AS/ECON 4070 3.0AF   Answers to Assignment 2

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1. The marginal excess burden depends on the slope of the compensated demand curve. For example, equation (18.3) of the text (page 469) implies that the marginal excess burden of a tax on a good is proportional to the compensated elasticity of demand. So if someone were to use the uncompensated (ordinary) demand curve to measure the marginal excess burden, the magnitude (and direction) of the error depends on the relation between the compensated own–price elasticity of demand, and the uncompensated own–price elasticity of demand.

But there is a relation between those elasticities, derived in intermediate microeconomics (AS/ECON 2300) : the Slutsky equation. (See, for example, chapter 5 of Nicholson, or chapter 8 of Varian.) The Slutsky equation says that the relation between the derivative of uncompensated demand with respect to a good's price, and the derivative of the compensated demand is

$$\frac{\partial Q}{\partial P} = \frac{\partial Q}{\partial P}|_{comp} - Q\frac{\partial Q}{\partial M}$$

where the first term on the right side refers to the price derivative of the compensated demand, and the second term is the quantity times the derivative of quantity demanded with respect to income.

For a normal good, one for which the quantity demanded increases with income, both terms on the right hand side are negative. (That's why a normal good can't be a Giffen good.) So the uncompensated derivative will be more negative than the compensated derivative. Since the compensated and uncompensated own–price elasticities are just these derivatives multiplied by P/Q, then for a normal good the uncompensated own–price elasticity is *greater* than the compensated price elasticity. Using the uncompensated demand curve will lead to an over–estimate of the marginal excess burden.

If the income elasticity of demand were zero, then the compensated and uncompensated own–price elasticities would be the same, and there would be no error. Only if the good were inferior would using the uncompensated elasticity result in an under–estimate of the marginal excess burden.

2. In this case, quantity demanded of videos depends on the price of books, and quantity demanded of books depends on the price of videos, so that the simple "inverse elasticity" Ramsey rule (on page 489) **cannot** be applied. Since neither the quantity demanded of videos nor the quantity demanded of books depends on the price of some untaxed good, the "Corlett–Hague" rule (pages 489–90) also does not apply.

Instead, the appropriate test of whether the commodity tax system is optimal is whether the tax system makes the quantity demanded of each good fall by the same proportion (as prescribed by the "proportional reduction" Ramsey rule presented on pp. 487–488 of the text). Initially, with no taxes, and with $P_V = 4$ and $P_B = 20$, the quantity demanded of videos is $40 - 3(4) + 20 = 48$, and the quantity demanded of books is $100 + 4 - 2(20) = 64$. The taxes (\$1.40 on videos, \$2.10 on books) raise the prices of videos and books to \$5.40 and \$22.10 respectively. The quantity demanded of videos falls to $40 - 3(5.40) + 22.10 = 45.90$ and the quantity demanded of books falls to $100 + 5.40 - 2(22.10) = 61.20$. Demand for videos has fallen by 2.10, and demand for books has fallen by 2.80. Since $2.10/2.80 = 48/64$, quantity demanded of each good has fallen by the same proportion (about 4.4 percent).

Therefore the tax system satisfies the "equi–proportionate reduction" Ramsey rule (equation 19.9, page 488 in the text), and could be optimal.

3. The cash grant is the tax rate, times the average income, since that is the average tax yield per person, and the tax collected per person will equal the grant paid per person. So

$$cg = \tau 30(1 - 3\tau^2) = 30[\tau - 3\tau^3]$$

To maximize the tax grant, take the derivative of this tax yield with respect to the tax rate, and set it equal to zero.

$$\frac{\partial cg}{\partial \tau} = 30[1 - 9\tau^2] = 0$$

implying

$$\tau^2 = \frac{1}{9}$$

or

$$\tau = \frac{1}{3}$$

In this case, the efficiency loss from high marginal tax rates would be so high that the tax yield would be maximized at a rate of 33.33 percent.

4. The person's total expected payments — tax and penalty— if she under–reports by X will be

$$(0.4)(Y - X) + (0.15)(1 + bX + X^2)$$

if Y is her (true) total income. Choosing X to minimize these total payments means setting the derivative of this expression equal to 0 :

$$-0.4 + 0.15b + 2(0.15)X = 0$$

or

$$X = \frac{8 - 3b}{6}$$

If $b = 2/3$, then $X = 1$, and if $b = 1$, then $X = 5/6$.

However, part of her penalty is a fixed amount, 1. She could avoid this penalty by simply telling the truth (setting $X = 0$).

If she set $X = 0$, her total tax liability would be $(0.4)Y$.

If $b = 2/3$, and $X = 1$, she instead would have a total tax and penalty cost of

$$(0.4)(Y - 1) + (0.15)(1 + \frac{2}{3} + 1) = 0.4Y$$

She would be indifferent between under–reporting by \$1000 (remember, all numbers are in thousand dollars), or telling the truth.

If $b = 1$, and $X = 5/6$, then her total tax and penalty costs would be (in expected value)

$$(0.4)(Y - \frac{5}{6}) + (0.15)(1 + \frac{5}{6} + \frac{25}{36})$$

which equals $(0.4)Y - 21/720$. In this case, the tax saved by under–reporting is less than the expected penalty.

The same answers could be obtained by looking instead at the taxpayer as trying to maximize her tax savings — minus the expected value of any penalties — relative to reporting her true income. Her savings in tax are $(0.4)X$, and her expected penalty is $(0.15)(1 + bX + X^2)$.

Figure 1 depicts these net savings minus expected penalties $(0.4)X - (0.15)(1 + bX + x^2)$ as a function of X. Figure 2 depicts her marginal benefits from underreporting, which equal 0.4, and her expected marginal costs, which are $(0.15)(b + 2X)$. Figure 2 shows that $MC = MB$ at $X = 1$, if $b = 2/3$, and $MC = MB$ at $X = 5/6$ if $b = 1$. However, figure 1 shows that the taxpayer's expected penalties ar at least as large as her savings when $b = 2/3$, and are larger than her tax savings when $b = 1$.

So the correct answers are

i either $X = 0$ or $X = 1$ (but nothing else)

ii $X = 0$ ($X = 5/6$ is best if the person does under–report, but she's better off telling the truth here)

5. *i* If she tells the truth, she will pay a tax of 40 percent of \$10,000, or \$4000. If she does not report it, then she would have to pay the tax owing \$4000, plus half that (another \$2000), plus a fine of \$6000, \$12,000 in total.

If all she cares about is expected income, then she should tell the truth as long as the probability of being caught is greater than 1/3, since the costs if she is caught cheating are three times the taxes she'd pay if she reported honestly. Thus, her best action is "cheat if $a < 1/3$, report honestly if $a > 1/3$, it doesn't matter if $a = 1/3$".

ii In this case, any probability of being caught which is above 1/3 is excessive. As long as $a > 1/3$, the person won't cheat, so that any further expenditure by the government on enforcement would be wasted. The optimal probability of detection should be 1/3 (or just above 1/3, if there's a chance that the person would cheat if she were indifferent between cheating and reporting honestly).