AS/ECON 2350 Q Answers to Mid–term Exam February 2011

*Q*1. Solve for a perfectly competitive firm's profit–maximizing level of output, as a function of its input prices and output price, if its production function has the equation

$$f(x_1, x_2) = 2(\sqrt{x_1} + \sqrt{x_2})$$

*A*1. The firm's profit is $pf(x_1, x_2) - w_1x_1 - w_2x_2$, so that here it should choose input levels x_1 and x_2 so as to maximize

$$2p\sqrt{x_1} + 2p\sqrt{x_2} - w_1x_1 - w_2x_2$$

The first–order conditions for this maximization are

$$\frac{p}{\sqrt{x_1}} = w_1 \qquad (1-1)$$

$$\frac{p}{\sqrt{x_2}} = w_2 \qquad (1-2)$$

(which are just the conditions that $pMP_1 = w_1$ and $pMP_2 = w_2$). These two equations can be re–written

$$x_1 = (\frac{p}{w_1})^2 \qquad (1-3)$$

$$x_2 = (\frac{p}{w_2})^2 \qquad (1-4)$$

which are the firm's demands for the two inputs. Substituting from $(1-3)$ and $(1-4)$ into the production function $f(x_1, x_2) = 2(\sqrt{x_1} + \sqrt{x_2})$,

$$y = \frac{2p}{w_1} + \frac{2p}{w_2} \qquad (1-5)$$

or

$$y = 2p\frac{w_1 + w_2}{w_1w_2}$$

which is the firm's profit–maximizing level of output, as a function of its input prices and output price.

*Q*2. What quantities of inputs 1 and 2 will minimize a firm's cost of producing y units of output, if its production function is

$$f(x_1, x_2) = x_1 + \ln(x_2 + 1) \qquad ?$$

The condition for long–run cost minimization is $MP_1/MP_2 = w_1/w_2$. Here

$$MP_1 = 1 \tag{2 – 1}$$

$$MP_2 = \frac{1}{x_2 + 1} \tag{2 – 2}$$

so that

$$x_2 + 1 = \frac{w_1}{w_2} \tag{2 – 3}$$

or

$$x_2 = \frac{w_1}{w_2} - 1 = \frac{w_1 - w_2}{w_2} \tag{2 – 4}$$

which is the firm's conditional factor demand for input #2 (at least when $w_1 > w_2$). To find the demand for input #1, substitute from $(2 - 3)$ into the production function

$$y = x_1 + \ln(x_2 + 1) = x_1 + \ln w_1 - \ln w_2 \tag{2 – 5}$$

(where I have used the fact that $\ln(w_1/w_2) = \ln w_1 - \ln w_2$). Therefore, the firm's conditional factor demand for input #1 is

$$x_1 = y - \ln w_1 + \ln w_2 \tag{2 – 6}$$

*Q*3. Could a single–price monopoly ever choose an output level at which its own–price elasticity of demand was less than 1 in absolute value? Explain.

*A*3. No, a single–price monopoly always chooses an output level at which the own–price elasticity of demand exceeds 1 (in absolute value).

One way of explaining this rule is the monopoly's first–order condition $MR = MC$. Since

$$MR = p(\frac{|\epsilon| - 1}{|\epsilon|})$$

then if $|\epsilon| < 1$, $MR < 0$, which is inconsistent with the condition $MR = MC > 0$.

Another way of explaining this rule is to note that if demand is inelastic (with respect to the good's own price), then an increase in price reduces quantity demanded by a smaller percentage than the price increase. This fact, in turn, means that the firm's sales revenue, py, would increase if the firm increased its price, if the demand were inelastic. But that means that the original policy can't be optimal : increasing the price would increase the firm's sales revenue, and decrease its costs (since it would not have to produce so much of the good).

*Q*4. Suppose that a monopoly can offer its customers several different varieties of a product, varying in their level of quality, and in their price.

The monopoly believes that it serves two different types of customer, with one type much more willing to pay for quality than the other. But it cannot identify directly the type of any individual customer.

What quality levels of product should it offer, and what prices should it charge for them?

*A*4. This is a question about second–price degree discrimination. Let q_1^* and q_2^* be the efficient levels of quality for the two groups, the quality levels for which

$$p_1(q_1^*) = MC(q_1^*)$$

$$p_2(q_2^*) = MC(q_2^*)$$

where $p_1(\cdot)$ and $p_2(\cdot)$ represent the marginal willingness to pay for a little more quality of the two groups, and $MC(\cdot)$ is the marginal cost of a little more quality. If $CS_1(q)$ and $CS_2(q)$ represent the two consumer types' total surplus from a product of quality level q (so that $p_1(q)$ and $p_2(q)$ are the derivatives of $CS_1(q)$ and $CS_2(q)$ with respect to q), then the monopolist would sell the quality level q_1^* at a price of $CS_1(q_1^*)$ to group 1, and q_2^* at a price of $CS_2(q_2^*)$ if it could identify the customers' types directly.

In the question, though, it cannot identify the types directly, and therefore must offer the same menu of packages to all customers. If type–1 customers are the ones with the **higher** willingness to pay for quality, then the monopoly's profit–maximizing menu consists of a quality level $q < q_2^*$, at a price of $CS_2(q_2)$, which is selected by type–2 customers, and a quality level q_1^*, at a price less than $CS(q_1^*)$, which is selected by type–1 customers. (The exact price of this higher–quality product is $CS_1(q_1^*) - CS_1(q_2) + CS_2(q_2)$.)