AS/ECON 4070 3.0AF Answers to Assignment 2 November 2005

$Q1$. What should the optimal tax rate be on good Z, if good X cannot be taxed, and if the commodity taxes on goods Y and Z are optimal, if the tax rate on good Y is 10%, and if the economy's one consumer's preferences can be represented by the utility function

$$u(X, Y, Z) = X + 40 \ln Y + 100\sqrt{Z}$$

where (X, Y, Z) is her consumption of the 3 goods, if the net–of–tax price of each good is 1?

$A1$. To solve for the optimal commodity tax rates, first the consumer's demand functions must be obtained.

The optimizing consumer must choose quantities of the goods so that her marginal rate of substitution between any 2 goods must equal the ratio of the goods' prices. So

$$MRS_{YX} = \frac{MU_Y}{MU_X} = \frac{40}{Y} = \frac{P_Y}{P_X} \tag{1-1}$$

which implies that

$$Y = 40\frac{P_X}{P_Y} \tag{1-2}$$

Equation $(1-2)$ defines the consumer's demand function for Y. Here equation $(1-2)$ is both her compensated demand function, and her uncompensated demand function, since quantity demanded of good Y does not depend on her income.

The own–price–elasticity of demand for good Y is 1 here.

Also

$$MRS_{ZX} = \frac{MU_Z}{MU_X} = \frac{50}{\sqrt{Z}} = \frac{P_Z}{P_X} \tag{1-3}$$

so that

$$Z = 2500(\frac{P_X}{P_Z})^2 \tag{1-4}$$

Equation $(1-4)$ defines the consumer's (compensated and uncomepnsated) demand function for good Z. Here the own–price–elasticity of demand for Z is 2, since

$$-\frac{\partial Z}{\partial P_Z}\frac{P_Z}{Z} = -(-\frac{5000(P_X)^2}{(P_Z)^3})(P_Z)\frac{(P_Z)^2}{2500(P_X)^2} = 2$$

Equations $(1-2)$ and $(1-4)$ show that demands for goods Y and Z are independent of each other's price. Therefore, it is appropriate here to use the "inverse elasticity" form of the Ramsey rule for optimal commodity taxation. The tax rates on goods Y and Z should be inversely proportional to the compensated own price elasticities of demand. The optimal tax rate on good Z must be half as large as the tax rate on good Y, if the commodity tax system is optimal. So if the tax rate on good Y is 10%, the tax rate on good Z must be 5%.

$Q2$. What is the relation between the optimal commodity tax rates on goods Y and Z if good X cannot be taxed, and if the commodity taxes on goods Y and Z are optimal, if the economy's one consumer's preferences can be represented by the utility function

$$u(X, Y, Z) = X + (YZ)^a$$

where $0 < a < \frac{1}{2}$, if the net–of–tax price of each good is 1 ? Explain briefly.

$A2$. As in question #1, a good start is to look at the conditions for utility maximization by the consumer. But here, equating MRS_{YZ} to the price ratio does not give the demand function for good Y right away. Here

$$MRS_{YX} = \frac{MU_Y}{MU_X} = aY^{a-1}Z^a = \frac{P_Y}{P_X} \qquad (2-1)$$

Unfortunately, the rightmost equality in $(2-1)$ has both Y and Z in it — so it cannot be solved so simply for the demand function for good Y. Similarly

$$MRS_{ZX}\frac{MU_Z}{MU_X} = aY^aZ^{a-1} = \frac{P_Z}{P_X} \qquad (2-2)$$

To get the optimal tax system, divide equation $(2-1)$ by $(2-2)$. Or take the remaining MRS,

$$MRS_{YZ} = \frac{MU_Y}{MU_Z} = \frac{Z}{Y} = \frac{P_Y}{P_Z} \qquad (2-3)$$

Equation $(2-3)$ says that, when the consumer chooses her optimal bundle, the ratio of quantities of the 2 goods Y and Z must be inversely proportional to the gross–of–tax prices that the consumer pays.

The "equi–proportional" version of the Ramsey rule says that, if the commodity tax system is optimal, then quantity demanded of each taxed good should fall by the same proportion. That means : if the commodity tax system is optimal, then Z/Y should not change, since Z and Y should fall by the same percentage. If Z/Y is unchanged, then equation $(2-3)$ says that P_Y/P_Z must remain the same.

Thus : if the commodity tax system here is optimal, the price ratio of goods Y and Z should be unchanged by the taxes. That is, the optimal commodity tax system must tax goods Y and Z at the same rate.

[Note : the demand functions for goods Y and Z can actually be obtained from equations $(2-1)$ and $(2-2)$; it just takes some more work. From equation $(2-3)$,

$$Z = \frac{P_Y}{P_Z}Y \qquad (2-4)$$

Substitute for Z from $(2-4)$ into $(2-1)$ to get

$$aY^{a-1}[\frac{P_Y}{P_Z}Y]^a = \frac{P_Y}{P_X} \qquad (2-5)$$

or

$$Y = a^{1/1-2a} P_X^{1/(1-2a)} P_Y^{-(1-a)/(1-2a)} P_Z^{-a/(1-2a)} \tag{2-6}$$

which implies that

$$Z = a^{1/1-2a} P_X^{1/(1-2a)} P_Y^{-a/(1-2a)} P_Z^{-(1-a)/(1-2a)} \tag{2-7}$$

is the demand function for good Z. Equations $(2-6)$ and $(2-7)$ imply that demands for goods Y and Z are not independent : quantity demanded of good Y depends on the price of good Z (and vice versa).

So the "inverse elasticity" version of the Ramsey rule is not appropriate here. By coincidence, this (wrong) rule does give the right answer, since both goods here happen to have the same own–price–elasticity of demand, namely $(1-a)/(1-2a)$. But that is just a lucky coincidence.]

*Q*3. Suppose that a small imaginary country consists of two groups of people. "High–ability" people, comprising 20% of the country's work force, can earn an annual income of 80, whereas the other 80% can earn an annual income of 30.

The government must raise tax revenue averaging 16 per person, using a flat tax, in which each person's tax liabilities (whether she is high–ability or not) are

$$T \equiv \tau(Y - E)$$

where Y is her annual reported income, τ the marginal tax rate, E the exemption level, and T the person's tax liabilities.

People do not get to choose how many hours to work in this little economy. However, they can choose whether or not to work in the commercial sector, or the "cash only" sector. If they work in the "commercial" sector, they earn their regular income (80 or 30, depending on whether they are "high–ability" or not), and all income is reported to the tax authorities. If they work in the "cash only" sector, they make only half as much money (40 or 15, depending on whether or not they are "high–ability"). But none of their income from the "cash only" sector gets reported to the tax authorities.

So each person has to choose one sector or the other, and chooses whichever job gives her the highest net income.

In this economy, which choice of flat tax system (τ, E) would be best for the ("low–ability") majority?

*A*3. Here the high–ability people will choose to work in the cash–only sector if and only if it yields them a higher net income than working in the commercial sector. Working in the commercial sector earns them 80, minus their taxes. Working in the cash–only sector earns them 40. So they will be willing to work in the commercial sector only if their total taxes (if they work in the commercial sector) are 40 or less.

So 40 is the most tax revenue (per person) which can be collected from high–ability people. Any attempt to collect more tax revenue from them would just drive them out of the commercial sector, and result in no taxes at all being collected from them.

If the tax system collects 40 from each high–ability person, how much must it collect from each other person? The required tax yield is 16 per person. So if the tax system collects T_0 dollars from each low–ability person, then the average tax revenue collected per person would be

$$(0.2)(40) + (0.8)(T_0) \tag{3 - 1}$$

since 20 percent of the population is of high ability. The value of T_0 which makes expression $(3-1)$ equal to 16 is $T_0 = 10$.

So it appears that the best that the low–ability people can do is to have a tax system which collects 40 from each high–ability person, and 10 from each low–ability person. But two things might be checked here. First, would the low–ability people want a system which collected less than 40 from each high–ability person? No, because then the tax system would have to collect more then 10 from each other person. Second, would the low–ability people be wlling to work in the commercial sector? If their tax liabilities were 10, then their net earnings from the commercial sector would be $30 - 10 = 20$, which is more than the 15 that they could earn in the cash–only sector.

So the best tax system for the low–ability majority here is one in which each of them pays 10 in taxes, and in which each of the high–ability people pays 40.

What tax system is that? The marginal rate τ and the exemption level E must satisfy

$$\tau(80 - E) = 40 \tag{3 - 2}$$

$$\tau(30 - E) = 10 \tag{3 - 3}$$

Subtracting $(3 - 3)$ from $(3 - 2)$ then implies that $\tau = 0.60$, and substituting back into $(3 - 2)$ shows that $E = \frac{40}{3} = 13.333$.

[The version of the assignment on my web page was actually different : there the income of low–ability people was 40 if they worked in the commercial sector, and 20 if they worked for cash only.

In that case, they still would want to get the maximium possible tax revenue from the high–ability people, and that amount would still be 40. As well, collecting 40 in tax revenue from each high–ability person would still imply a collection of 10 from each low–ability person. Since $40 - 10 = 30 > 20$, low–ability people would prefer to work in the commercial sector. But now the tax rate and exemption level would obey the equations

$$\tau(80 - E) = 40 \tag{3 - 2'}$$

$$\tau(40 - E) = 10 \tag{3 - 3'}$$

Subtracting $(3-3')$ from $(3-2')$ then implies that $\tau = 0.75$, and substituting back into $(3-2')$ shows that $E = \frac{80}{3} = 26.667$.]

*Q*4. According to the Haig–Simons (or "comprehensive") definition of income, what would the annual taxable income be for the following person?

She earns $100,000 in salary. She uses public transit to travel to work, for which she buys a transit pass which costs $1000 per year. She also owns a snowmobile, which she uses for recreation in winter. The annual cost of fuel, insurance, and depreciation on the snowmobile is $1000. In the city, she takes taxis when going to movies, restaurants, and other entertainment : she spends $1000 per year on taxi fares.

At the beginning of July, her uncle died, and she inherited his vacation property, which is worth $100,000. The uncle had rented the property out to another couple (for the whole year), at a rent of $2000 per month. The couple are still renting. The maintenance expenditure and taxes on the vacation property are $1000 per month.

At the beginning of the year, she owned stock which was worth $150,000. During the year, the stock decreased in value by $10,000. She also bought some additional shares, for $20,000, in a new public offering, during the year.

She lives in an apartment, on which she spends $18,000 a year in rent.

*A*4. According to Haig–Simons principles, a person's net labour income should be included in taxable income : net of any expenses incurred as part of earning the income. So a transit pass would be deductible, if the pass was used for travel to and from work. However, costs of transportation which is not part of earning income are not deductible. Hence, in this example, the cost of the transit pass would be deductible from income, but not the costs of snowmobile operation, nor the taxi fares.

Haig–Simons principles also say that any increase in a person's real wealth are part of the person's income in that year. That includes bequests received. So the entire value of the inherited vacation property must be included as part of taxable income (if income is calculated using the Haig–Simons definition).

In addition, net rental income is a part of a person's taxable income. In this case, the person received the income for 6 months (she inherited the property at the beginning of July), so that 6 months' rent – net of the maintenance expenditure and taxes — should be included in income.

Capital gains on assets owned by the person are a part of Haig–Simons income, since the definition is "what a person could consume in the year if her wealth remained unchanged". By the same argument, capital losses on assets which she owns are deductible.

New purchases of stock do not enter into Haig–Simons income : what people do with their income (spend it or save it) does not matter using the Haig–Simons definition. For the same reason, rental expenses (or any other consumption expenditures) are not a deduction from Haig–Simons income.

So for this person, her Haig–Simons income for the year would be her salary income of \$100,000, minus the \$1000 cost of the transit pass, plus the \$100,000 bequest received, plus \$6000 in net rental income (\$2000 per month for 6 months, minus 6 months' expenditure of \$1000 per month), minus the \$10,000 capital losses on her stock, for a total annual income of \$195,000.

*Q*5. According to the Haig–Simons (or "comprehensive") definition of income, what would the annual taxable income be for the following person?

He earned \$80,000 in salary. Of that salary, \$5,000 went into a company pension plan. In addition, his employer contributed \$5,000 into his account in the company pension plan.

He owns his own house, which was worth \$400,000 at the beginning of the year, and \$450,000 at the end of the year. His annual property taxes on the house were \$5000. He spent \$10000 a year on maintenance, utilities and insurance on the house. He also has a \$300,000 mortgage on the house, on which he paid \$15,000 in interest. He estimates that the house would rent for \$35,000 a year if it were rented to someone else.

Late in the year, he also sold a valuable painting he owned, for \$210,000. He had bought the painting in 1990 for \$120,000. At the beginning of the year, the value of the painting had been appraised at \$225,000.

*A*5. The Haig–Simons definition of income is the amount that a person could spend on consumption in the year, without changing the value of his wealth. So an employer's contribution to a pension plan is part of Haig–Simons income : his pension plan benefits are part of his wealth ; company contributions increase his wealth ; therefore he could increase his consumption expenditure without changing his wealth.

His own contributions, from his own income, to a pension plan, do not affect his Haig–Simons income. Haig-Simons income is unaffected by how the income is allocated between consumption and saving.

Under the Haig–Simons definition of income, the "imputed" rent from living in his house must be included as part of his income : it is part of the value of his consumption. The value to him of getting to live in his own house is the amount of annual rent that someone would pay to live in this house. But, under Haig–Simons principles, any expenses (taxes, maintenance, or mortgage interest) of owning a house would be deductible from the "imputed" income from living in the house.

Since the value of his house is part of his wealth, any increase in that value is part of his Haig–Simons income, to be included in the year in which the increase occurs. With a painting (or any other form of wealth), the capital gain on the asset should reported in the year in which the gain occurs ("accrues").So, under Haig–Simons principles, the capital gains on the painting which had occurred prior to this year (totalling \$105,000 [225000 − 120000]) were already reported in earlier years. He would be entitled to a capital loss equal to the fall in the value of the painting in the current tax year.

Therefore his Haig–Simons income would be the \$80,000 in salary, plus the employer's contribution of \$5000, plus the capital gain of \$50,000 on his house, plus the imputed rental income of \$35,000 from living in the house, minus the expenses \$5000 for taxes, \$10,000 for maintenance, and \$15,000 for mortgage interest, minus his capital loss of \$15,000 on the painting in the current year, for a total of \$125,000.