AS/ECON 2350 S2 N   Answers to Mid–term Exam   July 2017

time : 1 hour

Do all 4 questions. All count equally.

$Q1$. "Monopoly is inefficient because the monopoly's owner makes high profits, and the monopoly's customers pay high prices". Discuss.

$A1$. The key points : (i) **Single–price** monopoly is inefficient (but not necessarily price–discriminating monopoly) ; (ii) the inefficiency is **not** due to the monopoly's owner's high profits.

The efficient quantity of output for an industry is the level of output y^* for which $P(y^*) = MC(y^*)$, where $P(\cdot)$ is the inverse demand curve for the monopoly's product, and $MC(\cdot)$ the marginal cost curve. That is the output level which maximizes the sum of aggregate consumers' surplus, and aggregate profits of firms producing the good.

Decreasing total sales to some y below y^* is inefficient, because the units of output between y and y^* are valued by consumers more than the cost of producing the units.

In particular, a single–price monopoly chooses a level of output $y^M < y^*$, at which $MR(y^M) = MC(y^M)$, where $MR(\cdot)$ is the marginal revenue curve corresponding to the inverse demand curve $P(\cdot)$. It must be true that $MR(y) < P(y)$ (for any output level y), since

$$MR(y) = P(y) + P'(y)y$$

and $P'(y) < 0$ if the inverse demand curve slopes down.

The added profit a single–price monopoly gains in lowering output from $P^* \equiv MC(y^*)$ to y^M is the extra revenue from the higher price — $[P(y^M) - P^*]y^M$ — minus the lost profits from units between y^M and y^* — which is the area above the marginal cost curve between y^M and y^*, up to a height of P^*. [That's area A, minus area C, in Varian's figure 25.2.]

Consumers lose $[P^M - P^*]y^M$, **plus** the area between the inverse demand curve and the price P^*, between y^M and y^*, in moving from efficiency to single–price monopoly. [That's the area B in Varian's figure 25.5.]

So in Varian's figure 25.5, moving from efficiency to a single price monopoly increases owners' profits by $A - C$, and lowers aggregate consumers' surplus by $A + B$. Single–price monopoly is inefficient because the owners' gain $A - C$ must be **less** (by $B + C$) than consumers' loss $A + B$.

If a monopoly could price discriminate perfectly, then it would sell the efficient quantity y^*. It would gain the area $A+B$ in profit, compared to a perfectly competitive industry. This gain equals the total consumers' loss, so that a perfectly price discriminating monopoly would be efficient.

[A monopoly that price discriminates, but not perfectly, would also be inefficient. It might be more or less efficient than a single–price monopoly. For example, a monopoly which charged two different prices to two different groups would actually be less efficient than a single–price monopoly, provided that it did not produce a higher total quantity of output than the single–price monopoly.]

*Q*2. If the market demand curve for the product of some duopoly had the equation

$$Y = 24 - p$$

where $Y = y_1 + y_2$ was the total quantity produced by the two firms in the industry, and p the price paid by buyers, and if each firm (firm #1 and firm #2) could produce the product at zero cost,

(a) What is the equation of firm 2's reaction function, if it chose its own quantity y_2, taking as given firm #1's quantity y_1?

(b) What quantities of the good would each firm produce in the Cournot–Nash equilibrium (when each firm chooses its quantity, taking the other firm's quantity as given)?

*A*2. If the (regular) demand function for the good has the equation

$$Y = 24 - p$$

then the inverse demand function has the equation

$$P(Y) = 24 - Y$$

That means that, if firms #1 and #2 choose output quantities y_1 and y_2, then the price that each firm will receive for its output is

$$P(y_1 + y_2) = 24 - y_1 - y_2$$

Firm #2's profit is its revenue minus its total costs of production. But here each firm can produce the good for nothing, so that

$$\pi_2 = P(y_1 + y_2)y_2 = (24 - y_1 - y_2)y_2 = (24 - y_1)y_2 - (y_2)^2$$

If firm #2 takes y_1 as given, then maximizing π_2 with respect to its own output y_2 means setting the derivative of π_2 with respect to y_2 equal to zero. That means

$$24 - y_1 - 2y_2 = 0$$

or

$$y_2 = 12 - \frac{y_1}{2} \tag{a}$$

Equation (a) is the equation for firm #2's reaction function.

If firm #1 maximized its profits, taking y_2 as given, then it would have a reaction function

$$y_1 = 12 - \frac{y_2}{2} \tag{$r1$}$$

In Cournot–Nash equilibrium, each firm is on its reaction function, so that the equilibrium quantities y_1^E and y_2^E must satisfy both equations (a) and equation $(r1)$. So, substituting for y_2 from (a) into $(r1)$ yields

$$y_1 = 12 - \frac{1}{2}[12 - \frac{y_1}{2}] \tag{$r1e$}$$

Multiplying both sides of equation $(r1e)$ by 4, it becomes

$$4y_1 = 48 - 24 + y_1 \tag{$r1e2$}$$

or

$$y_1^E = 8 \tag{b1}$$

substituting from ($b1$) into (a),

$$y_2 = 12 - \frac{8}{2} = 8 \tag{b2}$$

so that $y_1 = y_2 = 8$ in the Cournot–Nash equilibrium.

*Q*3. Write down the payoff matrix of the following game :

The players are two sellers, who each have 2 used cell phones to sell. Each seller has no use at all for either of the used cell phones that he or she owns ; he or she wants to sell them. There are 2 identical potential buyers. Each buyer is willing to pay up to \$10 for a used cell phone ; each buyer wants to buy at most one phone ; each buyer will buy from the cheapest seller (if the cheapest seller charges a price of \$10 or less).

Each seller must choose a price to ask for her or his cell phones : the price must be one of $\{\$5, \$10, \$15\}$. When a seller picks a price, this is a commitment to sell each phone for that price, to whatever buyer is willing to buy. If both sellers choose the same price (of \$10 or less), buyer #1 buys from seller #1 and buyer #2 buys from seller #2.

The 2 sellers choose their prices (from the set of possible prices $\{\$5, \$10, \$15\}$) independently, and simultaneously.

[**note** : You are **not** required to solve this game, just to write down the payoff matrix for the game.]

*A*3. The strategies for player #1 are the three possible prices, \$5, \$10 and \$15, and player #2 also has those same three strategies. Charging \$15 leads to no revenue, since neither buyer is willing to pay \$15. Any lower price p ($p \in \{5, 10\}$) will give a seller revenue of $2p$ if it is the lowest–priced, p if the two sellers are tied, and 0 if she asks a higher price than the other seller.

So the payoff matrix is :

	\$5	\$10	\$15
\$5	$(5,5)$	$(10,0)$	$(10,0)$
\$10	$(0,10)$	$(10,10)$	$(20,0)$
\$15	$(0,10)$	$(0,20)$	$(0,0)$

$Q4$. Find **all** the Nash equilibria to the game with the following payoff matrix.

	L	M	R
t	$(1,1)$	$(2,1)$	$(3,0)$
b	$(0,0)$	$(5,5)$	$(10,2)$

$A4$. A Nash equilibrium is a **pair** of strategies, one for each player, such that neither player can do better by changing her strategy, given what her rival is doing.

In a payoff matrix, a Nash equilibrium is a row and column, such that player #1 cannot increase her payoff by changing the row (given the column chosen by player #2), and that player #2 cannot increase his payoff by changing the column (given the row chosen by player #1).

In this example (t, L) and (b, M) are both Nash equilibria.

(t, L) : if player #2 chooses column L, than player #1 cannot do better than choosing the top row, since the bottom row would give her a payoff of $0 < 1$; if player #1 chose row t, then player #2 gets a payoff of 1 from choosing the column L : he cannot do better than that, given player #1's choice of t, since M also gives him a payoff of 1, and R gives him a payoff of 0

(b, M) : if player #2 chose column M, then player #1 gets a payoff of 5 from choosing row b, which is greater than the payoff of 2 she would get from her other choice, row t ; on the other hand, if player #1 chooses row b, then player #2 should pick column M, since that gives him a payoff of 5, as opposed to 0 from L and 2 from R

No other pair of strategies is a Nash equilibrium : at (t, M) player #1 would like to move down to b ; at (t, R) player #1 would like to move down to b ; at (b, L) player #1 would like to move up to t ; at (b, R) player #2 would like to move left to M.

[And there is no other Nash equilibrium in mixed strategies in this game. (Since M is a weakly dominant strategy for player #2, if player #1 were to mix between her 2 strategies, then player #2 would always find M gives him a higher expected payoff than L or R, so that he would never be willing to mix among his strategies if player #1 mixed among her 2 strategies.)]