AS/ECON 4070 3.0AF Answers to Assignment 2 November 2004

Q1. What should the optimal tax be on good 2, if good 1 is taxed at 26 cents per unit, if the consumer's compensated demand functions for goods 1 and 2 can be written

$$X_1 = 24 - 2P_1 + P_2$$

$$X_2 = 36 + P_1 - 4P_2$$

(where X_i is the quantity demanded of good i, and P_i is the price paid by the consumer for good i), if the net–of–tax prices of the two goods are $4 each, and if the only goods which can be taxed are goods 1 and 2?

A1. In this case, the inverse elasticity Ramsey rule will **not** be very accurate, since quantity demanded of each taxed good depends on the price of the other taxed good, so that demands are not independent. The **equi–proportional** version of the Ramsey rule should be used.

Initially, the prices of each of the taxable goods, 1 and 2, are $4. Therefore, the quantities demanded of the goods are

$$X_1 = 24 - 2(4) + 4 = 20$$

$$X_2 = 36 + 4 - 4(4) = 24$$

Suppose a unit tax of t_1 is placed on good 1, and a unit tax of t_2 on good 2. Then, from the definition given of the compensated demand functions, the changes in quantities demanded (if people were compensated for the damage done by the taxes) are

$$\Delta X_1 = 20 - [24 - 2(4 + t_1) + (4 + t_2)] = 2t_1 - t_2 \qquad (1-1)$$

$$\Delta X_2 = 24 - [36 + (4 + t_1) - 4(4 + t_2)] = 4t_2 - t_1 \qquad (1-2)$$

The equiproportional form of the Ramsey rule says that the commodity taxes will be optimal if

$$\frac{\Delta X_1}{X_1} = \frac{\Delta X_2}{X_2}$$

which, from equations $(1-1)$ and $(1-2)$, implies that

$$\frac{2t_1 - t_2}{20} = \frac{4t_2 - t_1}{24} \qquad (1-3)$$

Equation $(1-3)$ can be written

$$80t_2 - 20t_1 = 48t_1 - 24t_2 \qquad (1-4)$$

or

$$56t_2 = 68t_1$$

which implies that

$$t_2 = \frac{17}{26} t_1 \tag{1 – 5}$$

Equation $(1-5)$ implies that the optimal unit tax on good 2 should be 17 cents, if the tax on good 1 is 26 cents.

Q2. Suppose that each person's preferences could be represented by the utility function

$$U(X, H) = X - aH^2$$

where X was her consumption (in dollars per week), H was numbers of hours per week which she worked, and a was some positive constant. People are free to choose their hours of work so as to maximize their utility, subject to the constraint that their weekly consumption equal their weekly net–of–tax income.

What would a person's weekly income be, if she received an hourly wage of w, and were subject to an income tax with a marginal rate of t, and if E dollars of her weekly income were exempt from the tax?

A2. The income tax schedule implies that she would have a net of tax income of $(1-t)[wH-E]$ if she chose to work H hours a week. That means that her consumption expenditure X would be $(1-t)[wH-E]$, so that the level of utility she would get, if she chose to work H hours per week, would be

$$(1-t)[wH-E] - aH^2 \tag{2 – 1}$$

Maximizing expression $(2-1)$ with respect to H implies that

$$(1-t)w = 2aH$$

or

$$H = \frac{w(1-t)}{2a} \tag{2 – 2}$$

would be her optimal choice of hours worked, if she had a wage of w per hour, and faced a marginal tax rate of t. Therefore, her weekly gross income wH would be

$$Y(w,t) \equiv wH = (1-t)\frac{w^2}{2a} \tag{2 – 3}$$

and her weekly net income would be

$$\frac{[w(1-t)]^2}{a} + tE$$

Q3. If every person in the economy had the preferences described in question #2 above, but if people's wages w varied over the population, what marginal tax rate t^* would maximize the revenue collected by the tax (for a given exemption level E)?

Suppose now that the tax schedule had to raise a fixed amount of revenue, so that the exemption E would have to adjust when the marginal tax rate t were changed. Would it be optimal to set the marginal tax rate above t^*? Below t^*? Explain briefly.

A3. From question 2, the taxable income of a person of wage w is $(1-t)w^2/2a$. That means that the taxes collected from a person whose wage was w would be

$$R(w,t) \equiv tY(w,t) = t(1-t)\frac{w^2}{2a} - tE \tag{3 − 1}$$

How does this tax revenue vary with the marginal tax rate t? Differentiating $(3-1)$ with respect to t yields

$$\frac{\partial R}{\partial t} = (1-2t)\frac{w^2}{2a} - E \tag{3 − 2}$$

So, the tax revenue from a person of wage w is maximized by setting the tax rate which makes the right side of equation $(3-2)$ equal zero, or

$$t^*(w) = 0.5 - \frac{aE}{w^2}$$

Notice that, no matter what the person's wage rate, tax revenue would decrease with the tax rate t if the tax rate were greater than 50 percent, due to the reduction in the person's labour supply.

The tax rate which maximizes the tax revenue for the whole economy is the average $t^*(w)$ for the whole economy. [At least if each person makes more than the exempt amount. If people with incomes below E did not get a payment from the government, then the tax revenue collected from them would be 0, independent of the tax rate t.]

Now the government has two policy instruments, the tax rate t, and the exemption level E. It is probably more convenient here to consider the **equivalent tax credit** $C \equiv tE$, instead of the exemption, as the second policy instrument. Then each person's labour supply is still $(1-t)w^2/2a$, but now the tax revenue collected from a person of wage w can be written

$$t(1-t)\frac{w^2}{2a} - C \tag{3 − 3}$$

so that the tax rate which maximizes the revenue collected from any person — holding constant the tax credit C — is $t = 0.5$. (That result is obtained by differentiating expression $(3-3)$ with respect to t, and setting the derivative equal to zero.)

If the government has a fixed revenue requirement, then any increase in revenue caused by a change in t will lead to an increase in the tax credit C. For example, if $t < 0.5$, then a small increase in t will raise revenues collected, enabling the government to raise C, so that the tax collections exactly meet the revenue requirement.

But if $t > 0.5$, then increasing the tax rate t would actually **decrease** revenue collected, due to the decrease in labour supply. That would force the government to lower the tax credit, in order to meet its revenue requirement.

How does the utility of a typical person vary with the tax system? From question #2, if the marginal tax rate is t, then she chooses to work $(1-t)w/2a$ hours per week, and will have a net income X, after tax, of $(1-t)Y(w,t)+C$, where $Y(w,t)$ is defined by equation $(2-3)$. If the tax rate changes, then the change in her utility would be

$$\frac{\partial X}{\partial t} - 2a\frac{\partial H}{\partial t}$$

Since $X=(1-t)Y(w,t)-C$, then this change in utility is

$$-Y(w,t)+(1-t)\frac{\partial Y}{\partial t} - 2a\frac{\partial H}{\partial t} \qquad (3-4)$$

Since $Y = wH$, this expression equals

$$-Y(w,t)+\frac{\partial H}{\partial t}[w(1-t)-2a] \qquad (3-5)$$

Equation $(2-2)$ implies the term in the second square brackets must be zero : an increase in the marginal tax rate must make her worse off. [Which should not be surprising : it lowers her net hourly wage.]

So, other things equal, no–one wants to face a higher marginal tax rate.

But other things are not equal. Changing the marginal tax rate leads to a change in the tax credit, because of the government's tax revenue requirement. So the overall change in a person's well–being, when the marginal tax rate changes, is

$$-Y(w,t)+\frac{dC}{dt} \qquad (3-6)$$

where dC/dt is the change in the tax credit caused from the tax rate increasing (and from the fixed revenue requirement). If $t > 0.5$, then both terms in $(3-6)$ are negative : raising the marginal tax rate not only has a direct negative effect, but it also forces a reduction in the exemption level.

Thus everyone agrees that the marginal tax rate should be no greater than 0.5 : further increases in t above t^* make everyone worse off. But for tax rates below t^* there is a tradeoff : a higher marginal rate implies a higher credit. Equation $(3-6)$ shows that the magnitude of the direct effect of a higher tax rate depends on a person's wage : the term $Y(w,t)$ is higher for higher–wage people (who are also higher–income people). Low–wage people will want a tax rate less than, but close to 0.5, since the direct effect of increasing the marginal rate on their well–being is relatively small, compared to the benefit of a higher credit. People with higher wages would want a low marginal rate (in fact they would want a negative marginal rate), since the benefit of a higher credit is relatively unimportant for them.

So the optimal marginal tax rate should not be above 0.5. But how much below t^* it should be will depend on how important the well–being of people of different wages is.

Now the last few paragraphs can be done in terms of the (original) **exemption** instead of the equivalent credit : for a tax rate t, an exemption of E is exactly the same as a credit of tE. So the conclusion is similar. If the tax rate is below 0.5, then there is a trade–off : low–wage people want the tax rate increased (an tE increased as well), while high–wage people would prefer a reduction in t (even if it meant a reduction in tE). Any tax rate below 50% might be optimal, depending on whose well–being matters most. But no tax rate above 50 percent can be optimal here : once t reaches 50%, further increases reduce the tax yield, and thus necessitate a fall in tE, making everyone worse off.

Q4. According to the Haig–Simons (or "comprehensive") definition of income, what would the annual taxable income be for the following person?

She earns $60,000 in salary. She uses public transit to travel to work, for which she buys a transit pass which costs $1500 per year. She has a car which she does not use for commuting. The annual cost of gasoline, insurance, and depreciation on the car is $2000.

At the beginning of the year, her mother gave her ownership in two time–share vacation properties in Florida, each worth $25,000 (all dollar figures are in Canadian dollars). But hurricane damage and the falling $US reduced the value of one of the properties to $20,000. The other property actually appreciated in value, to $27,000. Late in the year, she gave the less–valuable vacation time—share property (the one worth $20,000) to her sister (and kept the more valuable one for herself). At the beginning of the year, she owned stock which was worth $100,000. During the year, the stock decreased in value by $5,000. She also bought some shares, for $12,000, in a new public offering, during the year.

She lives in an apartment, on which she spends $16,000 a year in rent.

A4. First of all, the costs of transportation. Under the Haig–Simons definition, work–related expenses should be deducted from taxable income. (Why? An increase in work–related expenses would lower the value of what she could consume in a year while holding constant her wealth.) So in this question, the $1500 cost of the transit pass should be deductible from income. There is no tax deduction (but no added tax liability) for the expesnes associated with the car she uses for recreation : whether she spends her income on car trips, or movies (or saving) does not affect her tax liability under the Haig–Simons criterion.

So after accounting for her transport expenses (but before any of the other items in the question), her taxable income is at $58,500 : her salary minus her commuting costs.

Gifts received should count as part of taxable income under the Haig–Simons criterion. So the value of the two condominia ($25,000 each) received should be added to taxable income. The Haig–Simons criterion includes capital gains (net of capital losses) in income in the year in which they accrue, so she has a net capital loss of $3000 to subtract from income : the gain of $2000 on the one condominium, minus the loss of $5000 on the other. Just as gifts received must be added to income under the Haig–Simons criterion, gifts given to someone else can be subtracted. So she can deduct the actually value — $20,000 — of the gift she made to her sister.

So accounting for the condominia increases her taxable income by $27,000 : $50,000 in gifts minus a \$3000 capital loss minus a \$20,000 gift. [This also could be done by just looking at the value of the property which she actually held at the end of the year.]

All capital losses on stock can be deducted from taxable income under the Haig–Simons criterion. But new purchases of stock are neither a deduction nor an addition : under the Haig–Simons criterion it does not matter if she uses her income to buy stock or to buy clothes.

Similarly, the amount she pays in rent does not matter for her Haig–Simons income : spending her income on accommodation, rather than travel, or food, or new stock purchases, does not affect the value of what she could consume while holding constant her wealth.

So, under the Haig–Simons criterion, her taxable income would be \$80,500 : her salary, minus commuting costs, plus the value of gifts received, minus the value of gifts given, plus net capital gains (on property and on stock) : $60000 - 1500 + 2(25000) - 5000 + 2000 - 20000 - 5000$.

Q5. According to the Haig–Simons (or "comprehensive") definition of income, what would the annual taxable income be for the following person?

He earned \$100,000 in salary. Of that salary, \$10,000 went into a company pension plan. In addition, his employer contributed \$5000 into his account in the company pension plan.

He invested \$50,000 in a business run by his brother. By the end of the year, the business was bankrupt, and his investment worthless.

He owns his own house, which was worth \$400,000 at the beginning of the year, and \$420,000 at the end of the year. His annual property taxes on the house were \$2000. He spent \$8000 a year on maintenance, utilities and insurance on the house. He also has a \$200,000 mortgage on the house, on which he paid \$15,000 in interest. He estimates that the house would rent for \$40,000 a year if it were rented to someone else.

He also had to pay \$25,000 a year in alimony to his ex–wife.

A5. The money the person put into his own pension plan is neither an addition to, nor a subtraction from, his taxable income under the Haig–Simons criterion : putting the money into a pension plan, rather than into a vacation, does not affect the value of what he could consume while holding constant the value of his wealth. But the \$5000 his employer contirbuted must be included in his taxable income : it is a taxable benefit, since it increases the value of what he could consume while holding constant the value of his wealth. So, under the Haig–Simons criterion his taxable employment earnings are \$105,000.

All losses from his investment in a business which went bankrupt should be deductible from income under the Haig–Simons criterion. That's \$50,000 here.

Under the Haig–Simons criterion, the imputed income from owner occupancy must be included in taxable income. If the house could rent for \$40,000 a year, that is the value to him of living there. From this, associated expenses (\$2000 in taxes, \$8000 in maintenance, and \$15000 in mortgage interest) can be deducted, so that his net imputed income from living in the house is $40000 - 2000 - 8000 - 15000 = 15000$.

Capital gains must be included in Haig–Simons income, when it accrues, so the \$20,000 increase in the value of his house is also a part of taxable income. Alimony would be treated the same under Haig–Simons principles, as gifts given to someone else. So the \$25,000 alimony payment can be deducted from taxable income.

Therefore his taxable income, under the Haig–Simons criterion, is \$65,000 : $100000 + 5000 - 50000 + 40000 - 2000 - 8000 - 15000 + 20000 - 25000 = 65000$.