

Do **all 5** questions. All count equally

1. Find all the efficient allocations in the following 2–person, 2–good, 2–input economy.

The 2 goods, food and clothing, are produced using labour and machinery as inputs. There is a fixed quantity of 150 person-hours of labour available, which can be allocated between food and clothing production. There is also a fixed quantity of 130 machine-hours of machinery available, which can be allocated between food and clothing production.

The total quantity X of food produced depends on how much labour and machinery are used in the food industry :

$$X = \frac{1}{2}(L_X + K_X)$$

where L_X is the number of person-hours of labour used in food production, and K_X is the number of machine-hours of machinery used in food production.

The total quantity Y of clothing produced is

$$Y = \sqrt{L_Y K_Y}$$

Person 1's preferences can be represented by the utility function

$$u_1 = \ln x_1 + \ln y_1$$

where x_1 and y_1 are person 1's quantities consumed of food and of clothing respectively. Person 2's preferences can be represented by the utility function

$$u_2 = \ln x_2 + \ln y_2$$

2. An imaginary economy has 2 million people. Each person's preferences in this economy can be represented by the utility function

$$u = xy$$

where u is the person's utility, x her consumption of wheat, and y her consumption of barley.

There is only 1 input to production in this economy, land. A hectare of land can be used to grow 1 tonne of wheat, or to grow 1 tonne of barley.

One million of the people in the economy are poor, and own 1 hectare of land each. The other one million each own 5 hectares of land. (So that there are 6 million hectares of land in total, available for wheat or barley growing.)

Small competitive firms rent land from the people, and use the land to grow wheat and barley, which they sell on competitive markets. [There is no home production : the only way that people can get wheat or barley is to buy it from firms.]

The government taxes wheat sales. They confiscate half the wheat sold in any transaction. So if a firm sells 2 tonnes of wheat to a person, they must also give 2 tonnes of wheat to the government as well.

The government takes all the wheat that it collects, and divides it equally among the poor people.

What is the competitive equilibrium allocation in this economy?

3. Is the equilibrium allocation in question #2 above Pareto optimal?

Explain precisely why or why not.

4. What is the incidence of a 100% tax (calculated as a percentage of the net [before-tax] price) on haircuts, if the market for haircuts is perfectly competitive, if the supply curve in the market has the equation

$$Q^s = \sqrt{\frac{p_s}{2}}$$

and the demand curve has the equation

$$Q^D = \frac{16}{P^D}$$

where p_s is the price received by sellers, P^D is the price paid by buyers, Q^s is the quantity supplied, and Q^D is the quantity demanded?

5. What would be the incidence of a \$6 unit tax in the following market?

The market demand curve for the good has the equation

$$Q^D = 15 - P^D$$

where P^D is the price paid by buyers, and Q^D the quantity demanded.

There is a single firm which can produce as much or as little of the good as it wants, at a constant cost of \$5 per unit.

There are many other firms. But each other firm can only produce the good at a cost of \$8 per unit. (These other firms also produce under constant returns to scale.)

The single low-cost firm sets its price to maximize its profit, knowing that it will not sell any of the good if it charges a higher price than the other firms.

[You may assume that the customers all buy from the low-cost firm if the low-cost firm charges exactly the same price as the high-cost firms.]