AS/ECON 4070 AF   Answers to Assignment 1   October 2001

1. Yes, the allocation will be efficient, since the tax in this question is a tax on the value of people's endowments. This is a lump–sum tax.

In an exchange economy, an allocation is efficient if (and only if) everyone's marginal rates of substitution are the same.

What would be the budget problem faced by people in this economy? Let P_x and P_y be the prices of food and clothing. Then a type–1 person (a person with an endowment of food, but not clothing) faces the following problem. Her total income is $M^1 \equiv (0.75)P_x + g$, where g is her share of the government tax revenue. The reason for this? She has an endowment of 1 unit of food, worth P_x, and the government taxes 25 percent of that. So her budget line has the equation

$$P_x x + P_y y = (0.75)P_x + g \equiv M^1$$

A type–2 person (one with an endowment of clothing) has an income of $M^2 \equiv (0.75)P_y + g$, and so has a budget line with equation

$$P_x x + P_y y = (0.75)P_y + g \equiv M^2$$

That means that the demand for food of the two types of people (from the demand functions given in the question) are

$$x_1 = \frac{(0.75)P_x + g}{2P_x} = 0.375 + \frac{g}{2P_x}$$

$$x_2 = \frac{(0.75)P_y + g}{2P_x} = 0.375\frac{P_y}{P_x} + \frac{g}{2P_x}$$

and the demands for clothing are

$$y_1 = \frac{(0.75)P_x + g}{2P_y} = 0.375\frac{P_x}{P_y} + \frac{g}{2P_y}$$

$$y_2 = \frac{(0.75)P_y + g}{2P_y} = 0.375 + \frac{g}{2P_y}$$

What are the people's marginal rates of substitution? If $U(x_i, y_i) = x_i y_i$, then

$$MRS^i = \frac{U_x}{U_y} = \frac{y_i}{x_i}$$

so that

$$MRS^1 = \frac{P_x}{P_y} = MRS^2$$

and everyone's indifference curve has the same slope, so that the allocation is Pareto optimal.

This is not necessary to answer the question, but the actual allocation can be computed here. Since g is the total tax revenue divided equally among all people, then

$$g = (0.25)\frac{P_x + P_y}{2} = \frac{P_x + P_y}{8}$$

In an equilibrium allocation, total demand for food must equal the total supply of food. Here the total demand for food is

$$X^D = x_1 + x_2 = \frac{3}{8} + \frac{3}{8}(\frac{P_y}{P_y}) + \frac{g}{P_x} = \frac{3}{8} + \frac{3}{8}(\frac{P_y}{P_x}) + \frac{1}{8} + \frac{1}{8}(\frac{P_y}{P_x}) = \frac{1}{2}(1 + \frac{P_y}{P_x})$$

This total demand must equal the total supply of food, which is 1 (the endowment of the type–1 people). Therefore, if quantity supplied of food equals quantity demanded, then

$$\frac{1}{2}(1 + \frac{P_y}{P_x}) = 1$$

so that

$$P_x = P_y$$

(You can check that quantity demanded of clothing equals quantity supplied of clothing when $P_x = P_y$.) In this case,

$$g = \frac{P_x}{4} = \frac{P_y}{4}$$

and the actual allocation is

$$x_1 = x_2 = y_1 = y_2 = \frac{1}{2}$$

At this allocation, people's indifference curves have the same slopes (since they have the same preferences, and have the same consumption bundles), so that the allocation must be Pareto optimal.

2. If the only cost of education is that people cannot work while they are getting an education, then the allocation will be Pareto optimal, even if there is a proportional tax on all labour income.

The reason? The cost of a person choosing to stay in school for another week is the wage earnings she loses by staying in school. The benefit of another week's education is the increase in her wage in the future. In a world with no income tax, the person will stay in school up to the point at which the marginal increase in the present value of her lifetime earnings from another week in school exactly equals the wages she would earn that week, that is up to the point at which

$$40w = I$$

where I is the increase in the present value of her lifetime earnings caused by another week of education. (This marginal benefit is not a constant : most likely, the more education she acquires, the lower will be the increase in her earnings from another week of education.)

If all her income is taxed at the marginal rate t, then the cost of another week of education falls : it is only the net–of–tax wages that she forgoes. But the benefit of another week of education also falls : to the increase in the present value of her net–of–tax lifetime earnings. So her choice of how much education to acquire is to acquire education up to the point at which

$$(1-t)40w = (1-t)I$$

which is exactly the same condition as was optimal in the (efficient) world without taxes.

If there are other costs to education, aside from foregone income, then the proportional income tax will lead to an *inefficient* allocation. Suppose that another week of education imposes costs of c : the out–of–pocket costs of tuition and commuting, plus the dollar value she places on the sheer tedium of attending school. Suppose further that these costs are not tax–deductible. Then the efficient decision is to acquire education up to the point at which

$$40w + c = I$$

whereas if her income is taxed, she will acquire education up to the point at which

$$(1-t)40w + c = (1-t)I$$

or

$$40w + \frac{c}{1-t} = I$$

She will acquire an inefficiently low level of education, since she is sharing all of the benefits of the education with the tax collector, but only some of the costs.

3. First, the person's demand for coffee should be calculated. She chooses a consumption bundle (x, y) such that her marginal rate of substitution equals the price ratio. Here

$$MU_x = \frac{\partial U}{\partial x} = 2 - x$$

$$MU_y = \frac{\partial U}{\partial y} = 1$$

so that

$$MRS = 2 - x$$

Since y is her expenditure on all other goods and services, then $P_y = 1$, so that the price ratio is just P_x. Her optimal consumption bundle (x, y) must satisfy the equation

$$MRS = 2 - x = P_x$$

or

$$x = 2 - P_x$$

which defines her demand curve for coffee.

If there were no tax, and the price of coffee were \$1, then she would demand 1 cup of coffee per day. Since her income is \$100 per day, that leaves \$99 for expenditure on other goods and services, so that $(x, y) = (1, 99)$ when there is no tax. Her utility, if there is no tax, can be calculated as

$$U(1, 99) = 99 + 2(1) - \frac{1}{2}(1)^2 = 100.5$$

Now the tax raises the price of coffee to \$6. Her demand curve $x = 2 - P_x$ would then imply a *negative* consumption of coffee, which of course is impossible. If the price of coffee exceeds \$2, then she will choose not to consume any coffee, and to spend all her money on other goods and services. Thus the tax changes her consumption bundle to $(x, y) = (0, 100)$, and she gets a utility level of 100.

What is the cost of the tax to her?

One measure would be the *compensating variation* to the tax. How much would we have to pay her, if the tax were implemented, to get her back to her original utility level of 100.5? If her income were $100 + CV$, and coffee were taxed, she would still choose not to consume coffee, and she would spend all her income on other goods and services. So she would have $(x, y) = (0, 100 + CV)$, and a utility level of $100 + CV$. In order to get her back to her old level of utility, she would have to be compensated with $CV = 0.5$.

Another measure is the *equivalent variation* to the tax How much could be taken away from her, to have the same effect as the tax? The tax has the effect of lowering her utility from 100.5 to 100. What if instead the price of coffee stayed at \$1, and EV were taken away from her? With $P_x = 1$, she chooses to consume 1 cup of coffee per day, and to spend the rest on other goods and services. So she would choose $y = 100 - EV - 1$, leading to a utility level of

$$U = 100 - EV - 1 + 2(1) - \frac{1}{2}(1)^2 = 100.5 - EV$$

For her utility to fall to 100, it would have to be the case that $EV = 0.5$.

Notice that here the compensating and equivalent variations to the tax are the same. That is because her *quasi–linear* preferences imply that her demand for coffee is independent of income. In this case, there is only one compensated demand curve, and it equals the ordinary demand curve. (Recall from the Slutsky equation that ordinary and compensated demand derivatives are the same when the derivative of quantity of coffee demanded with respect to income is 0.)

Another way of calculating the cost is to use the area to the left of the demand curve, between the before– and after– tax prices. Here the demand curve has the equation $x = 2 - P_x$, so the demand curve is a straight line. When $P_x = 2$, $x = 0$, and when $P_x = 1$, $x = 1$. So the demand curve (ordinary, or compensated) connects the points $x = 0$, $P_x = 2$ and $x = 1$, $P_x = 1$. The area to the left of this demand curve, between the before–tax price of 1 and the after–tax price of 6, is a triangle with height 1 and width 1, so has an area of 0.5.

4. There are several ways to do this.

One is simply to solve directly :

The equilibrium condition is that quantity demanded equal quantity supplied, or

$$55 - 5(p_s + t) = 15p_s - 5$$

in this case. This equation simplifies to

$$20p_s = 60 - 5t$$

or

$$p_s = 3 - \frac{t}{4}$$

Therefore, when there is no tax, $p_s = P^D = 3$, and when there is a \$1 tax, $p_s = 2.75$ and $P^D = 3.75$. These results imply that buyers bear 75 percent of the cost of the tax, and sellers 25 percent.

A second way is to look at the slopes of the supply and demand functions : the share of the cost born by the seller equals

$$-\frac{\partial Q^D/\partial P^D}{\partial Q^s/\partial p_s - \partial Q^D/\partial P^D}$$

Here $\partial Q^D/\partial P^D = -5$ and $\partial Q^s/\partial p_s = 15$, so that the share of the cost born by the seller is $5/15 + 5 = 1/4$; sellers bear 25 percent of the cost and buyers 75 percent.

A third way is to use elasticities. The share of the cost born by the buyers is approximately

$$\frac{\epsilon_s}{\epsilon_s + \epsilon_D}$$

Here

$$\epsilon_s = 5\frac{p_s}{Q}$$

and

$$\epsilon_D = 15\frac{P_D}{Q}$$

so that the share born by sellers is approximately

$$\frac{15p_s}{15p_s + 5P^D}$$

Initially, with no tax, $p_s = P^D$, implying buyers bear about 75 percent of the tax, and sellers about 25 percent. (That's actually an exact answer : here the approximation in my elasticity formula happened to exactly cancel the approximation in assuming $p_s = P^D$.)

5. The main point is that demand for housing in the whole greater Toronto area should be much less elastic than demand for housing in King Township. There are many close substitute for houses in King Township, namely houses in nearby areas.

So the costs of a development levy in King Township should be born predominantly by owners of land in the township. The price of houses in King Township cannot rise very much in response to the levy, or prospective buyers would simply choose to buy somewhere else. If the tax cannot be shifted forward on to housing buyers, it must be shifted backwards. It cannot be shifted much onto suppliers of housing materials or onto workers in the construction industry. The supply of labour and of construction materials to the housing industry in King Township should be pretty elastic : if wages in construction fell much in King Township, compared to the rest of the greater Toronto area, then no–one would be willing to work on housing construction in King Township.

Since the supply of land in a single jurisdiction is (pretty well perfectly) inelastic, land owners would wind up bearing much of the cost of a development levy imposed in a single small jurisdiction.

For the whole greater Toronto area, the demand for housing is likely to be much less elastic, as is the supply of labour to the construction industry. The costs of a development levy are likely to be shared by housing buyers, workers, and land owners.