

Do **all 5** questions. All count equally

1. Find the equation of the production possibility curve in the following 2-good, 2-input economy.

Food and clothing are both produced using labour and machinery as inputs. The quantity produced of food is

$$X = \sqrt{L_X K_X}$$

where X is the quantity produced of food, L_X is the quantity of labour used in food production, and K_X is the quantity of machinery used in food production. The quantity produced of clothing is

$$Y = L_Y + 4K_Y$$

where Y is the quantity produced of clothing, L_Y is the quantity of labour used in clothing production and K_Y is the quantity of machinery used in clothing production.

The economy has a fixed total endowment of 200 units of labour and 90 units of machinery.

2. In the economy described in question #1, there are 290 people. There are 90 capitalists, each of whom owns one unit of machinery, and no labour. There are 200 workers, each of whom owns one unit of labour, and no machinery. Each of the 290 people has the same preferences, represented by the utility function

$$u(x, y) = xy$$

where x is the person's consumption of food, and y is her consumption of clothing.

Show that, if all firms and people behave as perfect competitors, there is an equilibrium in which the price of clothing is \$1, the price of food is \$4, the wage of each worker is \$1, and the return to each unit of machinery is \$4.

Is this equilibrium Pareto optimal? Explain briefly.

3. In the model described in questions 1 and 2, suppose now that the government imposes a high tax on the use of machinery in the clothing industry : clothing firms must pay a tax of \$3 to the government for every \$1 they spend on the use of machinery (that is, there is a tax of 300% imposed on the use of machinery in the clothing industry).

The revenue from this tax is distributed only to machinery owners, and it is distributed equally to them, so that each capitalist gets a fraction $1/90$ of the tax revenue.

Show that, if all firms and people behave as perfect competitors, there is an equilibrium in which the price of clothing is \$1, the price of food is \$2, the wage of each worker is \$1, and the return to each unit of machinery is \$1. Is this equilibrium Pareto optimal? Explain briefly.

4. What would be the approximate incidence of a unit tax of \$2 levied on sellers in a perfectly competitive market in which the quantity of the good demanded by buyers was

$$Q^D = 60 - (P^D)^2$$

and the quantity supplied by sellers was

$$Q^S = 10p_s - 36$$

where Q^D is the quantity demanded by buyers, Q^S the quantity supplied by sellers, P^D the price paid by buyers and p_s the price received by sellers?

5. Suppose that a good is produced by two firms, each of which has a constant marginal cost of production of \$6 per unit.

Consumers regard the two firms' products as perfect substitutes for each other, and will always buy from the cheaper firm (if the firms were to charge different prices from each other).

The aggregate demand for the product has the equation

$$Q^D = 240 - 30P^D$$

where Q^D is the quantity demanded by buyers, and P^D the price paid by buyers.

The two firms in this oligopolistic market behave as *Bertrand* duopolists. That is, firms each set their own prices, taking as given the price charged by the other firm (and anticipating how buyers will respond to their pricing decisions).

What will be the incidence of a \$1 unit tax levied on both sellers in this market?