1. To solve for a subgame perfect equilibrium, one always starts at the last stage of the game, and works backward. In this game, the extensive form diagram cannot be drawn completely, since player 1 has an infinite number of actions at each stage.

The last stage of the game would be the morning of the third day, which will only be reached if player 2 has rejected player 1's first two offers. Player 2 will accept any non–negative payment that player 1 offers on the third day : if player 2 rejects the offer, the game ends and he gets a payoff of 0.

Since player 2 will accept any non–negative offer if the negotiations continue to the third morning, then player 1's best response is to offer the minimum possible amount, if she gets to make an offer on the third morning. So her equilibrium strategy must involve offering 0 (or perhaps 1 cent, if she wants to make sure that player 2 doesn't flip a coin to decide).

These strategies mean that the payoffs to the two players will be $(1000, 0)$ if the first two offers get rejected.

Moving back to the second morning, what offers should player 2 accept? If an offer is made on the second morning, and if player 2 rejects that offer, then the offer made on the third morning by player 1 will be 0 (or 1 cent), and that offer will be accepted by player 2. So rejecting an offer on the second morning will lead to player 2 getting a payoff of 0, if both players play their equilibrium strategies on the third morning.

That means that player 2 should accept any non–negative offer on the second morning, since rejecting any offer means he will be offered nothing on the third morning.

Player 1's best response to this strategy is to offer nothing on the second morning, if she gets to make an offer then (again, perhaps offering 2 cents to make sure that player 2 does not reject). These strategies mean that the payoffs to the two players will be $(2000, 0)$ if the first morning's offer is rejected, and if each player plays her or his equilibrium strategy in the remainder of the game.

On the first morning, player 2 should accept any offer that gives him a non–negative payoff, since rejecting any offer just leads to a sub–game in which he will get a payoff of 0 in equilibrium. So, just as on the subsequent two mornings, player 2's best strategy is to accept any non–negative offer on the first morning. Player 1 should react to that strategy by offering next to nothing on the first morning.

Thus the sub–game perfect Nash equilibrium has player 1 offering 0 every time she gets to make an offer, and player 2 accepting any non–negative offer. When each player plays her and his equilibrium strategy, the outcome is an offer of next to nothing is made and accepted on the first morning, and the two players' payoffs are $(3000, 0)$.

2. Now player 1 has the last move. If the first two offers are rejected, on the third morning she will be willing to pay up to $1000 in rent, because that's what she'll earn from the booth in

the one remaining day. So her optimal strategy on the third morning is to accept any offer of rent less than or equal to \$1000.

Player 2's best strategy, if his first two offers are rejected, is to offer a rent of \$1000 on the third morning, since that is the highest level of rent which will not be rejected.

If the first two offers by player 2 are rejected, then the equilibrium on the third morning leads to payoffs of $(0, 1000)$.

On the second morning, player 1 should accept any offer of \$2000 or less, since if she rejects the second morning's offer, she will wind up paying rent of \$1000 on the third day, if both players play their equilibrium strategies in that subgame. Since player 1 is willing to pay up to \$2000, player 2 will offer a rent of \$2000 on the second morning if his first offer is rejected.

On the first morning, player 1 will agree to pay any amount of rent up to \$3000, since she will wind up paying \$2000 for two days' use of the booth if she rejects the first morning's offer. That means player 2 will offer a rent level of \$3000 on the first morning.

So the sub–game perfect Nash equilibrium to this game has player 2 offering a rent of \$1000 per day each morning, and player 1 agreeing to pay any rent up to \$1000 per day. The first day's offer is accepted in the equilibrium, and the players' payoffs are $(0, 3000)$.

note on questions 1,2 : In this very stylized model of bargaining, it really pays to get to make the offers. (Or it really hurts to be forced into a take–it–or–leave–it position.) What is most commonly used in game–theoretic models of bargaining is to assume the players get to alternate in making offers : player 1 gets to make an offer on the first morning ; if player 2 rejects this offer, he can make a counter–offer on the second morning ; if player 1 rejects this counter–offer, she can make a final offer ont he third morning, which player 2 gets to take or leave.

Given the logic of the answers to questions #1 and #2, the sub–game Nash equilibrium to this game is : player 2 will accept any positive rent offer if the game goes to a third day ; player 1 would offer 0 in rent on the third morning if the first two days' offers are rejected ; player 1 will be willing to pay a total rent of \$1000 for the two days, if she gets to accept or reject an offer on the second morning ; player 2 will offer to rent the booth for \$1000 for two days if he gets to make a counter–offer ont he second morning ; player 2 will accept any offer of \$1000 or more in rent for the whole three days ; player 1 will offer \$1000 for the whole three days on the first morning.

So the gains from bargaining do get distributed a little more evenly when players alternate offers : here the payoffs are $(2000, 1000)$ in equilibrium, and they'd be $(1000, 2000)$ if player 2 got to make the first offer.

3. If the players play "share" each period, they each get a payoff of 5 each period. If one player broke the implicit agreement, and played "fight", then that player would get a payoff of 8 in the period in which the agreement was broken. But if the other player's strategy was "play 'share' unless someone has played 'fight' ; play 'fight' if anyone has ever played 'fight' before", then that first breaking of the agreement would lead to both players fighting thereafter.

If that is the strategy played by each player, then playing "fight" would lead to a gain of 3 to the player who first played "fight", but would lose that player 2 in every subsequent period of the game.

Suppose α is the probability that the game is going to continue at least one more period... and α^2 the probability it's going to continue at least two more periods, and so on. Then the expected net gain from breaking the agreement is

$$3 - 2\alpha - 2\alpha^2 - 2\alpha^3 \cdots$$

Since

$$\alpha + \alpha^2 + \alpha^3 + \cdots = \frac{\alpha}{(1-\alpha)}$$

this expected net gain is

$$3 - 2\frac{\alpha}{1-\alpha}$$

Fighting will be profitable, given the other player's strategy, if this expected payoff is positive, or if

$$3 > 2\frac{\alpha}{1-\alpha}$$

which is the same thing as

$$3(1-\alpha) > 2\alpha$$

or

$$\alpha < 0.6$$

If each player is playing this "grim trigger" strategy, neither player will want to fight as long as the probability α that the game will continue for at least another period, is 0.6 or more.

4. The overall demand function, adding up demands of males and females, faced by a single–price monopoly, is

$$Q = 156 - 2p \qquad \text{if} \quad p \leq 36$$

$$Q = 120 - p \qquad \text{if} \quad 120 \geq p \geq 36$$

(There is a kink in the aggregate demand curve at $p = 36, Q = 84$, since quantity demanded by males cannot be negative.) To find the marginal revenue, the inverse demand curve is used, since

$$MR = p(Q) + p'(Q)Q$$

The inverse demand curve is

$$p = 78 - \frac{Q}{2} \qquad \text{if} \quad Q \geq 84$$

$$p = 120 - Q \qquad \text{if} \quad Q \leq 84$$

Since each of these segments is linear, there is a simple expression for marginal revenue : when $p = a - bQ$, then $MR = a - 2bQ$. So

$$MR = 120 - 2Q \qquad \text{if} \quad Q < 84$$

$$MR = 78 - Q \qquad \text{if} \quad Q > 84$$

Notice that the marginal revenue changes discontinuously at $Q = 84$, from -48 to -6. Notice as well that it is negative for all $Q > 84$, since MR here falls with Q.

The monopoly's marginal cost here is 24. The level of Q at which $MR = MC$, is the level of Q for which

$$120 - 2Q = 24$$

or

$$Q = 48$$

implying $p = 72$. The single–price monopoly would maximize profits by charging a price of 72, a price so high that none of the male customers choose to buy. Lowering the price enough to attract any male customers would lower the firms' profits too much, if it had to charge the same price to both men and women.

5. Now the firm has separate problems for each group.

For females, the demand curve is

$$Q = 120 - p$$

with an inverse demand curve

$$p = 120 - Q$$

and a marginal revenue curve

$$MR = 120 - 2Q$$

so that setting $MC = MR$ implies $Q_F = 48$ and $p_F = 72$, just as in question #4.

For males, the demand curve is

$$Q = 36 - p$$

with an inverse demand curve

$$p = 36 - Q$$

and a marginal revenue curve

$$MR = 36 - 2Q$$

so that setting $MR = MC$ implies $Q_M = 6$ and $p_M = 30$.

Because third–degree price discrimination here enables the monopoly to serve a new group of customers, it is actually Pareto–improving in this example. Females pay the same price with or without third–degree price discrimination ; males face a lower price and are thus made better off ; the monopoly's profits are increased, from 2304 to 2340.