AP/ECON 2350 Answers to Assignment 2
W 2010–11

*Q*1. What should firm #1 do to maximize its profits in the following modification of the quantity leadership (Stackelberg) model?

Firms 1 and 2 produce a homogeneous good ; firm #1 chooses its output first, followed by firm #2 (with firm #2 observing firm #1's output before making its own output choice) ; the inverse demand function for the good is

$$p = 24 - (y_1 + y_2)$$

But here the marginal cost of producing the good is **zero**. However each firm incurs a **fixed** cost of 16 if it chooses to produce any output at all. (This fixed cost can be avoided if the firm chooses to go out of business).

*A*1. If firm #2, the follower, chooses to produce anything at all, then firm #2's reaction function can be calculated as

$$y_2 = \frac{a - c}{2b} - \frac{y_1}{2} \tag{1}$$

since the demand curve is a straight line, and the marginal cost of production is constant. From the question, here $a = 24, b = 1, c = 0$, so that equation (1) becomes

$$y_2 = 12 - \frac{y_1}{2} \tag{2}$$

So if firm #1 produces an output of y_1, then equation (2) implies that total industry output $Y = y_1 + y_2$ is

$$Y = y_1 + y_2 = y_1 + 12 - \frac{y_1}{2} = 12 + \frac{y_1}{2} \tag{3}$$

so that the market price is

$$p = 24 - (y_1 + y_2) = 24 - (12 + \frac{y_1}{2}) = 12 - \frac{y_1}{2} \tag{4}$$

If firm #1 produces an output of y_1, then the profit earned by firm #2 is just its revenue minus its fixed costs, since its marginal production costs are zero. So firm #2 earns profits of

$$\pi_2 = py_2 - 16 = (12 - \frac{y_1}{2})(12 - \frac{y_1}{2}) - 16 = (12 - \frac{y_1}{2})^2 - 16 \tag{5}$$

Notice that the more that firm #1 produces, the lower are firm #2's profits.

But firm #2 only incurs fixed costs (of 16) if it chooses to produce some output. If its revenue (py_2) does not cover its fixed costs, then firm #2 is better off not producing anything at all, and leaving the market to firm #1. From equation (5), firm #2 can cover its costs if $\pi_2 > 0$, that is if

$$(12 - \frac{y_1}{2})^2 \geq 16 \tag{6}$$

which is the same thing as

$$12 - \frac{y_1}{2} \geq 4 \tag{7}$$

or

$$y_1 \leq 16 \tag{8}$$

So firm #1, acting as leader, can drive firm #2 right out of the market, if it picks such a high output that firm #2 cannot cover its fixed costs, no matter what y_2 it chooses. From equation (8), firm #1 must choose an output of 16 or more in order to drive out firm #2.

And that's what firm #1 should do in order to maximize its own profits. If it chooses an output level of 16 (or just slightly more), then firm #2 will choose not to enter the industry at all, leaving firm #1 with the whole market. In this case, what will firm #1 earn? Total industry output is 16 here — $y_1 = 16$ and $y_2 = 0$. So the price is $p = 24 - 16 = 8$, and firm #1's profits are $py_1 - 16$, or $128 - 16 = 112$.

That's the best that firm #1 can do. Increasing output above 16 will keep firm #2 out, but lower firm #1's profits. [If $y_2 = 0$, firm #1's profits are $(24 - y_1)y_1 - 16 = 24y_1 - (y_1)^2 - 16$, which *decrease* with y_1, at least when $y_1 > 16$.] On the other hand, if firm #1 produces an output smaller than 16, firm #2 will choose to produce a positive level of output, $y_2 = 12 - \frac{y_1}{2}$. In this case, equation (4) says that firm #1's profits would be

$$\pi_1 = py_1 - 16 = (12 - \frac{y_1}{2})y_1 - 16 = 12y_1 - \frac{(y_1)^2}{2} - 16 \tag{9}$$

The derivative of expression (9) with respect to y_1 is

$$\frac{\partial \pi_1}{\partial y_1} = 12 - y_1 \tag{10}$$

which is maximized at $y_1 = 12$. So if firm #1 were to produce less than 16 units of output, inducing firm #2 to enter the industry, then firm #1's best strategy would be to produce 12 units. In that case, expression (9) says that firm #1's profit would be $12(12) - \frac{(12)^2}{2} - 16 = 56$. Firm #1 will earn more money from choosing $y_1 = 16$ and driving out firm #2, than from choosing $y_1 = 12$ and picking its best point on firm #2's reaction curve.

Here the fixed costs are high enough that it becomes profitable for firm #1 to expand its output in order to shut down its rival. This would not happen if there were no fixed costs, or even if fixed costs were small. In this latter case, shutting down firm #2 requires too much output from firm #1, which drives down the price to far to make such a strategy worthwhile.

*Q*2. Suppose everything is the same as in question #1, except that the firms choose their output quantities **simultaneously** (as in the Cournot model).

Find an equilibrium in this model.

*A*2. As long as both firms are covering their fixed costs, they'll behave like "standard" Cournot duopolists with marginal costs of 0. Equation (2) above is firm #2's reaction curve to firm #1's output, and firm #1's reaction to firm #2's output is

$$y_1 = 12 - \frac{y_2}{2} \tag{11}$$

The Cournot–Nash equilibrium is a pair of outputs (y_1, y_2) such that firm #2's choice of output y_2 is on its reaction function (2) to y_1, and that firm #1's choice of output y_1 is on its reaction curve (11) to y_2.

The solution, in general, to the 2–firm Cournot–Nash equilibrium with linear demand and constant marginal costs is $y_1 = y_2 = \frac{a-c}{3b}$. But this result can be derived directly here, by substituting from (2) into (11) to get

$$y_1 = 12 - \frac{1}{2}[12 - \frac{y_1}{2}] \tag{12}$$

or

$$y_1 = 12 - 6 + \frac{y_1}{4} \tag{13}$$

so that

$$\frac{3y_1}{4} = 6 \tag{14}$$

or $y_1 = 8$. If $y_1 = 8$, then (2) says that $y_2 = 12 - \frac{8}{2} = 8$.

So, if there were no fixed costs, the Cournot–Nash equilibrium here would be

$$y_1 = y_2 = 8 \tag{15}$$

But the fixed costs have no effect on firms' behaviour, once they have decided to produce some positive amount of output. So if both firms have decided to enter the industry, then the only equilibrium outcome is for both of them to produce an output of 8.

Will they both want to enter? If $y_1 = y_2 = 8$, then $p = 24 - (8 + 8) = 8$, so that each firm's profits will be $py_i - 16 = 8(8) - 16 = 48 > 0$. Therefore, an equilibrium to the Cournot [simultaneous quantity–setting] game here is for each firm to produce 8 units of output.

In fact, this outcome $y_1 = y_2 = 8$ is the only equilibrium outcome here. Firm #2 would want to enter, and produce $y_2 > 0$, as long as $y_1 < 16$, as shown in the answer to question #1. So the only way in which there could be a Cournot equilibrium in which $y_2 = 0$ would be if $y_1 \geq 16$. But firm #1 would never choose to produce $y_1 \geq 16$ if $y_2 = 0$, and if they both chose output levels simultaneously : equation (11) shows that firm #1 would want to choose $y_1 = 12 < 16$ if it were to believe that $y_2 = 0$.

So $y_1 = 8 = y_2$ is actually the only possible equilibrium outcome if both firms were to choose their output levels simultaneously here.

*Q*3. Find the equilibrium in the following Cournot (simultaneous quantity–setting) duopoly model, in which the firms have **different** production costs.

Firms choose their output levels simultaneously.

The inverse demand function for the good that they produce is

$$p = 24 - (y_1 + y_2)$$

Firm #1's total cost of producing y units of output is

$$TC_1(y) = 6y$$

and firm #2's total cost of producing y units of output is

$$TC_2(y) = 12y$$

*A*3. The demand curve here is a straight line, and each firm has a constant marginal cost, so the formula for a firm's reaction curve can be used. Firm #2's reaction to firm #1's output is $y_2 = \frac{a-c_2}{2b} - \frac{y_1}{2}$ where here $a = 24$, $b = 1$ and c_2 is firm #2's marginal cost, which is 12. So here firm #2's reaction curve has the equation

$$y_2 = 6 - \frac{y_1}{2} \tag{16}$$

Similarly, firm #1's reaction to firm #2's output is $y_1 = \frac{a-c_1}{b} - \frac{y_2}{2}$ where $a = 24, b = 1, c_1 = 6$, so that

$$y_1 = 9 - \frac{y_2}{2} \tag{17}$$

The Cournot equilibrium is the output combination (y_1, y_2) which is on both firm's reaction curves ; the combination which satisfies both equation (16) and (17). Substituting for y_2 from (16) into (17),

$$y_1 = 9 - \frac{1}{2}[6 - \frac{y_1}{2}] \tag{18}$$

or

$$y_1 = 9 - 3 + \frac{y_1}{4} \tag{19}$$

which means that

$$\frac{3}{4}y_1 = 6 \tag{20}$$

So that in the Cournot–Nash equilibrium

$$y_1 = 8 \tag{21}$$

Substituting from (21) into (16), in equilibrium

$$y_2 = 6 - \frac{8}{2} = 2 \tag{22}$$

So the equilibrium has firm #1 producing 8 units of output, and firm #2 producing 2 units of output. Perhaps not surprisingly, the lower–cost firm chooses to produce more in equilibrium.

In this equilibrium, the price is $24 - 8 - 2 = 14$, so that firm #1 makes profits of $(14)(8) - (6)(8) = 64$, and firm #2 makes profits of $(14)(2) - (12)(2) = 4$.

*Q*4. What output levels y_1 and y_2 maximize the combined profits of the two firms in question #3 above?

*A*4. Notice that firm #2's costs of production, for any levels of output, are higher than firm #1's. Maximization of combined profits means having all the production done by firm #1. [That is, if $y_2 > 0$, the firms' combined profits could be increased by lowering y_2 by some amount x, and raising y_1 by the same amount x, reducing costs by $12x - 6x$.]

So the policy which maximizes combined profits is to choose a level of output y_1 so as to maximize

$$(24 - y_1)y_1 - 6y_1$$

This expression is maximized if $y_1 = 9$ — which is exactly firm #1's profit–maximizing reaction to $y_2 = 0$.

The output plan $y_1 = 9, y_2 = 0$ leads to a price of $p = 24 - 9 = 15$, and total profits of

$$(15)(9) - (6)(9) = 81$$

This example illustrates some of the problems in collusion among oligopolists. The combined profits are highest if all sales are allocated to the lowest–cost firm. But firm #2 has no incentive to cooperate with this plan unless it can somehow be compensated. Firm #2 makes positive profits (only 4, but $4 > 0$) if firms do not collude, as in question #3. To agree to help in the collusion, somehow firm #2 must be compensated. Maybe firm #1 would have to agree to bribe firm #2 not to produce anything. Or maybe firm #1 will have to let firm #2 do some of the production, sacrificing overall industry profits in the interests of getting firm #2 to join in the collusion. [For example, the plan $y_1 = 7, y_2 = 1.5$ would leave each firm with higher profits than it would get if they behaved non–cooperatively, as in question #3.]

*Q*5. Write down the payoff matrix to the game described below, and find **all** the Nash equilibria to the game.

The two firms choose prices simultaneously, for a homogeneous good, as in the Bertrand model of duopoly.

But each firm's price has to be an integer : either \$1, \$2, \$3, or \$4.

The market demand curve for the firms' output is

$$Y = 4 - p$$

and each firm's total cost of producing y units of output is

$$TC(y) = y$$

*A*5. First of all, the payoff to firm #1 will be zero if it chooses a higher price than firm #2 — and vice versa. The higher–priced firm gets no customers.

Secondly, total industry demand will be 0 if the lower price (of the two firms' prices) is 4, so that a firm setting a price of 4 must earn profits of 0, since it will have no sales.

If the lower price is 3, then industry demand will be 1. So total industry profits will be 2 1 unit, sold for $3, which costs $1 to produce. If both firms were to charge $3, then they would split that profit.

If the lower price is 2, then industry demand is 2, and total industry profits are 2 (2 units, selling for $2 each, which cost $1 each to produce).

If the lower price is 1, then price equals marginal cost, and industry profits will be zero.

From this information, the payoff matrix to the game is

	1	2	3	4
1	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
2	$(0,0)$	$(1,1)$	$(2,0)$	$(2,0)$
3	$(0,0)$	$(0,2)$	$(1,1)$	$(2,0)$
4	$(0,0)$	$(0,2)$	$(0,2)$	$(0,0)$

There are **two** Nash equilibria (in pure strategies) to this game. The "standard" Bertrand equilibrium here is for each firm to charge a price of 1, since 1 is the marginal cost. And $p_{=}1, p_2 = 1$ is a Nash equilibrium, since neither firm can increase its profits above 0 if the other firm is charging 1 as a price. But a price of 2 is actually a dominant strategy here. For example, for firm #1, a price of 2 is better than any other price, if the other firm set $p_2 = 2$ or $p_2 = 3$, and is just as good as any other if the other firm set a price of $p_2 = 1$ or $p_2 = 4$.

[And there are no other Nash equilibria to this game, not even in mixed strategies.]