Do all 5 questions. All count equally

1. If a person's preferences can be represented by a utility function

$$
u(X, Y, Z)=Z+20 \ln X Y
$$

where $X, Y$ and $Z$ are the person's quantities consumed of food, clothing and other goods, and if the net-of-tax price of each of the goods is 1 , and if the person's income is 60 , what would be the total excess burden of a tax of $\$ 1$ on good $X$ and a tax of $\$ 3$ on good $Y$ ?
2. A person chooses how much to work when young, how much to consume when young, and how much to consume when old.

If she works $H$ hours per week when young, and spends $C_{Y}$ on consumption when young, then she can save $w H-C_{Y}$ per week. The interest rate on her saving is $r$ (so that she gets to consume $(1+r) S$ per week when old, if she saves $S$ per week when young). Her wage rate is $w$ per hour.

Her preferences can be represented by the utility function

$$
u\left(C_{Y}, C_{O}, H\right)=20 \ln C_{Y}+20 \ln C_{O}-H
$$

where $C_{O}$ is her consumption expenditure per week when old.
The government needs to raise a certain amount of revenue $R$, from sales taxation. It can tax consumption in each period. Since the government can borrow or lend at the interest rate $r$, it is the present value of tax revenue which matters ; the present value of tax collections

$$
T_{Y}+\frac{T_{O}}{1+r}
$$

must equal $R$, where $T_{Y}$ and $T_{O}$ are the sales tax revenue collected in the two periods of the person's life.

What is the relation between the sales tax rate when the person is young, and the sales tax rate when she is old, if the government wants to raise the required present value of revenue at minimum harm to the person?
3. Suppose everything is the same as in the previous question (\#2) - the same preferences, the same government revenue requirement, the same wage rate and interest rate. The only difference is in the government's tax instruments. Now the only taxes it can levy are a proportional sales tax on the person's wage earnings when young, and a proportional tax on her interest income when old.

What is the relation between the wage tax rate when the person is young, and the tax rate on interest income when she is old, if the government wants to raise the required present value of revenue at minimum harm to the person?
4. Suppose that the populace of a small imaginary country consists of two groups of people. "High-ability" people, comprising $40 \%$ of the country's work force, can earn an annual income of 60 , whereas the other $60 \%$ can earn an annual income of 45 .

The government must raise tax revenue averaging 12 per person, using a flat tax, in which each person's tax liabilities (whether she is high-ability or lower-ability) are

$$
T \equiv \tau(Y-E)
$$

where $Y$ is her annual reported income, $\tau$ the marginal tax rate, $E$ the exemption level, and $T$ the person's tax liabilities. If a person reports income less than $E$, she pays no tax (but gets no money back from the government.)

People do not get to choose how many hours to work in this little economy. However, they can choose whether or not to work in the commercial sector, or the "cash only" sector. If they work in the "commercial" sector, they earn their regular income ( 60 or 45 , depending on whether they are "high-ability" or not), and all income is reported to the tax authorities. If they work in the "cash only" sector, they make only half as much money ( 30 or 22.5 , depending on whether or not they are "high-ability"). But none of their income from the "cash only" sector gets reported to the tax authorities.

So each person has to choose one sector or the other, and chooses whichever job gives her the highest net income.

In this economy, which choice of flat tax system $(\tau, E)$ would be best for the ("low-ability") majority?
5. According to the Haig-Simons (or "comprehensive") definition of income, what would the annual taxable income be for the following person?

She earned $\$ 90,000$ in salary. Of that salary, $\$ 5,000$ went into a company pension plan. In addition, her employer contributed $\$ 5,000$ into her account in the company pension plan.

She owns her own house, which was worth $\$ 500,000$ at the beginning of the year, and $\$ 600,000$ at the end of the year. Her annual property taxes on the house were $\$ 10000$. She spent $\$ 10000$ a year on maintenance, utilities and insurance on the house. She also has a $\$ 300,000$ mortgage on the house, on which she paid $\$ 15,000$ in interest. She estimates that the house would rent for $\$ 45,000$ a year if it were rented to someone else.

She leases a car for $\$ 10,000$ a year, and spends another $\$ 4000$ on insurance, gasoline, and maintenance for the car. She drives 20,000 kilometres per year, 10,000 on trips to and from work, and 10,000 on trips for shopping or entertainment.

