AS/ECON 4070 3.0AF   Answers to Assignment 1   October 2005

$Q1$. What are **all** the allocations on the contract curve for the following 2–person 2–good exchange economy?

Person 1's preferences can be represented by the utility function

$$u(x_1, y_1) = x_1 y_1$$

Person 2's preferences can be represented by the utility function

$$U(x_2, y_2) = 160x_2 - (x_2)^2 + 160y_2$$

The economy's total endowment of good X is 40, and the economy's total endowment of good Y is 60.

$A1$. To find the Pareto optimal allocations, first expressions must be derived for the two people's marginal rates of substitution. Each person's MRS, the slope of her indifference curve, is the ratio of the marginal utilities of the 2 goods. Differentiating, for person 1

$$MU_x^1 \equiv \frac{\partial u}{\partial x_1} = y_1$$

$$MU_y^1 \equiv \frac{\partial u}{\partial y_1} = x_1$$

so that

$$MRS^1 \equiv \frac{MU_x^1}{MU_y^1} = \frac{y_1}{x_1} \qquad (1-1)$$

Similarly

$$MU_x^2 \equiv \frac{\partial U}{\partial x_2} = 160 - 2x_2$$

$$MU_y^2 \equiv \frac{\partial U}{\partial y_2} = 160$$

so that

$$MRS^2 \equiv \frac{MU_x^2}{MU_y^2} = \frac{160 - 2x_2}{160} \qquad (1-2)$$

There are 40 units of good X, so that

$$x_2 = 40 - x_1 \qquad (1-3)$$

An allocation will be Pareto efficient if $MRS^1 = MRS^2$, which from equations $(1-1)$ – $(1-3)$ means that

$$\frac{y_1}{x_1} = \frac{160 - 2(40 - x_1)}{160} \qquad (1-4)$$

Equation $(1-4)$ can be re–written

$$y_1 = \frac{1}{2}x_1 + \frac{1}{80}(x_1)^2 \qquad (1-5)$$

Equation $(1-5)$ defines an upward–sloping curve in the Edgeworth box ; it is the equation of (most of) the contract curve. When $x_1 = 0$, equation $(1-5)$ indicates that $y_1 = 0$; the curve goes through the bottom left corner of the Edgeworth box. When person 1 has the enitire endowment of good X, $x_1 = 40$, equation $(1-5)$ says that $y_1 = 40$, so that the point $(40, 40)$, on the right edge of the Edgeworth box, is on the contract curve. Between these points $(0, 0)$ and $(40, 40)$, the curve slopes up, at an increasing rate.

What happens if $x_1 = 40$, and if $y_1 > 40$? Then $MRS^1 > MRS^2$. Person 1 would like to exchange some of her good Y for some good X from person 2. But person 2 doesn't have any X. That means that the allocation (x_1, y_1, x_2, y_2) is efficient, if $x_1 = 40$, $y_1 > 40$ (and $x_2 = 0$, $y_2 = 60 - y_1$). So whole contract curve is the set of allocations (with $0 \leq x_1 \leq 40$) satisfying equation $(1-5)$, plus the right edge of the Edgeworth box, above the point $x_1 = 40, y - 1 = 40$.

The accompanying figure illustrates.

*Q*2. What is the competitive equilibrium allocation for the following 2–million–person, 2–good exchange economy, in which the government levies an excise tax?

There are 2 million people in the economy, each of whom has preferences which can be represented by the utility function

$$u(x_i, y_i) = x_i + 36 \ln y_i$$

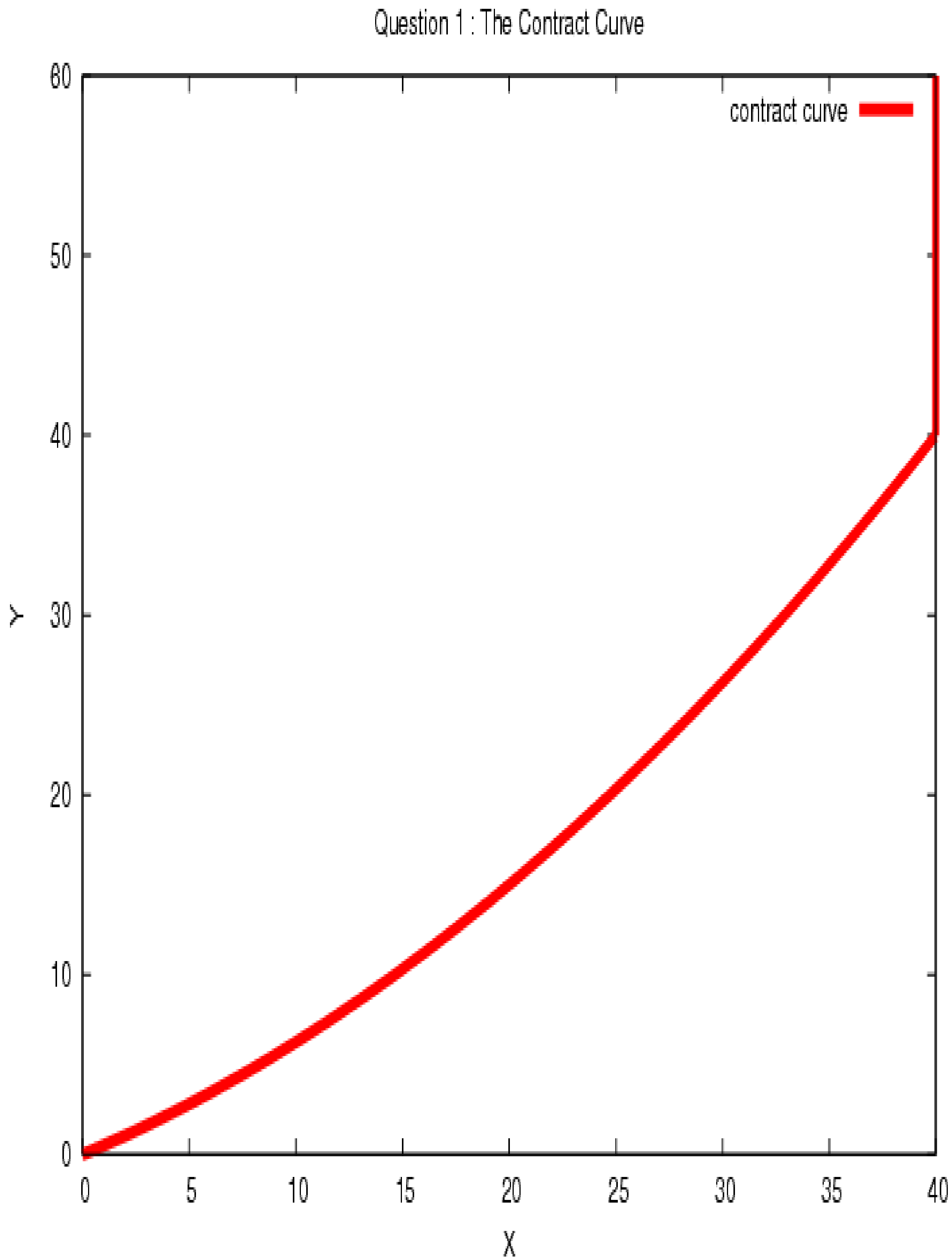
where x_i is person i's food consumption, y_i is person i's clothing consumption, and "ln" refers to the natural logarithm function. (Recall that the derivative of $\ln y_i$ with respect to y_i is $1/y_i$.)

One million of the people have an endowment of 72 units of food, and no endowment of clothing. The other one million people each have an endowment of 72 units of clothing and no food.

The government taxes *sales* of clothing in the following fashion. Each time one person sells any clothing, the government confiscates half the clothing. The government than takes all the clothing which it confiscates, and divides it up equally among all the people.

*A*2. To find the answer here, the people's demand functions must be calculated.

But first, it is important to distinguish between the price a person gets from selling clothes, and the price a person must pay if she is buying clothes. Because the governments confiscates half the clothing sold in any transaction, if p_y is the price that the seller gets from selling clothes, then $P^Y = 2p_y$ would be the price the buyer pays. That is the seller's price p_y must be half the buyer's price P^Y, since the government is taxing half the proceeds.


Question 1 : The Contract Curve
contract curve
Y
X
0
10
20
30
40
50
60
0
5
10
15
20
25
30
35
40

To find people's demands for clothing, note that each person's marginal substitution can be calculated as follows.

$$MU_x^i = 1$$
$$MU_y^i = \frac{36}{y_i}$$

for any person i. Therefore

$$MRS^i \equiv \frac{MU_x^i}{MU_y^i} = \frac{y_i}{36} \tag{2-1}$$

for any person i.

A person's preferred consumption bundle is the bundle which is on her budget line, and for which her MRS equals the price ratio. Thus for a person who chooses to **buy** clothing, her optimal consumption bundle will satisfy the equation

$$\frac{y_i}{36} = \frac{p_x}{P^Y}$$

so that

$$y_i = 36\frac{p_x}{P^Y} \tag{2-2}$$

and for a person who chooses to sell clothing

$$y_i = 36\frac{p_x}{p_y} \tag{2-3}$$

where p_x is the price of food (which is the same for buyers and sellers).

Since 1 million people are endowed with no clothing, they each must be clothing buyers. Since 1 million people are endowed with no food, they must be clothing sellers. The market for clothing will be in equilibrium, only if the amount of clothing that sellers wish to consume, defined by equation $(2-3)$, added up over 1 million clothing sellers, plus the amount of clothing that buyers wish to consume, defined by equation $(2-2)$, added up over 1 million clothing buyers, equals the available quantity of clothing. [Alternately, each person who has an endowment of 72 units of clothing will want to sell $72 - 36(p_x/p_y)$ units of clothing ; this planned sale must equal the quantity demanded by some person with no endowment of clothing.]

So in equilibrium

$$36\frac{p_x}{P^Y} + 36\frac{p_x}{p_y} = 72 \tag{2-4}$$

or (using the fact that $P^Y = 2p_y$)

$$\frac{p_x}{p_y}(\frac{1}{2} + 1) = \frac{p_x}{p_y}\frac{3}{2} = 2 \tag{2-5}$$

Which implies that, if the clothing market is in equilibrium, then it must be the case that

$$p_y = \frac{3}{4}p_x \tag{2-6}$$

so that

$$P^Y = \frac{3}{2}p_x \qquad (2-7)$$

(Since only relative prices matter, **any** pair of prices (p_x, p_y) which satisfy equation $(2-6)$ will ensure that the market for clothing is in equilibrium.)

How much clothing actually gets bought and sold? Let Q be the amount of clothing that a person with no food sells. Half that amount is confiscated, and distributed equally. So a person who starts with no endowment of clothing, actually gets $Q/4$ units from the government : $Q/2$ units are confiscated per transaction, and that is split equally among all 2 million people.

A person with no endowment of clothing wants to consume $36p_x/P^Y$ units of clothing. When $p^Y/p_x = 3/2$, this total demand is 24. So the amount that she must buy is this total demand of 24, minus the clothing which has been given to her by the government, $Q/4$. What she buys is half the amount that each seller sells, $Q/2$ (since the government confiscates half the clothing sold). Therefore

$$\frac{Q}{2} = 24 - \frac{Q}{4} \qquad (2-8)$$

or

$$Q = 32$$

The amount she actually buys is half that (since the government confiscates half) : $q = 16$.

How much food must she sell to pay for the clothing she buys? If she sells s units of food, she gets $p_x s$. The cost of the food she buys is $16P^Y$. So

$$p_x s = 16P^Y$$

or

$$s = 16\frac{P^Y}{p_x} \qquad (2-9)$$

Since $\frac{P^Y}{p_x} = 3/2$, equation $(2-9)$ implies that she must sell

$$s = 24$$

units of food.

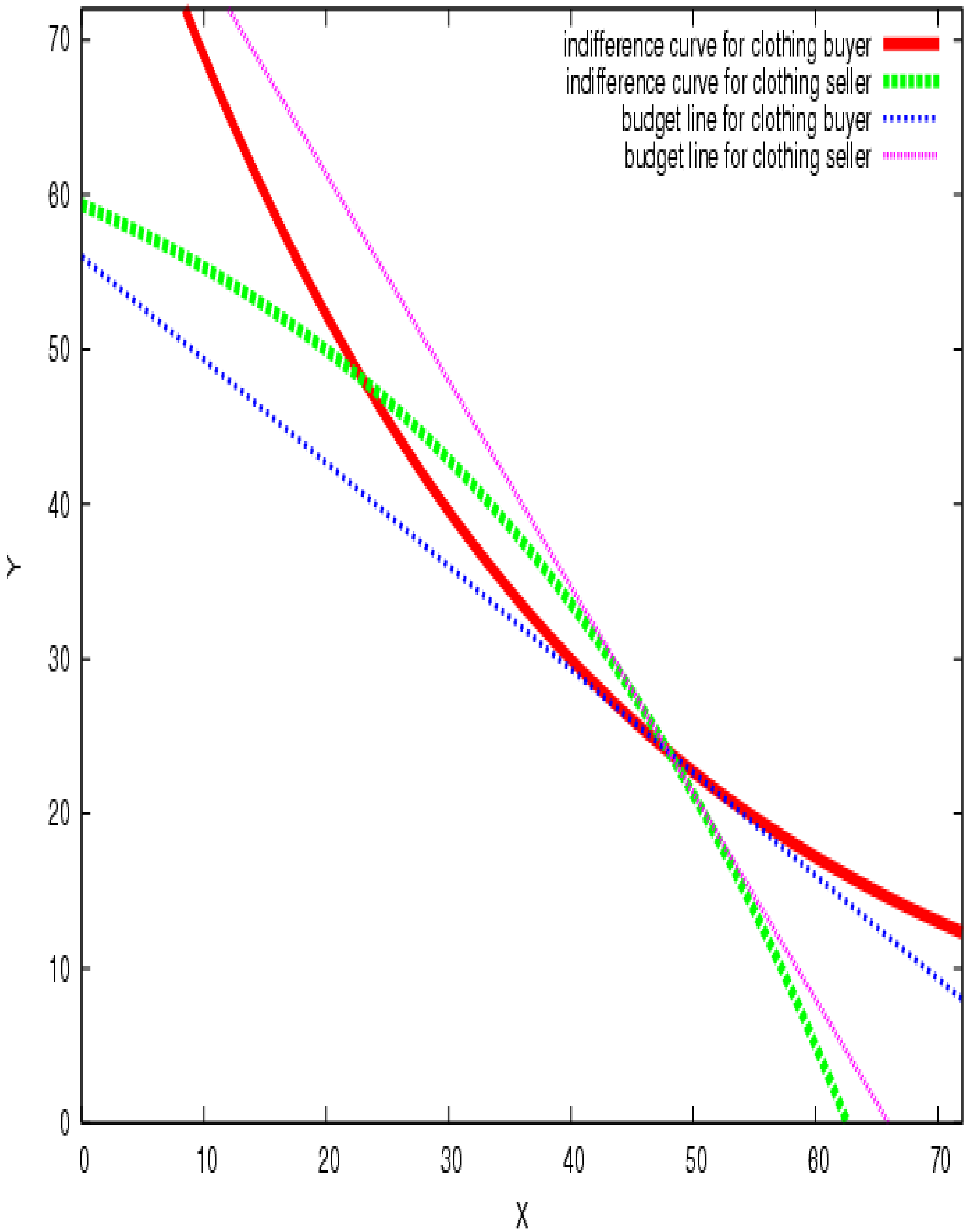
Similarly, those who are endowed with clothing can afford to buy 24 units of food each, from the money they earn selling 32 units of clothing each.

People who are endowed with food have $72-24 = 48$ units of food left, after selling some. The have a total food clothing consumption of 24. People who are endowed with clothing each buy 24 units of food. They have a total clothing consumption of $36p_x/p_y = 48$. Therefore the equilibrium allocation is

$$(x_1, y_1) = (48, 24)$$

$$(x_2, y_2) = (24, 48)$$

Question 2


indifference curve for clothing buyer
indifference curve for clothing seller
budget line for clothing buyer
budget line for clothing seller
0
10
20
30
40
50
60
70
0
10
20
30
40
50
60
70
X
Y

if type 1 people are those with an endowment of food, and type 2 people are those with an endowment of clothing.

Here $MRS^1 = \frac{2}{3} \neq \frac{4}{3} = MRS^2$ so that the allocation is not Pareto optimal. The figure shows that clothing sellers' indifference curves have a different slope at the equilibrium allocation that those of clothing buyers.

*Q*3. What is the competitive equilibrium to an economy which is exactly the same as the one described in question #2, if the government did not levy any taxes, or redistribute any goods?

*A*3. Now each person, whether buyer or seller, faces the same price for clothing, since there are no taxes on sales.

Whether or not she has any clothing endowment, a person's demand for clothing will be determined from the condition $MRS^i = p_x/p_y$, where p_y is now the price faced by both buyers and sellers. From the answer to question #2 above, that condition implies that

$$y_i = 36\frac{p_x}{p_y} \qquad (3-1)$$

Equation $(3-1)$ says that each person, regardless of endowment, will now demand the same quantity of clothing. [1] Since the endowment of clothing per person is 36 (2 million people in total, half of them with an endowment of 72 units), quantity demanded of clothing equals quantity available if and only if

$$36\frac{p_x}{p_y} = 36 \qquad (3-2)$$

or

$$p_x = p_y$$

(Again, since only *relative* prices matter, any pair of prices (p_x, p_y) will clear the clothing market, if $p_x = p_y$.)

To get 36 units of clothing, type–1 people must sell 36 units of food (since $p_x = p_y$ in equilibrium). So their food consumption is $72 - 36 = 36$. Type–2 people buy 36 units of food with the money they earn from selling 36 units of clothing. So the equilibrium allocation is

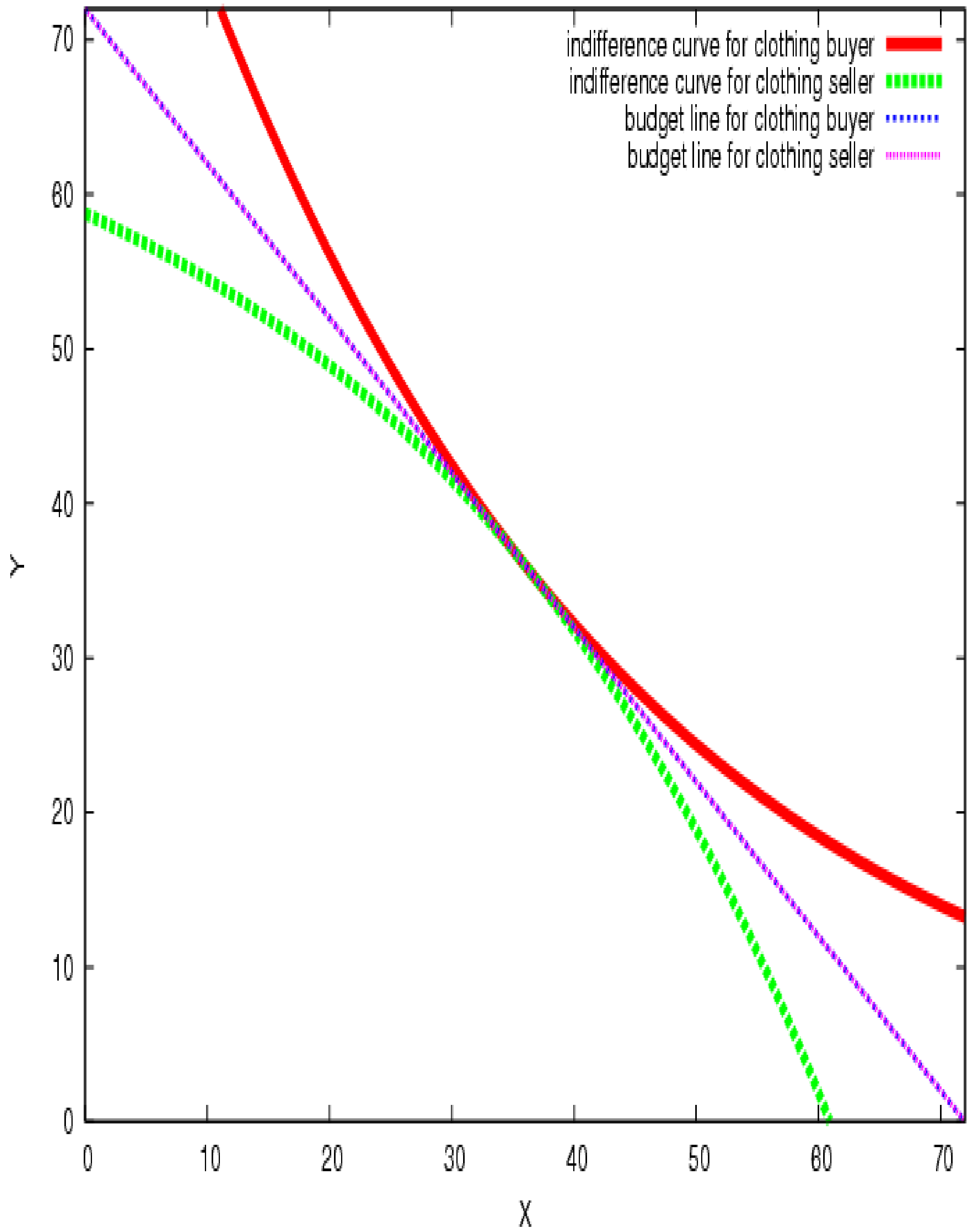
$$(x_1, y_1) = (36, 36)$$

$$(x_2, y_2) = (36, 36)$$

Calculating the utility levels $x_i + 36 \ln y_i$, for each allocation, $u_1 = u_2 = 165$ in this "laissez–faire" equilibrium, compared to $u_1 = 162.41$, $u_2 = 163.36$ in the allocation in question #2, it can

[1] This special property, that demand for y depends only on prices, and not on a person's income, holds only when utility can be written in the form $u(x, y) = x + f(y)$ for some function $f(y)$. Here $f(y) \equiv 36 \ln y$.

Question 3



indifference curve for clothing buyer
indifference curve for clothing seller
budget line for clothing buyer
budget line for clothing seller
70
60
50
40
30
20
10
0
Y
0
10
20
30
40
50
60
70
X

be seen that in this example at least, the tax on clothing sales makes every person worse off, even when the government redistributes all the proceeds directly to the people.

*Q*4. What is the incidence of a tax of \$6 per shirt in a perfectly competitive market for shirts, if the supply curve in the market has the equation

$$Q^s = 2p_s$$

and the demand curve has the equation

$$Q^D = \frac{288}{P^D}$$

where p_s is the price received by sellers, P^D is the price paid by buyers, Q^s is the quantity supplied, and Q^D is the quantity demanded?

*A*4. Quantity supplied will equal quantity demanded if and only if

$$2p_s = \frac{288}{P^D} \qquad (4-1)$$

which can be written

$$2p_s(p_s + t) = 288 \qquad (4-2)$$

where t is the unit tax on the good (since $P^D = p_s + t$). Expanding equation $(4-2)$,

$$(p_s)^2 + tp_s - 144 = 0 \qquad (4-3)$$

defines the net–of–tax price price p_s as a function of the tax t per shirt.

If there is no tax on shirts, then $t = 0$, and equation $(4-3)$ can be written $(p_s)^2 = 144$, or

$$p_s = 12$$

So the price of a shirt would be \$12 if there were no tax. Since equation $(4-3)$ is a quadratic function of the price p_s, it actually can be solved explicitly for p_s as a function of t :

$$p_s = -\frac{t}{2} + \frac{1}{2}\sqrt{t^2 + 4(144)} \qquad (4-4)$$

If the tax were \$6 per shirt, then formula $(4-4)$ implies that $p_s = 9.3693$, so that $P^D = 15.3693$. Since the price paid by buyers has gone up by $15.3693 - 12 = 3,3693$, buyers bear a fraction $3.3693/6 = 0.561$ of the tax burden. So buyers bear about 56% of the tax, and sellers about 44%.

But fairly similar results can be obtained using approximation formulae, using only the fact that $p_s = 12$ when there is no tax. When $p_s = P^D = 12$, and $Q^s = Q^D = 24$, then the slope of the supply curve is

$$\frac{\partial Q^s}{\partial p_s} = 2 \qquad (4-4)$$

$$\frac{\partial Q^D}{\partial P^D} = -\frac{288}{(P^D)^2} = -\frac{288}{144} = -2 \tag{4 – 5}$$

So using the slopes of the supply and demand curves, at the no–tax equilibrium suggests that buyers and sellers should each bear about 50% of the burden of the tax, not a great approximation, but not a bad one either. Using the elasticity of supply and demand, evaluated at $p_s = P^D = 12$, would give the same approximation. (Here the elasticity of supply, and the elasticity of demand, both equal 1.)

*Q*5. What would the incidence of a tax of $100 on each laptop computer sold in Canada, if laptops were produced by a five–firm oligopoly, in which the firms colluded to fix prices so as to maximize the total profits of all the firms, if each firm could produce laptops (in unlimited quantities) at a price of $1500 each, and if the total quantity Q^D of laptops demanded in Canada was

$$Q^D = 1000000 - 200P^D$$

where P^D was the price paid by consumers?

*A*5. If the 5 firms selling the good colluded so as to maximize profits, then they are behaving exactly the same as if the entire industry were run by a single monopoly. In each case, they are choosing a price so as to maximize the total profit of the industry.

So the tax incidence here would be the same as the tax incidence levied on an industry which is a (single–price) monopoly.

In this case, the demand function for the good has a demand function which is a straight line. The cost of producing a laptop is $1500, independent of the scale of operations, so that the industry's marginal cost curve is a horizontal line, at a height of $1500.

So the following result can be applied : if a single–price monopoly faces a linear demand curve, and operates under constant returns to scale, then it will bear exactly 50 percent of the burden of any tax.

[The question could also be solved directly. The cartel will choose a price p_s to maximize its total profits, which are

$$(1000000 - 200(p_s + t))(p_s - 1500) \tag{5 – 1}$$

the product of the number of laptops sold, and the profit per laptop. Maximizing $(5 - 1)$ with respect to p_s means setting its derivative equal to zero

$$1000000 - 400p_s - 200t = 0 \tag{5 – 2}$$

Equation $(5 - 2)$ implies that $p_s = 3250$ when $t = 0$, and $p_s = 3200$ when $t = 100$, so that a tax of $100 per laptop reduces the industry's net–of–tax price by $50.]