AS/ECON 2350 N   Answers to Mid–term Exam   February 2011

$Q1$. **One** of the following four statements is true. Which one? Explain why the statement is true. (You do **not** need to explain why any of the other three statements are false.)

(i) An industry cannot be perfectly competitive if all firms have U–shaped average cost curves.

(ii) An industry cannot be perfectly competitive if the technology in the industry exhibits increasing returns to scale at all levels of output.

(iii) An industry can be perfectly competitive only if all firms produce using constant returns to scale.

(iv) An industry must be perfectly competitive if all firms' marginal cost curves slope up.

$A1$. Statement (ii) is the true one.

There are several ways to explain this.

One is to note that large firms will have a cost advantage over small firms if the technology exhibits increasing returns to scale at all levels of output.

More specifically, suppose that the input mix $(x_1, x_2, \ldots, x_n)$ maximizes profits for a perfectly competitive firm, and that these input levels are bigger than zero. Then profits from this input mix would be

$$\pi \equiv pf(x_1, x_2, \ldots, x_n) - w_1 x_1 - w_2 x_2 - \cdots - w_n x_n \geq 0$$

Since increasing returns to scale imply that $f(tx_1, tx_2, \ldots, tx_n) > tf(x_1, x_2, \ldots, x_n)$ when $t > 1$, therefore the profit from using the input mix $(tx_1, tx_2, \ldots, tx_n)$ would be

$$\begin{aligned}\tilde{\pi} =& pf(tx_1, tx_2, \ldots, tx_n) - tw_1 x_1 - tw_2 x_2 - \cdots - tw_n x_n \\ >& tpf(x_1, x_2, \ldots, x_n) - tw_1 x_1 - tw_2 x_2 - \cdots - tw_n x_n \\ =& t\pi \\ \geq& \pi\end{aligned}$$

So $\tilde{\pi} > \pi$, which contradicts the assumption that the original input mix $(x_1, x_2, \ldots, x_n)$ maximized profits. In other words, there can be no input mix which maximizes profits for a perfectly competitive firm, if there are increasing returns to scale at all levels of output.

Another way of showing that there cannot be perfect competition when there are increasing returns to scale is to use the cost function. If there are increasing returns to scale, then the long–run average cost must **decrease** with the quantity y of output. Whenever $AC' < 0$, then $MC < AC$. So if perfectly competitive firms set $p = MC$, and there are increasing returns to scale, then $AC > MC = p$, so that firms would lose money, which is inconsistent with long–run equilibrium.

As long as we have decreasing returns to scale somewhere, or constant returns to scale, then $MC \geq AC$, so that long–run equilibrium is possible in perfect competition. That means that statements (i), (iii) and (iv) are false.

$Q2$. What is the long–run cost function for a firm with a production function

$$f(x_1, x_2) = \min(x_1, 4x_2)$$

where x_1 and x_2 are the quantities used of the two inputs to production?

$A2$. The two inputs are **perfect complements** with this technology. Every unit of output requires 1 unit of input #1, and 1/4 unit of input #2.

So if w_1 is the cost of 1 unit of input #1, and w_2 is the cost of one unit of input #2, then the cost of producing one unit of output is $w_1 + \frac{w_2}{4}$. Since this expression is the cost of producing 1 unit of output, then the total cost of producing y units of output (there are constant returns to scale here) is

$$C(w_1, w_2, y) = w_1 y + \frac{w_2 y}{4}$$

$Q3$. Is it possible that a firm's short–run supply curve could have a negative slope? Explain.

$A3$. No. Firms' supply curves must slope up, since they consist of the upward–sloping part of the firm's marginal cost curve (above the firm's minimum average variable cost).

More formally, suppose that the firm chooses to supply y units of the good, at a price of p, and suppose that $y' < y$. The fact that the firm chooses to supply y, rather than y', means that $py - TVC(y) > py' - TVC(y')$, or

$$p(y - y') > TVC(y) - TVC(y')$$

Now if $p' > p$, then

$$p'(y - y') > p(y - y') > TVC(y) - TVC(y')$$

so that the firm will always make more profit supplying y rather than y' if $p' > p$. That means that the firm will never choose to supply $y' < y$ at a price of $p' > p$, so that the firm must choose to supply a quantity of y or more when the price is greater than p : its supply curve must slope up.

*Q*4. What prices should a monopoly charge in Canada and the United States, if it can charge different prices in Canada and the United States, if the demand curve for its product is

$$y_C = 12 - p_c$$

in Canada, and

$$y_{US} = 24 - p_{US}$$

in the US, if its total cost of producing Y units of output is

$$TC(Y) = 2Y \qquad ?$$

*A*4. This is an example of third–degree price discrimination, and the monopoly's profit–maximizing strategy here is to set $MR_C = MC = MR_{US}$, where MR_C and MR_{US} are the marginal revenues in the two countries.

Here, the fact that $TC(Y) = 2Y$ means that the firm's marginal cost is a constant 2. The fact that demand curves in both Canada and the United States are straight lines means that the marginal revenue curves in each country have twice the slope of the demand curves, and start at the same height :

$$MR_C(y_C) = 12 - 2y_c$$

$$MR_{US}(y_{US}) = 24 - 2y_{US}$$

So the firm should choose its output quantities in the two countries so that

$$12 - 2y_C = 2$$

$$24 - 2y_{US} = 2$$

or

$$y_C = 5$$

$$y_{US} = 11$$

That means, from the demand curves, that the prices charged in the two countries should be

$$p_C = 12 - y_c = 12 - 5 = 7$$

$$p_{US} = 24 - y_{US} = 24 - 11 = 13$$