AS/ECON 4070 Answers to Assignment 1 October 2003

1. Since the question stated that there was no production, it refers to a pure exchange economy. In an exchange economy, the only economic activity is allocation of the existing stock of consumption goods among the people.

In an exchange economy, an allocation is Pareto efficient if and only if the marginal rate of substitution (MRS) between any two goods is the same for any two people.

So what must be checked is whether each person has the same MRS between food and clothing, when people are subject to this 25 percent tax, and when they choose how much to consume in competitive markets.

If person i has an endowment $(\bar{x}_i, \bar{y}_i)$ of food and clothing, and if she receives no money from the government, then she chooses her consumption (x_i, y_i) of food and clothing so as to get to the highest indifference curve in her budget set. Her budget constraint is that the value of all her consumption, plus the taxes she pays, not exceed the value of her endowment, or

$$(1.25)(p_x x_i + p_y y_i) \leq p_x \bar{x}_i + p_y \bar{y}_i \qquad (i)$$

where p_x is the price of food, and p_y is the price of clothing. (The factor 1.25 in the equation above is there because she must pay a tax of 25 percent on the value of all her consumption.)

The budget line defined by the constraint (i) is a line with a slope of $1.25p_x/1.25p_y = p_x/p_y$ if I graph food consumption on the horizontal axis and clothing consumption on the vertical. The person will choose a consumption bundle where her MRS equals the slope of this budget line, so she will choose a consumption bundle for which $MRS_i = p_x/p_y$.

Now consider person j, an old person, who is receiving some tax revenue from the government. Let R be the amount of this tax revenue he receives . Then his budget constraint is

$$(1.25)(p_x x_j + p_y y_j) \leq px\bar{x}_j + p_y \bar{y}_j + R \qquad (j)$$

Again, this defines a budget line with a slope of $1.25p_x/1.25p_y = p_x/p_y$; the tax revenue he receives shifts out his budget line, but does not change its slope.

So when the older person j chooses his preferred consumption bundle (x_j, y_j), he too will pick a bundle such that his MRS equals the slope p_x/p_y of his budget line.

Therefore, in any competitive equilibrium, $MRS_i = MRS_j$ for any two people i and j. Since all people face the same relative prices, they all choose allocations with the same MRS. Collecting tax money from everyone, and giving it only to the old people, will change the equilibrium. Compared to an equilibrium with no taxes at all, it will move the economy to a new allocation which is better for old people and worse for young people. But it does so in an efficient way, since it moves to an allocation on the contract curve, at which people's indifference curves are tangent in the Edgeworth box diagram.

2. At any Pareto optimal allocation inside the Edgeworth box, the two people's indifference curves must be tangent. The set of all such Pareto optimal allocations is the contract curve in the Edgeworth box.

The slope of a person's indifference curve is her MRS, the ratio of the marginal utilities of the two goods.

Since person 1's utility function is $\ln x_1 + \ln y_1$, then her marginal utilities are $MU_x^1 = 1/x_1$ and $MU_y^1 = 1/y_1$ — the marginal utility of a good is the derivative of the utility function with respect to the consumption of the good, and the derivative of the function $f(z) = \ln z$ is $1/z$.

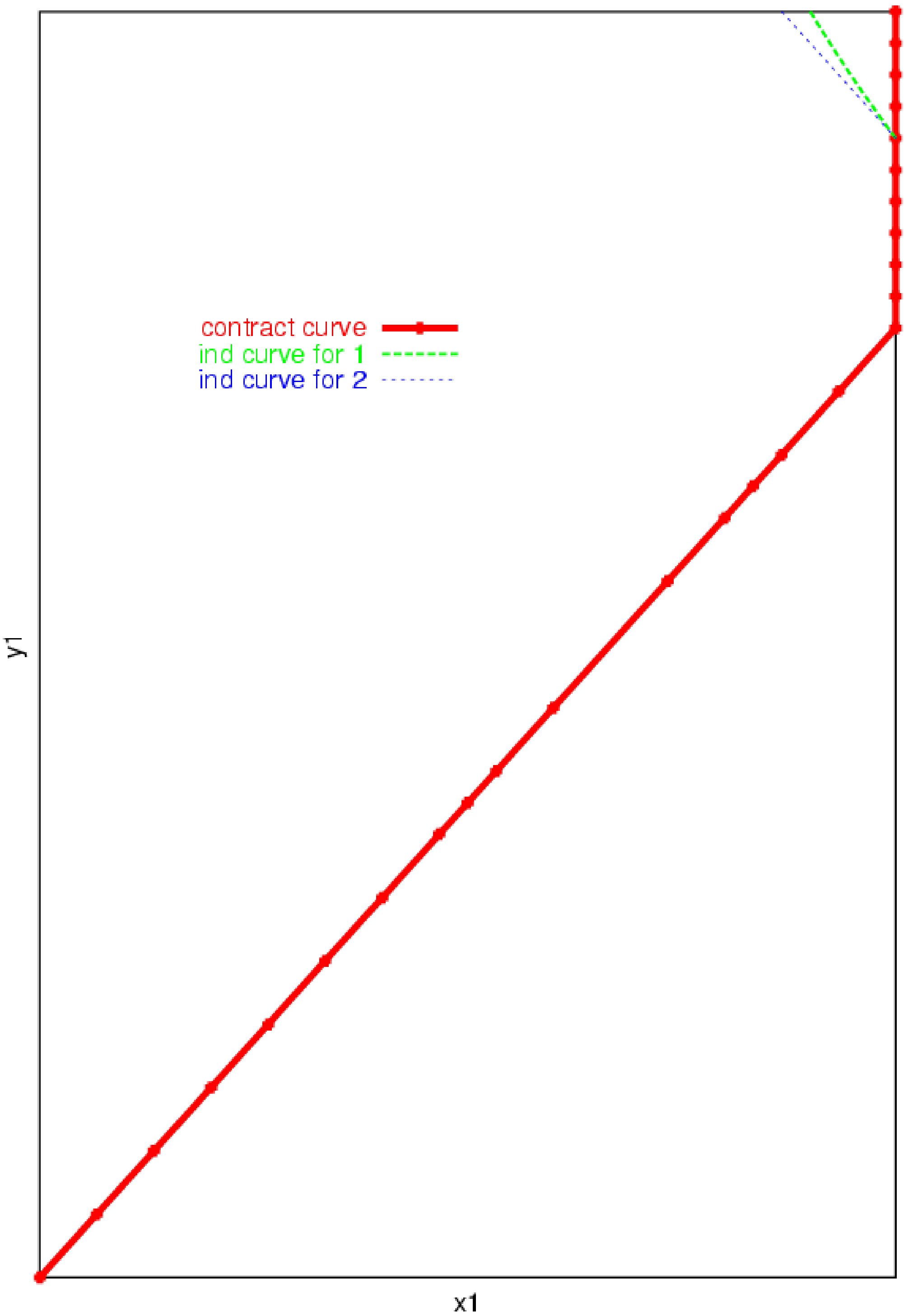
Person 1's MRS is the ratio of the marginal utilities, $MU_x^1/MU_y^1 = y_1/x_1$.

Person 2's utility function is $x_2 + y_2$, so for him $MU_x^2 = 1$ and $MU_y^2 = 1$. This person's MRS equals 1, no matter what he consumes. That is, this person regards the two goods as *perfect substitutes* : his indifference curves are straight lines.

So when will $MRS_1 = MRS_2$? Whenever $x_1/y_1 = 1$, which is the same as $x_1 = y_1$. So the contract curve, in this case, is a straight line in the Edgeworth box. Measuring from the bottom left corner, the contract curve starts at the bottom left corner $(0, 0)$, and going up through points such as $(1, 1)$, $(3, 3)$, $(4.113, 4.113)$ until it hits the right edge of the Edgeworth box at the point $(15, 15)$.

The trickier part, is what happens "after" the contract curve hits the right edge of the Edgeworth box, at the point $x_1 = 15$, $y_1 = 15$ (and $x_2 = 0, y_2 = 5$). The answer is that the contract curve continues up the right side of the Edgeworth box, through points such as $(15, 16)$ and $(15, 18)$, until it hits the top right corner of the box.

Why are points such as $(15, 18)$ Pareto optimal? Because at such a point, the indifference curves of person 1 are steeper than those of person 2. (At $(15, 18)$, $MRS_1 = 18/15 > MRS_2 = 1$.) In other words, the allocations which are better for both people would lie below and to the right of $(15, 18)$, points which are impossible, since they're outside the Edgeworth box. Drawing indifference curves for the two people through $(15, 18)$, it can be seen that no allocations in the box are better for both people than $(15, 18)$: points below the curve $\ln x_1 + \ln y_1 = \ln 15 + \ln 18$ are worse for person 1, and points above the line $x_2 + y_2 = 2$ are worse for person 2.


contract curve
ind curve for 1
ind curve for 2
y1
x1

3. If wine is in perfectly elastic supply, then the buyers are going to bear all of the tax.

But the question asked what is the cost of that tax to them. In other words, how much would this buyer pay to avoid this tax (the equivalent variation to the tax)? or how much would we have to compensate the buyer for the damage done by the tax (the compensating variation to the tax)?

To answer either of those questions, the buyer's demand function for wine must be derived.

The buyer chooses her preferred consumption bundle by finding the point on her budget line at which her MRS equals the price ratio. Here her utility function is $x + 40y - 2y^2$, so that the marginal utility of x consumption equals 1, and the marginal utility of wine consumption is $40 - 4y$. So her marginal rate of substitution of wine for other goods is $MU_y/MU_x = 40 - 4y$. Setting this MRS equal to the price ratio implies that

$$40 - 4y = P_y$$

where P_y is the price per bottle of wine (including any taxes). This equation can be re–arranged into

$$y = (40 - P_y)/4 \qquad (D)$$

which is the equation of her demand curve for wine.

Notice that information about the person's income was needed in deriving this demand curve. That's because her preferences are *quasi–linear* : as discussed, for example, in Varian's intermediate micro textbook, quasi–linearity of preferences means that her income elasticity of wine demand is zero. Her quantity demanded of wine does not depend on her income ; any changes in her income result in her changing only her consumption of other goods.

From equation D, it follows that she would demand $y = 5$ bottles of wine if there were no tax (so that the price P_y were 20), and $y = 2$ bottles of wine if it were taxed at \$12 a bottle.

Notice that the demand curve D is a straight line. So the area under the demand curve, betwen the prices of 20 and 32, equals 42 : it's the sum of a rectangle which has height 12 and length 2, and a right–angle triangle of height 12 and length 3.

Is 42 the compensating variation for the tax, or the equivalent variation to the tax? It's both. Since quantity demanded of wine is independent of income, it is also true that the compensated demand curve does not shift with the person's level of utility. Because of the quasi–linear preferences, there is only one demand curve, D.

The compensating and equivalent variations can be calculated directly. Suppose that the person had income M. Then when wine sold for \$20 a bottle, she would buy 5 bottles of wine, leaving $M - 100$ to spend on other goods x. So her utility, $x + 40y - 2y^2$ would be $M - 100 + 40(5) - 2(5)^2 = M + 50$. When wine is taxed, and costs \$32 a bottle, she buys 2 bottles, leaving $M - 64$ for other goods, and giving her a utility level of $M - 64 + 40(2) - 2(2)^2 = M + 8$. Taxing wine has lowered her utility by 42. So if we increased her income from M to $M + 42$, after putting in the tax, then her utility would be $(M + 42) - 64 + 40(2) - 2(2)^2 = M + 50$, so that 42 is the compensating variation to the tax. On the other hand, if we took away 42 from the person, instead of taxing wine, then her utility would go from $M + 50$ to $(M - 42) - 100 + 40(5) - 2(5)^2 = M + 8$, the same as it would be if we taxed wine. Therefore, 42 is also the equivalent variation to the tax.

4. A major error on my part in setting up this question : the quantity demanded turns out to be negative after the tax, which is impossible.

Ignoring this enormous problem, the equilibrium could be solved explicitly by noting that the quantity demanded must equal the quantity supplied, so that

$$120 - 3(p_s + t) = 7p_s - 180$$

where t is the tax per blank CD.

Solving,

$$10p_s = 300 - 3t$$

or

$$p_s = 30 - \frac{3t}{10}$$

When there is no tax, $p_s = 30$, and when there is a tax of fifty cents, then $p_s = 30 - 150/10 = 15$. Thus a tax of fifty cents causes the supply price to fall by 15 cents, and the buyers' price $P^D = p_s + t$ to rise by 35 cents (from 30 cents to 65 cents). Hence buyers bear 70 percent of the tax.

The elasticity formula gives a fair approximation here. For example, if the elasticities are evaluated at the original, no–tax equilibrium, at which $p = 30$ and $Q = 30$, then $\epsilon_s = 7$ and $\epsilon_D = 3$, so that

$$\frac{\epsilon_D}{\epsilon_s + \epsilon_D} = 0.3$$

which is exactly the answer. Evaluating elasticities at the after–tax equilibrium also gives a good approximation — except that both ϵ_s and ϵ_D are negative, since $Q = -75 < 0$.

Given the glitch in the question, a couple of other approaches are certainly sensible. One would be to assume that the quantity cannot be negative, so that the tax results in the market shutting down : at any p_s below 25.71, a quantity of zero will be supplied, and at any P_D above 40 a quantity of zero will be demanded. So it would be "as if" p_s fell from 30 to 25.71, and P^D increased from 30 to 40.

Another approach would be to assume that the P^D and p_s in the equations are measured in dollars, not cents, so that $t = 0.5$ instead of $t = 50$. In that case, the new p_s is $30 - 3(.5)/10 = 29.85$, and $P^D = p_s + t = 30.35$, so that buyers bear 70 percent of the tax.

5. Who could possibly bear this tax? Given the assumptions, the only possible groups which could bear the tax are cranberry juice buyers, cranberry juice firms, sugar suppliers, or cranberry growers.

Juice buyers cannot bear much of the tax, since the demand is relatively elastic with respect to the demand price.

If the production function exhibited constant returns to scale, so that each additional 10 kilograms of sugar, and 1 kilogram of cranberries, yielded exactly the same quantity of juice, then the marginal cost of production would be constant. (For example, if the price of sugar were p_s per kg, and the price of cranberries p_c per kg, and each 10 kg of sugar and 1 kg of cranberries produced 10 litres of juice, then the marginal cost of each 10 litres of juice would be $10p_s + p_c$.) With constant marginal cost, there would be no economic profits earned by firms in a competitive cranberry juice industry. So firms would not bear any of the tax, since they are earning zero economic profits to start with.

On the other hand, if the cranberry juice industry were not competitive, or if it were supplied under decreasing returns to scale, then firms supplying juice might see their profits fall after the tax, and might bear some of the tax.

What about suppliers of the inputs, cranberries and sugar? Even though sugar is being taxed, sugar suppliers cannot bear much of the tax if the supply is elastic. Suppose, for example, that the world price of sugar were 50 cents a kg, and that Canada were too small to influence that price. Then the price paid by juice producers in Canada has to be this world price : if they were to pay a price less than 50 cents, then sugar suppliers would sell their sugar elsewhere, where they can get 50 cents a kilo.

The most likely outcome? Even though they are not officially being taxed, cranberry growers will bear most of the tax. Putting the tax on sugar use in cranberry juice manufacture will raise the cost of cranberry juice production — other things equal. But if the cost of production goes up, then the price goes up. And the elastic demand by juice drinkers says that the price can't go up very much : cranberry juice drinkers would switch to some other kind of juice if the price went up much.

So the cost can't go up. It's also impossible for cranberry juice manufacturers to substitute for sugar, at least according to the story told in the question.

So if the price were to go up, and quantity demanded were to fall, then the demand for the raw material cranberries would fall. That would tend to drive down the market price of cranberries, so that cranberry producers bore most of the tax.

In the extreme case — perfectly elastic cranberry juice demand, perfect competition and constant returns to scale in cranberry juice production, and perfectly elastic supply of sugar — then cranberry producers would bear all of the tax. For example, if initially $p_s = 100$ and $p_c = 200$, and a tax of $t = 10$ were placed on the use of sugar in the cranberry industry, then p_c would fall from 200 to 100. The cost to cranberry juice producers (and to buyers of juice) would stay the same : $10(100) + 200 = 1200$ initially, and $10(100) + 10(10) + 100 = 1200$ after the tax of 10 cents per kg of sugar, and the fall in the price of cranberries.

More generally, cranberry producers will certainly bear a lot of the tax. Cranberry juice firms might bear some of the tax if they were earning economic profits from production, but sugar suppliers and cranberry juice drinkers could only bear a small fraction of the tax.