AS/ECON 4070 3.0AF Answers to Assignment 1 October 2000

1. The allocation resulting from the government tax on clothing sales will **not** be Pareto efficient. The reason : people's marginal rates of substitution of food for clothing will not all be the same as each others'.

For example, suppose that the government collects its 25 percent from the people who have just bought clothing. Consider person 1, who has an endowment $(\bar{x_1}, \bar{y_1})$ of food and clothing, and wants to buy more clothing. If p_x and p_y are the prices of food and clothing on the market, then the amount of clothing y_1 she actually gets to consume must equal

$$y_1 = \bar{y_1} + \frac{3}{4}\frac{p_x[\bar{x_1} - x_1]}{p_y} + y_G \tag{1}$$

where y_G is the amount of clothing she gets from the government. Why? The first term on the right side of equation (1) is the amount of clothing she owns originally ; the third term is the amount she is given by the government ; the second term is the amount she gets from purchasing : she sells food, earning her $p_x[\bar{x_1} - x]$ in money, she can buy $(p_x/p_y)[\bar{x_1} - x_1]$ units of clothing with this money, and she only gets to keep three–quarters of her purchases.

I can re–write equation (1) as

$$p_y y_1 + \frac{3}{4} p_x x_1 = p_y[\bar{y_1} + y_G] + \frac{3}{4} p_x \bar{x_1} \tag{2}$$

or

$$p_y y_1 + \frac{3}{4} p_x x_1 = M_1 \tag{3}$$

where M_1 is defined as the right side of equation (2) ; M_1 can be regarded as her "income" ; it is something over which she has no control, being the value of her endowment plus the value of what she gets from the government.

Now consider person 2, who does not buy clothes, but instead sells clothes to pay for his food. His clothing consumption is

$$y_2 = \bar{y_2} + y_G - \frac{p_x}{p_y}[x_1 - \bar{x_1}] \tag{4}$$

since he must sell clothing to buy food. I can re–write equation (4) as

$$p_y y_2 + p_x x_2 = M_2 \tag{5}$$

where

$$M_2 = p_y[\bar{y_2} + y_G] + p_x \bar{x_2}$$

is his exogenous "income".

So from equation (3), person 1's budget line has a slope of $\frac{3p_x}{4p_y}$, and from equation (5), person 2's budget line has a slope of $\frac{p_x}{p_y}$. When they choose how much to buy and sell, person 1 will choose a consumption bundle where her $MRS_1 = \frac{3p_x}{4p_y}$, while person 2 will choose a consumption bundle where his $MRS_2 = \frac{p_x}{p_y}$. Their indifference curves will not be tangent ; the "tax" on clothing sales has driven a wedge between their MRS's.

To get these results, I had assumed that person 1 was a buyer of clothing, and person 2 was a seller. But if there is any exchange at all — if people do not just consume what they own — there must be someone who is buying clothing and someone who is selling clothing, so my assumption makes sense. I also assumed that the government collected its "tax" from the clothing buyer. If it collected the tax from the seller instead, then the sellers' MRS would be $3p_x/4p_y$ and the buyers' MRS would be p_x/p_y, so again MRS's would not be the same for everyone.

The main point is : however the government collects its 25 percent, there will be a difference in the terms of trade between how much clothing sellers of food get for each kilo of food sold, and how much clothing buyers of food must give up for each kilo of food bought.

2. *i* The person will always choose a consumption bundle where her marginal rate of substitution equals ratio of prices she faces. By definition

$$MRS = \frac{MU_x}{MU_y}$$

When $U(x, y) = xy$, $MU_x = y$ and $MU_y = x$ so that

$$MRS = \frac{y}{x}$$

If $x = 12$ and $y = 12$, then her MRS equals 1, which equals the price ratio when $p_x = 1$ and $p_y = 1$. If $x = 12$ and $y = 16/3$, then

$$MRS = \frac{y}{x} = \frac{16}{3}/12 = \frac{4}{9}$$

When $p_x = 1$ and $p_y = 2.25$, then

$$\frac{p_x}{p_y} = \frac{1}{2.25} = \frac{4}{9}$$

So in each case, her MRS equals the price ratio she faces.

It also can be checked that she can exactly afford the consumption bundle she chooses in each case :

$$1 \cdot 12 + 1 \cdot 12 = 24$$

$$1 \cdot 12 + (2.25)(\frac{16}{3}) = 1 \cdot 12 + \frac{9}{4}\frac{16}{3} = 24$$

ii If the tax were imposed, she would face prices of 1 and 2.25, choose the consumption bundle $(12, \frac{16}{3})$, and attain a utility level of

$$12(\frac{16}{3}) = 64$$

What would happen if she paid EV dollars to avoid the tax? She would face prices of $(1, 1)$, and so would choose $x = y$, since her MRS will equal 1 only when $x = y$. So in this case, she would pick $x = \frac{24-EV}{2} = y$, and would have utility of

$$xy = (\frac{24 - EV}{2})^2$$

If this utility were to equal 64 (the utility she would get if the tax were imposed), then it must be the case that

$$(\frac{24 - EV}{2})^2 = 64$$

Since $64 = 8^2$, this is the same as

$$\frac{24 - EV}{2} = 8$$

or

$$EV = 8$$

This can be done graphically as well. Without the tax, the consumer was on the indifference curve through $(12, 12)$, the set of consumption bundles such that $xy = 144$. After the tax, she is on the indifference curve through $(12, 16/3)$, the set of consumption bundles such that $xy = 64$. The equivalent variation involves moving the original budget line — the one tangent to the "high" indifference curve at $(12, 12)$ — in parallel, until it is tangent to the "low" indifference curve. It will be tangent to this low indifference curve at $(8, 8)$, and how much the budget line has shifted in parallel can be read off the horizontal axis : the horizontal intercept has shifted from 24 to 16, a shift in of 8.

iii When there was no tax, she chose the consumption bundle $(12, 12)$, and got a utility level of 144. What level of compensation CV would bring her back to this level of utility if the tax were imposed? If she faces the prices $(1, 2.25)$, then her choice of consumption (x, y) would have to result in

$$\frac{y}{x} = MRS = \frac{1}{2.25} = \frac{4}{9}$$

So

$$y = \frac{4}{9}x$$

Her budget constraint, if she faced the tax, and were given the compensation CV, would be

$$x + 2.25y = 24 + CV$$

Substituting for y from the previous equation

$$x + (2.25)(\frac{4}{9})x = 24 + CV$$

Or

$$2x = 24 + CV$$

Then

$$y = \frac{4}{9}x = \frac{(24+CV)4}{2(9)} = \frac{2(24+CV)}{9}$$

and her utility would be

$$xy = [\frac{24+CV}{2}][\frac{2(24+CV)}{9}] = \frac{(24+CV)^2}{9}$$

If the compensation CV brought her exactly back to her old level of utility of 144, then it would have to be true that

$$\frac{(24+CV)^2}{9} = 144$$

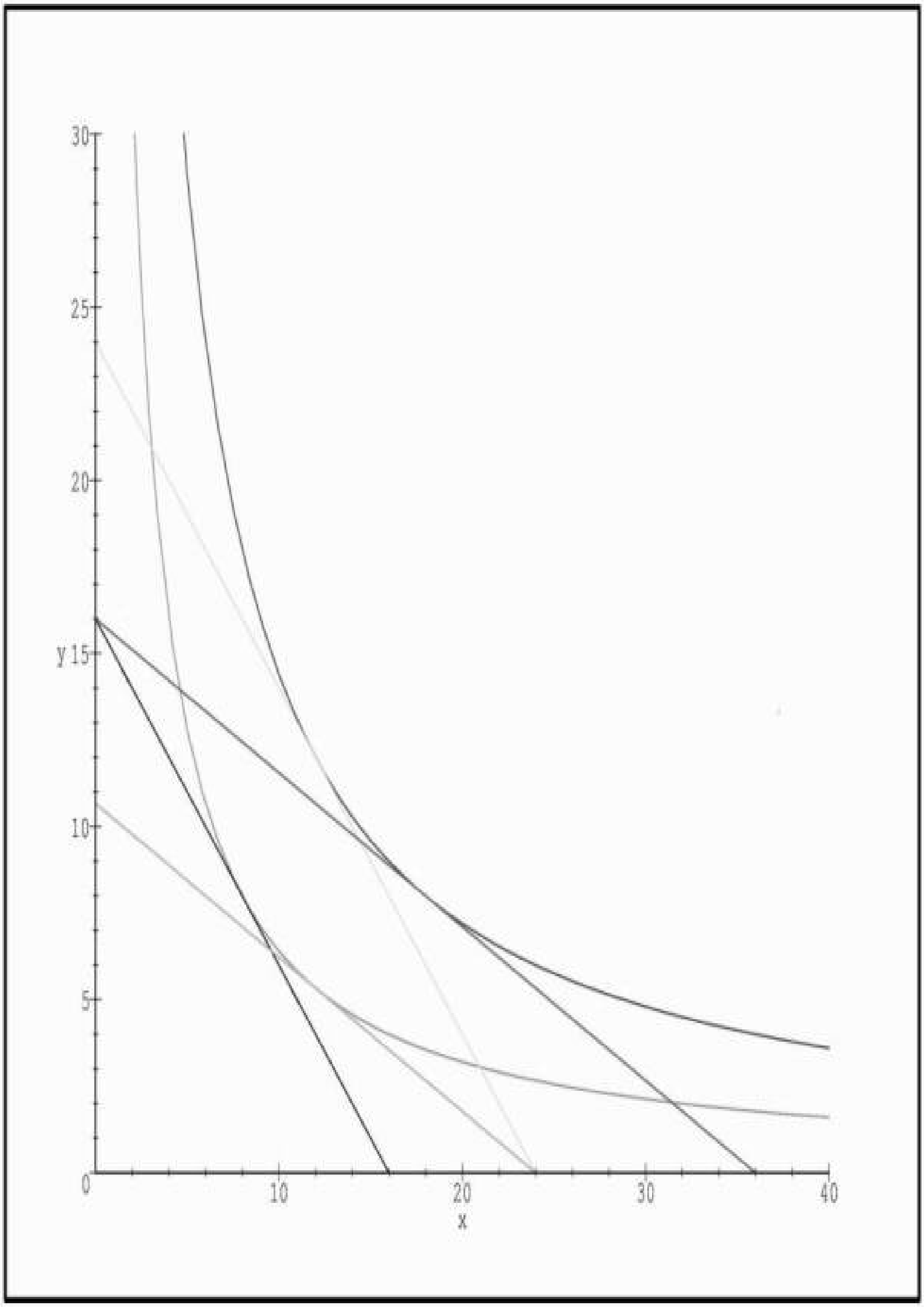
Since $144 = 12^2$, and $3^2 = 9$ this means

$$\frac{24+CV}{3} = 12$$

or

$$CV = 12$$

This can be done graphically as well. The tax moves the customer from the "high" indifference' curve $xy = 144$ to the "low" indifference curve $xy = 64$. After the tax is imposed, the person is at the point $(12, 16/3)$ on this low indifference curve. The CV is found by shifting out the budget line which is tangent to the low indifference curve at $(12, 16/3)$. This budget line has a slope of 2.25, because that is the slope of the indifference curve at that point. The compensation is supposed to shift out this budget line (with a slope of 2.25), until it is tangent to the high indifference curve (at $(18, 8)$). The amount of compensation can be read of the horizontal axis : the intercept of the budget line has shifted out from 24 to 36, a compensation of 12.



3. Without the tax, $p_s = P^D$, and the equilibrium price p, where the demand and supply curves cross, solves

$$180 - 2p = 4p$$

or

$$p_s = P^D = 30$$

A tax of \$6 implies that $p_s = P^D - 6$. Now quantity supplied will equal quantity demanded if

$$180 - 2P^D = 4p_s = 4(P^D - 6)$$

or

to the seller, now I am just willing to pay \$19, since paying \$19 to the seller means now that I am paying \$20 in total.

If the perfectly price–discriminating monopoly wants to sell any of the good, after the \$1 unit tax has been imposed on buyers, the monopoly must lower its price, by \$1 per unit. The seller bears 100 percent of the tax here, just as in part *i*.

5. *i* Firms here charge a price of $(1+m)c$, where c is their average cost of production. A unit tax of \$1 raises their costs to $1+c$, meaning that they would all raise their prices to $(1+m)(1+c)$, an increase in price of $(1+m)$ dollars, which is greater than the increase in the tax. That is, buyers would bear more than 100 percent of the tax here.

ii In this case, the firms' costs are unchanged, so that the price they charge would be unchanged, assuming they stuck with their fixed mark–ups. The net–of–tax price would stay at $(1+m)c$, but buyers would now have to pay the tax of \$1 in addition, so that the tax–inclusive price paid by buyers would increase by exactly the amount of the tax. Buyers would bear exactly 100 percent of the tax.

With the fixed mark–up rule, because sellers appear not to be behaving as profit maximizers, the statutory incidence of the tax does matter ; the buyers bear more of the tax when the statutory incidence is on the sellers.