AS/ECON 4070 Answers to Assignment 2 November 2001

1. If there were no taxes on the two goods, the prices of food and clothing would both be 1. Then the quantities demanded of the two goods would be

$$Q_F^0 = 18 - 2(1) - 1 = 15$$

$$Q_C^0 = 12 - 1 - 2(1) = 9$$

A 50% tax on both food and clothing would raise both P_F and P_C to 1.5, changing the quantities demanded of the two goods to

$$Q_F^1 = 18 - 2(1.5) - 1.5 = 13.5$$

$$Q_C^1 = 12 - 1.5 - 2(1.5) = 7.5$$

The "proportional reduction" version of the Ramsey rule (equation 19.9 of the text) states that the commodity tax system will be optimal if it reduces the compensated demand for all taxed goods by the same proportion.

In this case the proportional reductions in (compensated) quantities demanded are

$$\frac{\Delta Q_F}{Q_F} = \frac{Q_F^1 - Q_F^0}{Q_F^0} = -\frac{1.5}{15} = -0.1$$

$$\frac{\Delta Q_C}{Q_C} = \frac{Q_C^1 - Q_C^0}{Q_C^0} = -\frac{1.5}{9} \approx -0.1667$$

Since the tax system reduces the quantity demanded of clothing by a much greater proportion than the quantity demanded of food, then the tax system is not optimal.

(Note 1 : It would be equally valid to calculate the proportional reductions in demand as proportions of the after–tax quantities Q_F^1 and Q_C^1. Calculating the reductions this way, the proportional reduction in food demand would be about 0.11, and the proportional reduction in clothing demand would be 0.2, so that again the proportional reduction in clothing demand is much higher.)

(Note 2 : It is not valid to use the "inverse elasticity" form of Ramsey's rule here [equation (19.11) of the text], because the demand for food depends on the price of clothing, and the demand for clothing depends on the price of food.)

2. Here Q_F^0 and Q_C^0 are exactly as in question 1, 15 and 9 respectively. A tax of 0.7 on food and of 0.1 on clothing would reduce the quantities demanded to

$$Q_F^1 = 18 - 2(1.7) - 1.1 = 13.5$$

$$Q_C^1 = 12 - 1.7 - 2(1.1) = 8.1$$

so that the proportional reductions in quantities demanded are

$$\frac{\Delta Q_F}{Q_F} = \frac{Q_F^1 - Q_F^0}{Q_F^0} = -\frac{1.5}{15} = -0.1$$

$$\frac{\Delta Q_C}{Q_C} = \frac{Q_C^1 - Q_C^0}{Q_C^0} = -\frac{0.9}{9} = -0.1$$

The proportional reduction is exactly the same, so that the commodity tax system would be optimal in this case.

(Note 3 : Notes 1 and 2 from question #1 would also apply here.)

3. The tax yield from 1 person would be t times the person's income, or

$$tz[1 - 3t^2]$$

The tax collection from this particular person would, therefore, be maximized by choosing a tax rate t so as to make

$$t[1 - 3t^2]$$

as large as possible.

This maximand equals

$$t - 3t^3$$

Its derivative with respect to the tax rate t is

$$1 - 9t^2$$

Setting the derivative equal to zero, means finding a tax rate such that

$$1 - 9t^2 = 0$$

or

$$t^2 = \frac{1}{9}$$

meaning that a tax rate of

$$t = \frac{1}{3}$$

maximizes tax yield from this person.

But this tax rate would maximize the tax collections from each person, regardless of her value of z, so that a tax rate of $t = 1/3$ maximizes the total tax yield.

(Taking the second derivative of $t[1 - 3t^2]$ gives an expression $-18t$, which must be negative if $t > 0$, so that the solution derived above, $t = 1/3$, must be a maximum for tax yield, not a minimum.)

4. If a person's "original" (before tax) income z equalled zero, then her only source of income would be what she got from government redistribution. That is, her income would equal R, the per capita revenue collected from the income tax with the constant marginal rate t.

If the government cared only about the well–being of the worst–off person, then it would want to set the tax rate t so as to maximize this tax revenue R per capita.

If the original average level of income in the country were $\bar{z}$, then expression $(*)$ implies that

$$R = t\bar{z}[1 - 3t^2]$$

Making R as large as possible thus means making $t\bar{z}[1 - 3t^2]$ as large as possible.

But that is exactly the problem solved in question #3 above, finding the tax rate which maximizes the total tax yield, namely

$$t = \frac{1}{3}$$

5. If the person were risk neutral, she would want to equate the marginal benefit from tax evasion with the marginal cost of tax evasion.

Since she faces a constant marginal income tax rate of t, each dollar of income which she does not report saves her t dollars in taxes. So the total benefit TB from underreporting her income by E would be

$$TB = tE$$

meaning that her marginal benefit of tax evasion is

$$MB = t$$

The total cost of underreporting E dollars in income, if she is risk neutral, would be the *expected* amount she would have to pay. That's the amount she would have to pay, times the probability ρ that she is caught. The amount she would have to pay is the tax owing, tE, plus the extra penalty $(\alpha E)(tE)$. Therefore

$$TC = \rho(tE + (\alpha E)(tE) = t\rho(E + \alpha E^2)$$

That means the marginal cost of underreporting by another dollar, is the derivative of TC with respect to E, or

$$MC = t\rho(1 + 2\alpha E)$$

Here the marginal cost of evasion rises with the amount evaded.

The optimal amount of evasion would be the level of E which makes the marginal benefit equal the marginal cost, or the solution E^* to

$$t = t\rho(1 + 2\alpha E^*)$$

So that

$$1 = \rho(1 + 2\alpha E^*)$$

or

$$E^* = \frac{1-\rho}{2\alpha\rho}$$

This value is *independent* of the tax rate t, since changes in t affect the marginal benefit and the marginal cost by the same proportion.

(Here the optimal level E^* of evasion will decrease with the probability ρ of being caught, and with the parameter α which measures the magnitude of the extra penalty amount.)