio of the marginal products of the two inputs.

Here, in the food industry

$$MP_L^F = 1$$

$$MP_K^F = 1$$

so that

$$RTS^F = 1$$

In the clothing industry

$$MP_L^C = (0.5)(L_C)^{-0.5}(K_C)^{0.5}$$

$$MP_K^C = (0.5)(L_C)^{0.5}(K_C)^{-0.5}$$

so that

$$RTS^C = \frac{K_C}{L_C}$$

Efficiency in production requires

$$RTS^F = RTS^C$$

or

$$K_C = L_C$$

Substituting back in the production function for clothing

$$C = \sqrt{L_C K_C} = \sqrt{(L_C)^2} = L_C = K_C$$

and

$$F = L_F + K_F = (120 - L_C) + (180 - K_C) = (120 - C) + (180 - C)$$

so that the equation of the production possibility frontier is

$$F = 300 - 2C$$

The production possibility frontier here is a straight line, since the set of efficient allocations in the Edgeworth box is the diagonal of the box, and since both outputs are produced under constant returns to scale.

5. Take any point (F, C) on the country's production possibility frontier. Since the country can buy and sell food and clothing at prices of 8 and 1 respectively then it could move from (F, C) to any point (F', C) with

$$8F + C = 8F' + C'$$

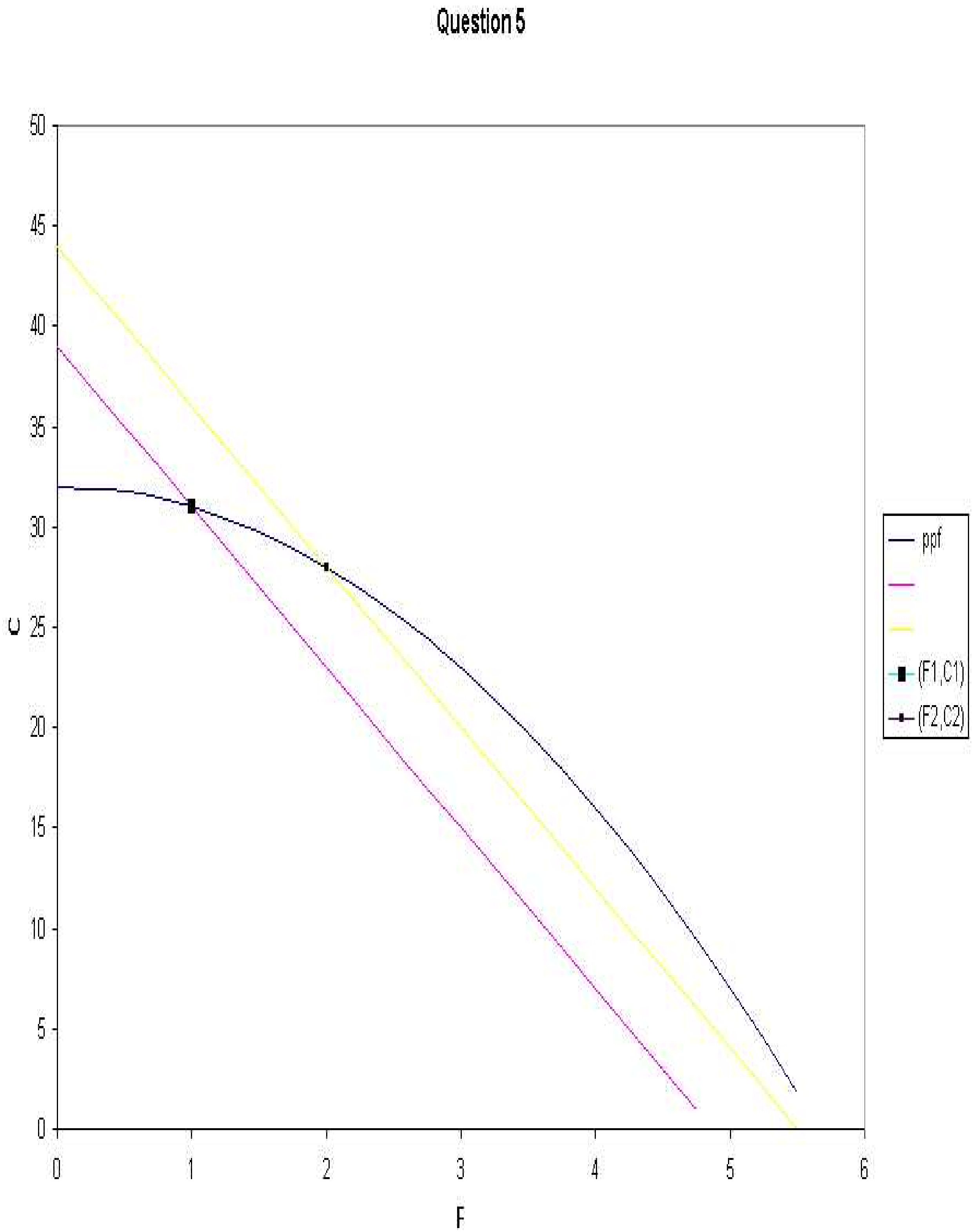
by trade with the rest of the world.

(For example, if $F' > F$, it could buy $F' - F$ units of food, at a cost of $8(F' - F)$, by selling $C - C'$ units of clothing. The value of imports will equal the value of exports if $8(F' - F) = (C - C')$ or $8F' + C' = 8F + C$.)

So if the country produces some combination (F, C) on its production possibility frontier, the available consumption quantities are quantities (F', C') on a line through (F, C), with slope 8, if I graph F on the horizontal and C on the vertical.

To get to consume as much as possible, the country should produce a production combination (F, C) which moves this line through (F, C) as far out as possible. (Figure 3 shows that producing

(F_1, C_1) leads to consumption possibilities which are better than producing at (F_2, C_2).)



The best possible consumption possibilities will arise if the country produces (F, C) so that the line with slope 8 is tangent to the production possibility frontier.

The production possibility frontier has equation

$$C = 32 - F^2$$

so that its slope is

$$-2F$$

The optimal production point is therefore where the absolute value of this slope equals 8, or

$$F = 4$$

so that producing $F = 4$, $C = 16$ yields the best possible consumption possibilities.