AS/ECON 2350DW   Answers to Mid–term   April 2001

1. In this question, food is produced using only labour, and using no capital. Therefore, al the available capital should be used in producing the other good, clothing, since capital is productive in making clothing, and not productive in making food. That means, if K_C denotes the amount of capital used in the clothing industry, and K_F the amount of capital used in the food industry, that

$$K_F = 0 \quad K_C = 80$$

since there are 80 units of capital available in total.

The question also stated that the production function for food was

$$F = L_F$$

This means that

$$L_C = 100 - L_F = 100 - F$$

since there are 100 units of labour available.

Since the production function for clothing is

$$C = \sqrt{L_C K_C}$$

therefore

$$C = \sqrt{80(100 - F)}$$

is the equation for the production possibility frontier, since it expresses the quantity produced of clothing as a function of the quantity produced of food.

Although the question did not require this information, it also can be checked that this PPF slopes down, since

$$\frac{dC}{dF} = -\sqrt{20}(100 - F)^{-0.5}$$

and that it gets steeper as we move down the PPF, since

$$\frac{d^2C}{dF^2} = -\sqrt{5}(100 - F)^{-1.5}$$

2. A risk–averse person would never invest any money in a risky asset, if the expected return was lower than the certain return on a safe asset.

One way of explaining this is to note that taking some of her money out of the safe asset, and putting into the risky asset, is a bet. Risk averse people will only take bets if the expected payoff from the bet is positive, which it will not be in this case.

Alternatively, if X denotes the amount of money she puts in the risky asset, W the wealth she has to invest, r_0 the return on the safe asset, and r the return ont he risky asset, then she will choose X so as to maximize her expected utility of wealth

$$EU[(1+r)X + (1+r_0)(W-X)]$$

The derivative of this expression with respect to X is

$$E\{U'[(1+r)X + (1+r_0)(W-X)][r-r_0]\}$$

At $X = 0$, this expression equals

$$[Er - r_0]U'[(1+r_0)W]$$

which will be negative if the expected return Er on the risky asset is less than the certain return r_0 on the safe asset.

A third way of seeing this result is diagrammatically, for example using the "state preference" diagram introduced in pages 227–233 of the text. For a risk–averse person, indifference curves are convex in this diagram. The slope of an indifference curve along the "certainty line" is $-\pi_g/(1-\pi_g)$ if π_g is the person's belief of the probability of the "good" event occurring (with wealth in the good event graphed along the horizontal axis, wealth in the bad event on the vertical). If the risky asset pays r_g in the "good" state of the world and r_b in the "bad" state of the world, and the safe asset pays r_0 (in both states of the world), then the person's budget line in the state preference diagram has slope $(r_b - r_0)/(r_g - r_0) < 0$.

If the safe asset has a higher expected return, then

$$r_0 > \pi_g r_g + (1-\pi_g)r_b$$

which implies that

$$\pi_g(r_g - r_0) < (1-\pi_g)(r_0 - r_b)$$

meaning that the slope of the person's indifference curve along the certainty line is less than the slope of the budget line, meaning that the person will not want to choose a point to the right of the certainty line. Being on the certainty line means having the same consumption in both states of the world — in other words, not putting any money in the risky asset. So the state preference diagram shows the person would never choose a "contingent wealth claim" involving positive holding of the risky asset.

3. The game has two Nash equilibria in pure strategies, (t, L) and (b, R). If player one plays t, then player two's best action is to play L ; L pays him 3 while the other two actions pay 0 or 1. But if player 2 plays L, then player one will want to play t ; t pays her 5 while b pays 2. So neither player would want to change his or her action unilaterally from (t, L). Similarly, if one played b

and two played R then one would not want to move up, and two would not want to move left, in the normal form matrix.

Are these two the only Nash equilibria? Notice that the strategy M is a *dominated* strategy for player two : whatever player one does, two gets a higher payoff from playing L (or R for that matter) than from playing M.

So any other Nash equilibria cannot involve player two playing M — even in a mixed strategy.

The only other possibility is a mixed strategy equilibrium, in which player one randomizes between t and b, and player two randomizes between L and R.

Player one's expected payoff if player two plays L with probability β and R with probability $1-\beta$ would be

$$5\beta + 1(1-\beta)$$

from playing t, and

$$2\beta + 2(1-\beta)$$

from playing b. So player one will be willing to randomize only if

$$5\beta + 1(1-\beta) = 2(\beta) + 2(1-\beta)$$

or $\beta = 3/4$

Player two's expected payoff if player one plays t with probability α and b with probability $1-\alpha$ would be

$$3\alpha + 2(1-\alpha)$$

from playing L, and

$$1\alpha + 10(1-\alpha)$$

from playing R. So player two will be willing to randomize between L and R only if $3\alpha + 2(1-\alpha) = \alpha + (10(1-\alpha)$, or $\alpha = 4/5$.

Therefore, there exists as well a mixed strategy Nash equilibrium, in which player one plays t with probability 4/5 and b with probability 1/5, and in which player two plays L with probability 1/4, (M with probability zero), and R with probability 3/4.