AS/ECON 4070 Answers to Assignment 2 November 2003

1. In this example, quantity demanded (X) of food depends on the price (P_Y) of clothing, and quantity demanded of clothing depends on the price of food. Therefore, the "inverse elasticity" form of the Ramsey rule should not be applied ; that's only valid when quantity demanded of each taxed good is independent of the price of other taxed goods.

What is valid is the "proportional reduction" version of the Ramsey rule : that the commodity taxes should reduce quantity demanded of each taxed good by the same proportion, if the taxes are optimal.

In this case, initially $P_X = P_Y = 24$, so that $\sqrt{P_X/P_Y} = \sqrt{P_Y/P_X} = 1$, and $\sqrt{P_X} \approx 4.9$. Plugging in to the definitions of quantities demanded for the two goods,

$$X \approx \frac{36}{4.9} + 1 = 8.35$$

$$Y = 1 + 144/24 = 7$$

If both goods were taxed at the same rate, 50 percent, then P_X would increase to 36, and so would P_Y, so that $\sqrt{P_X/P_Y} = \sqrt{P_Y/P_X} = 1$ and $\sqrt{P_X} = 6$. That means that now

$$X = 36/6 + 1 = 7$$

$$Y = 1 + 144/36 = 5$$

Quantities demanded of the two goods have not been reduced by the same proportion : the taxes have reduced the quantity demanded of food by about 16 percent, and reduced the quantity demanded of clothing by about 28 percent. So it would not be optimal to tax both food and clothing at 50 percent in this example.

2. *i* If the average income per person were $40(1-t)$ if the tax rate were t, then the taxes collected per person would be t times the average income, or $40t(1-t)$. The question stated that this tax revenue would be used to fund a grant G for each person, so that

$$G = 40t(1-t)$$

To maximize G, take the derivative of G with respect to t, and set it equal to zero, so

$$40 - 80t = 0$$

or

$$t = 0.5$$

So a tax rate of 50 percent would maximize the taxes collected per person, and thus would yield the highest possible grant. (Raising the tax rate above 50 percent would lower average income so much, as people altered their work and saving behaviour, that revenue would actually fall. For example $t = 0.5$ yields a tax revenue of $G = 10$, but raising the tax rate to 75 percent lowers the average income so much that revenue collected per person falls to 7.5.)

ii if a person's ability were a, then her gross income would be $a(1-t)$. She has to pay tax at a rate of t on that income, and receives a grant of G. So her after–tax income is $(1-t)$ times her gross income, or $a(1-t)^2$. Therefore, her net income, after she has paid her tax, and received a grant, is

$$a(1-t)^2 + G$$

If $a = 0$, then she has no taxable income, and she would prefer a tax rate which maximized her grant, so that the answer to part i implies that a tax rate of 50 percent would be best for her.

(More generally, since part i implied that $G = 40t(1-t)$ when the tax rate is t, for a person of ability a, her net income would be

$$a(1-t)^2 + 40t(1-t)$$

if the tax rate were t, and the tax rate which maximizes the net income for that person can be calculated by taking the derivative of $a(1-t)^2 + 40t(1-t)$ with respect to t and setting it equal to zero. This means that net income is maximized for a person of ability a with a tax rate of $(40-2a)/(80+2a)$.)

3. If she decides to hide H dollars in income, how much will her expected net income change? With probability 10/11, she will not get caught, and she will save tH on her taxes, if t is the marginal tax rate she faces. With probability 1/11 she is caught, which means that she does not save anything on her taxes (she has to pay the taxes owing), and she has to pay a fine of $F + H^2/10$. That is, if she is caught, instead of saving money on her taxes, she loses $F + H^2/100$, compared to reporting truthfully.

So the expected gain from hiding H dollars would be

$$\frac{10}{11}tH - \frac{1}{11}(F + \frac{H^2}{10})$$

If she does choose to evade taxes, she wants to choose the level H of evasion which maximizes her expected gain. Taking the derivative of the above expression with respect to H, and setting it equal to 0,

$$\frac{10}{11}t - \frac{1}{11}\frac{H}{5} = 0$$

so that a level of evasion of $H = 50t$ would maximize her expected gain. With a tax rate of 40 percent, she should choose $H = 20$ if she wants to maximize that expected gain — given that she does choose to evade taxes.

The expected marginal benefit of hiding a little more income here is $10t/11$, and the expected marginal cost is $(1/11)(2t/10)$ so that $H = 20$ is the level of evasion for which $MB = MC$.

But there is a fixed cost F which is part of the penalty. if she under–reports as much as a penny, she must pay F. In other words, choosing to evade a little income causes the expected total cost of evasion to jump : it's zero if she reports honestly, and it's a little more than F if she under–reports by a penny.

So a choice she also must make is whether to evade at all. If she evades, she should hide 20,000 dollars in income : the calculation above showed that $H = 20$ was the optimal amount to hide if she hides anything. But she now must choose whether to evade, by choosing $H = 20$, or not to evade at all.

Her expected gain from evasion is

$$\frac{10tH}{11} - \frac{F}{11} - \frac{H^2}{110}$$

With $t = 0.4$, and with H set at its optimal level of 20, this expected gain is

$$\frac{10(0.4)20}{11} - \frac{F}{11} - \frac{(20)^2}{110} = \frac{80}{11} - 2 - \frac{40}{11} = 1.64$$

if the fine is 22. So her expected gain from evasion is positive : she is better off setting $H = 20$ than reporting honestly.

When $F = 55$, then the expected gain is

$$\frac{10(0.4)20}{11} - \frac{F}{11} - \frac{(20)^2}{110} = \frac{80}{11} - 5 - \frac{40}{11} = -3.36$$

her expected gain is *negative*. She is better off reporting honestly. In this case, if she were to evade, and were to pick the best possible level of evasion to undertake ($H = 20$), there is a 10/11 chance she would gain 8000 dollars, and a 1/11 chance that she would have to pay a fine of 95,000 dollars. In this case, evasion is not worth it.

So the correct answers are : *i* $H = 20$ and *ii* $H = 0$.

Note : it is **not** not true that here $MC > MB$ at $H = 0$ as in figure 17.6 of the text. If she were to hide a very small amount of income, the marginal benefit of hiding a little more would be greater than the marginal cost ($MB \approx 0.36$ and $MC \approx 0$ if H is close to 0). The problem is : there is a fixed cost, which she can avoid only by reporting truthfully.

4. Under Haig–Simons principles, how income is spent does not matter. So the expenditures listed in the second paragraph of the question are not relevant. Neither is the amount which she invested in a mutual fund. What matters is her net income from employment, the net transfers she received, and the capital gains which have accrued.

Her net employment income is \$58,000 : the gross income minus commuting costs. Commuting costs are deductible from gross income, under the Haig–Simons principles, if they are a cost of earning income. (How they are treated by the Canadian tax code is a different matter.) Her net transfer income is \$10,000 : the gift she received minus the gift she gave. Her capital gains were \$20,000. So, her Haig–Simons income would be \$88,000.

5. Here the money that a person contributed to his own pension fund does not matter : that was how he spent his money, which is not relevant in the Haig–Simons definition. The \$40,000 in salary is part of his income. But so is the \$2000 contributed by his employer to his own pension plan : that's part of his compensation from his job.

He earned \$15,000 from trading on the internet, but his taxable income from that enterprise should be his net income, after costs of running the business are deducted. That's \$10,000 (revenue of \$45,000 minus \$30,000 in merchandise costs, minus \$5000 in other costs).

Since the house went up in value by \$50,000, he would have to report that capital gain as part of his income, under Haig–Simons principles.

Under Haig–Simons principles, imputed income from owner–occupied housing should also be included as part of income. The value of the housing he consumed was \$35,000 ; that's what the house would have earned on the rental market. From that, he would be entitled to deduct the costs of earning that imputed income, which were \$20,000. As well, \$6000 of the \$35,000 in annual rent was not housing services he consumed, but an input to his private business. [That is : either we can regard the value of housing services that he consumed as only \$29,000, or we can regard the \$6000 as rent which he paid to himself, so that it's part of his imputed income, but a deduction from the income of his collectibles business.) So his net imputed income from owner–occupied housing is only \$9000 : the value of housing services consumed minus the costs of earning the imputed income.

Finally, the alimony is a transfer to someone else, which should be deducted from his Haig–Simons income. So his total Haig–Simons income is: $42000 + 10000 + 50000 + 9000 - 25000$, or \$86,000.