AS/ECON 4070 3.0AF Answers to Assignment 1 October 2004

1. An allocation is on the contract curve if the two people's indifference curves have the same slopes at the allocation.

So the first step is to calculate the slopes of these people's indiffrence curves. The slope of an indifference curve, the MRS, is the ratio of the marginal utilities of the different goods. For person 1, differentiation of the utility function implies that

$$MU_X^1 = 1$$

$$MU_Y^1 = \frac{3}{y_1}$$

so that the slope of her indifference curve is

$$MRS^1 = \frac{MU_X^1}{MU_Y^1} = \frac{1}{(3/y_1)} = \frac{y_1}{3}$$

For person 2,

$$MU_X^2 = \frac{12}{x_2}$$

$$MU_Y^2 = 1$$

and

$$MRS^2 = \frac{12}{x_2}$$

Therefore, a feasible allocation (x_1, y_1, x_2, y_2) will be on the contract curve if $MRS^1 = MRS^2$, that is if

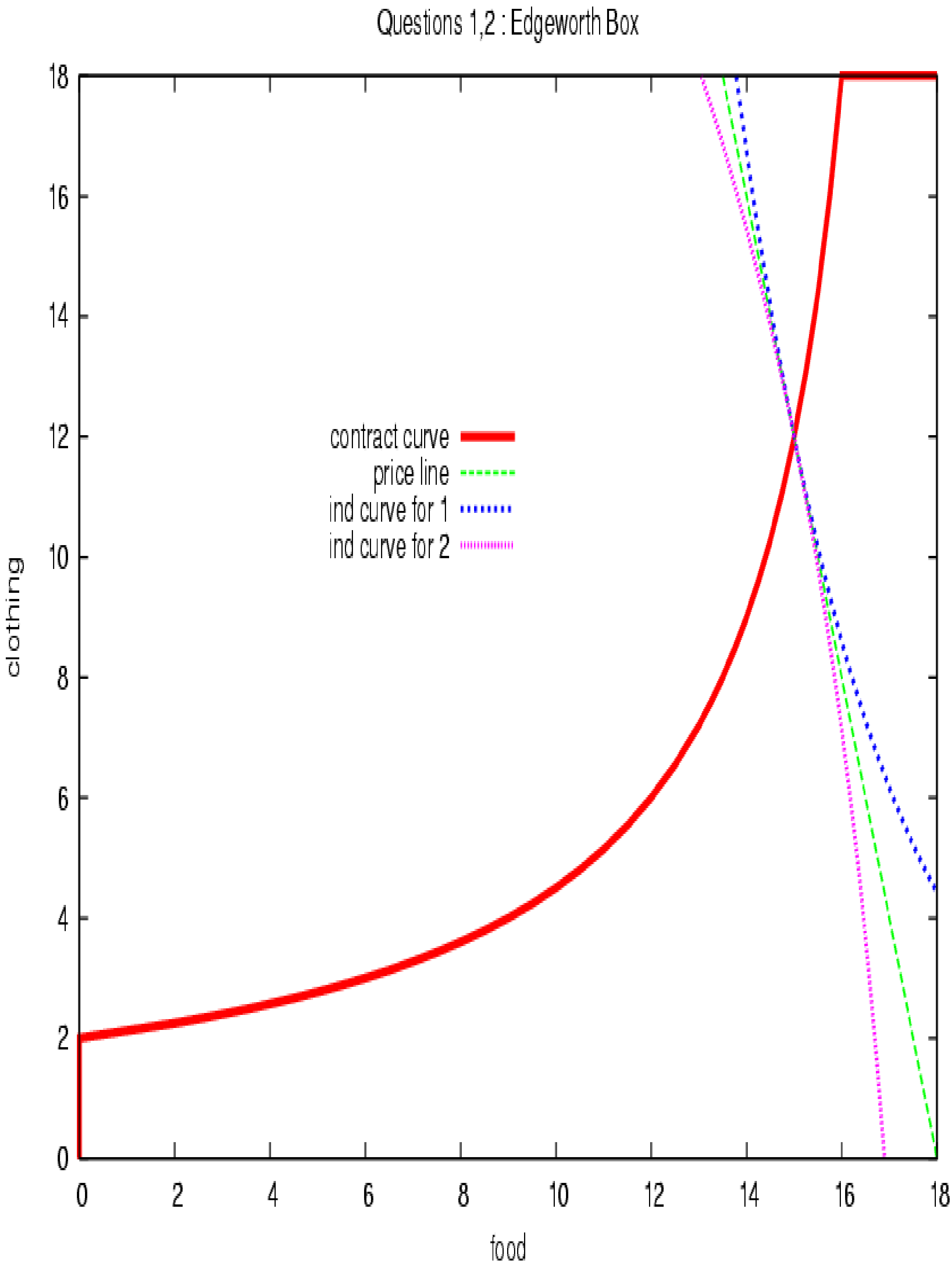
$$\frac{y_1}{3} = \frac{12}{x_2}$$

Since $x_1 + x_2 = 18$, then $x_2 = 18 - x_1$, so that the above condition can be written

$$y_1 = \frac{36}{18 - x_1} \qquad (cc)$$

Equation (cc) is the equation of the contract curve. It defines an upward–sloping curve in the Edgeworth box, going through points such as $(2, 0)$, $(6, 3)$, $(12, 6)$ and $(14, 9)$ — where the coordinates here are measured from the bottom left of the box.

This curve starts along the vertical axis, at $(2, 0)$, and hits the top of the Edgeworth box at the point $(16, 2)$. So how does it get to the corners of the box? It also should include the left edge of the box, between $(0, 0)$ and $(0, 2)$, and the top edge, between $(16, 18)$ and $(18, 18)$. The figure illustrates the contract curve.


Questions 1,2 : Edgeworth Box
contract curve
price line
ind curve for 1
ind curve for 2
clothing
food
0
2
4
6
8
10
12
14
16
18

2. What is the demand for good Y of a person of type #1? She wants to choose a consumption bundle (x_1, y_1) such that her indifference curve is tangent to her budget line. That means she wants a consumption bundle (x_1, y_1) such that $MRS^1 = p_x/p_y$. From the answer to question #1, $MRS^1 = y_1/3$, so that she will choose a consumption bundle such that

$$\frac{y_1}{3} = \frac{p_x}{p_y}$$

or

$$y_1 = 3\frac{p_x}{p_y} \tag{d1}$$

Equation $(d1)$ defines her demand function for good Y, as a function of the prices of the two goods. Since she has no endowment of good Y, this equation defines how much good Y she will want to buy.

Person #2 also wants to choose a consumption bundle at which his indifference curve is tangent to his budget line, so that $MRS^2 = p_x/p_y$, or

$$\frac{12}{x_2} = \frac{p_x}{p_y} \tag{dx2}$$

Unfortunately, this equation does not yet define his demand for good Y ; it defines his demand for good X. But the person must also be on his budget line : the value of his consumption of the two goods must equal the value of his endowment. Since his endowment is $(0, 18)$, an allocation (x_2, y_2) is on his budget line if

$$p_x x_2 + p_y y_2 = 18p_y \tag{b2}$$

Since his demand for good X has been defined by equation $(dx2)$, $12/x_2 = p_x/p_y$, which means that $x_2 = 12p_y/p_x$. Substituting this expression for x_2 in his budget line equation $(b2)$ means that

$$p_x \frac{12p_y}{p_x} + p_y y_2 = 18p_y$$

or

$$y_2 = 6 \tag{d2}$$

In other words, in this case, person #2 will consume exactly 6 units of good Y, whatever are the prices.

Since person 1 has an endowment of 18 units of good Y, he will want to sell $18 - 6 = 12$ units to person #1. That means, from equation $(d1)$, that

$$3\frac{p_x}{p_y} = 12$$

if (p_x, p_y) are equilibrium prices.

But only relative prices matter. Any price pair such as $p_x = 1, p_y = 1/4$, $p_x = 0.8, p_y = 0.2$, $p_x = 4, p_y = 1$ will clear the market for good Y.

Now if the market for good Y clears, so will the market for good X. [You can check this : person 1 has the demand function $x_1 = 15$, and person 2 has the demand function $x_2 = 12\frac{p_y}{p_x}$. So person 1 wants to sell 3 units of good X to person 2 ; person 2 will be willing to buy them only if $3 = 12(p_y/p_x)$, or $p_x = 4p_y$.]

If the price ratio is $p_x/p_y = 4$, then person #1 buys 12 units of good Y from person #2. From her budget line equation $p_x x_1 + p_y y_1 = 18p_x$, then

$$x_1 = 18 - \frac{p_y}{p_x}y_1 = 18 - \frac{1}{4}12 = 15$$

So the equilibrium allocation has $(x_1, y_1) = (15, 12)$ and $(x_2, y_2) = (3, 6)$. You can check that this allocation satisfies the equation of the contract curve given in question #1, so that the first fundamental theorem of welfare economics holds.

3. Recognizing that $x_2 = 18 - x_1$, and $y_2 = 18 - y_1$, a social planner with the welfare function $W = u - 1 + u - 2$ would want to maximize

$$x_1 + 3\ln y_1 + 12\ln 18 - x_1 + 18 - y_1$$

with respect to x_1 and y_1

Setting partial derivatives with respect to x_1 and y_1 equal to zero means

$$1 - \frac{12}{18 - x_1} = 0 \qquad (foX)$$

$$\frac{3}{y_1} - 1 = 0 \qquad (foY)$$

Equation (foX) is equivalent to

$$18 - x_1 = 12$$

or

$$x_1 = 6$$

and equation (foY) is equivalent to

$$y_1 = 3$$

Therefore the allocation which maximizes this Benthamite social welfare function is $x_1 = 6$, $y_1 = 3$, $x_2 = 12$, $y_2 = 15$.

Since any allocation which maximizes a social welfare function is Pareto optimal, you can check that this allocation is on the contract curve.

4. Since quantity demanded must equal quantity supplied,

$$Q^s = 12p_s - 6 = Q^D = 72 - 6P^D = 72 - 6p_s - 6t$$

if t is the unit tax per video.

The above equation,

$$12p_s - 6 = 72 - 6p_s - 6t$$

means that

$$18p_s = 78 - 6t$$

or

$$p_s = \frac{78}{18} - \frac{t}{3} \qquad (inci1)$$

Equation ($inci1$) says immediately that sellers in this market bear 1/3 of a unit tax, and buyers 2/3.

It also says that the net–of–tax price is $78/18 = 4.333$ when there is no tax at all, and $4.333 - 0.3333 = 4$ when there is a \$1 unit tax, confirming that sellers bear 1/3 of the tax.

You can also apply directly the formula, that buyers bear a share $d/(b+d)$ of the tax when the demand curve has the equation $a - bP^D$ and the supply curve $-c + dp_s$. Here $b = 6$ and $d = 12$, so that buyers bear 2/3 of the tax.

Finally, using elasticities gives a pretty good approximation : at the no–tax equilibrium

$$\epsilon_D = -\frac{\partial Q^D}{\partial P^D}\frac{P^D}{Q^D} = 6\frac{4.333}{47} = \frac{25}{47}$$

$$\epsilon_s = \frac{\partial Q_s}{\partial p_s}\frac{p_s}{Q_s} = 12\frac{4.333}{47} = \frac{50}{47}$$

so that $\epsilon_s/(\epsilon_D + \epsilon_s) = 50/75$, again indicating that buyers bear 2/3 of the tax.

5. If an ad valorem tax at the rate τ is put on the net–of–tax price, then

$$P^D = p_s(1+\tau)$$

so that the equilibrium condition $Q_s = Q^D$ can be written

$$p_s = \frac{576}{p_s(1+\tau)}$$

implying that

$$(p_s)^2 = \frac{576}{1+\tau}$$

or

$$p_s = \sqrt{(\frac{76}{1+\tau}}$$

With no tax, $\tau = 0.$, so that $p_s = 24$. With a 44 percent tax, $1 + \tau = 1.44$, so that $p_s = 20$. A 44 percent tax on a net price of 20 is \$8.80. So the tax raises the buyers' price from 24 to 28.80, and lowers the buyers' price from 24 to 20. That means buyers are bearing a share 4.80/8.80, or about 54% of the tax, and sellers about 46%.

The simplest elasticity formula doesn't give too bad an approximation here. The elasticity of supply here is

$$\epsilon_s = \frac{\partial Q_s}{\partial p_s}\frac{p_s}{Q_s} = 1$$

and the elasticity of demand is

$$\epsilon_D = -\frac{\partial Q^D}{\partial P^D}\frac{P^D}{Q^D} = 1$$

so the simplest elasticity formula says that each side of the market should bear 50% of the tax.