From the last part of subsection “b”, the excess burden can be defined as that area under the demand curve, minus the tax revenue collected. The tax revenue collected is just the tax times the quantity consumed. That’s the area of a rectangle : the height of the rectangle is the tax, and the width of the rectangle is the quantity purchased. The excess burden, being the difference between the equivalent variation and the tax yield, is thus the area to the left of the compensated demand curve, above a line at the height of the before–tax price, and to the right of the quantity consumed. That area will be a triangle if the compensated demand curve is a straight line. [This is illustrated in Figure 5.]

if the demand curve was a straight line, the magnitude of the excess burden would be the area of a triangle : the height of the triangle is the magnitude of the tax (p_x), and the width is the (compensated) change in the quantity demanded of the taxed good ($X^H(p_x, p_y, u) - X^H(p_x + t_x, p_y, u)$).

If the compensated demand curve is not a straight line, then this triangle is still a good approximate measure of the excess burden. Recall that the area of a triangle is one half the height times the base. So

$$EB = \frac{1}{2}[t][\Delta Q] \tag{35}$$

since the height of the triangle is the level of the tax, and the base is the change in the **quantity** of the good consumed, before and after the tax. (This is actually the difference between the quantity consumed at the net–of–tax and tax–inclusive prices, holding constant the person’s utility.) Now the change in the price is the tax that has been imposed,

$$t = \Delta P$$

The change in the quantity consumed in response to a price change can be written

$$\Delta Q = -\frac{\partial Q}{\partial P}\Delta P \tag{36}$$

It is sometimes convenient to put this in elasticity form. From the definition of elasticities,

$$\eta^c \equiv -\frac{\partial Q}{\partial P}\frac{P}{Q}$$

where η^c is the compensated own–price elasticity of demand (defined so as to have a positive sign), so that

$$-\frac{\partial Q}{\partial P} = -\frac{\partial Q}{\partial P}\frac{P}{Q}\frac{Q}{P} = \eta^c\frac{Q}{P} \tag{37}$$

That means that

$$EB = \frac{1}{2}t^2\eta^c\frac{Q}{P} \tag{38}$$

Finally, it is often convenient to use not the unit tax t, but the *ad valorem* tax rate

$$\tau = \frac{t}{P}$$

which means that

$$t = \tau P$$

so that the formula becomes

$$EB = \frac{1}{2}\tau^2 \eta^c PQ \tag{39}$$

(This is also equation 15.3 in Rosen, Wen, and Snoddon.)

This formula can be obtained directly from equation (25) (from subsection "b").

$$\frac{\partial EB}{\partial t_x} = -t_x \frac{\partial X^H(P_X, p_y, u_1)}{\partial P_X} \geq 0 \tag{25}$$

Now if the demand curve is a straight line, then its slope is a constant. So with a straight line demand curve,

$$\frac{\partial X^H(P_X, p_y, u_1)}{\partial P_X} = -d \tag{40}$$

for some constant $d \geq 0$. That mean that

$$\frac{\partial EB}{\partial t_x} = d t_x \tag{41}$$

so that

$$EB = \frac{d}{2}[t_x]^2 \tag{42}$$

Since $\frac{\partial X^H(P_X, p_y, u_1)}{\partial P_X} = -d$, therefore

$$d = \eta^c \frac{X^H}{P_X} \tag{43}$$

and substitution of (43) into (42) yields – exactly — expression (39) (since X^H is the quantity Q of the taxed good, and $t_X = \tau P$).

A couple of things to note about this formula. First of all, the higher is the (compensated) elasticity of demand, the bigger is the deadweight loss. That suggests, other things equal, that it is a bad idea to levy taxes at high rates on goods for which the demand is very elastic. The more elastic is demand, the more that a given tax rate will induce the consumer to substitute away from the good. And it is this substitution induced by the tax which is responsible for the deadweight loss.

Second, not only does the formula suggest that the excess burden goes up with the tax rate, it goes up with the square of the tax rate. That is, it goes up more than proportionally to the tax rate. This relation can be examined another way by considering what the **tax yield** is. If TY denotes the tax yield, then

$$TY = \tau PQ \tag{44}$$

so that

$$EB = \frac{1}{2}\tau\eta^C TY \qquad (45)$$

The ratio of the excess burden to the tax yield goes up with the tax rate. As the tax rate on a given good or group of goods rises, not only does the excess burden go up, but the excess burden per dollar of tax revenue raised goes up. For example, if $\eta^C = 1$, then if the tax rate were 5 percent, the formula says that the excess burden is only 2.5 percent of the tax yield. But if the tax rate were 40 percent, then we would be wasting 20 percent of the tax revenue due to the excess burden. This proportionality of the excess burden to the square of the tax rate also suggests that it's better, other things equal, to have lots of little tax distortions rather than one big distortion. Suppose, for example, that we could raise a given amount of revenue either by a 40 percent tax on one good, or by a 20 percent tax on that good, as well as a 20 percent tax on another good. The first method gives an excess burden of 20 percent of the tax revenue. The second method gives an excess burden in each market of 10 percent of the tax revenue raised. Therefore in aggregate, the sum of the excess burdens in the two markets will be 10 percent of the tax revenue raised in total. Spreading out the taxes between the two markets serves to reduce the overall excess burden.

So I should repeat the two lessons the formula gives, because they form the basis of **optimal commodity taxation** : concentrate taxes on goods which have inelastic demand, and spread the taxes out among a lot of goods, rather than one big tax on one good.

Now in making the case for spreading out the tax on a bunch of goods, I simply assumed that the total excess burden was just the sum of the excess burden triangles in each market. That assumption is equivalent to assuming that changing the tax on one good has effects only on the excess burden in the market for that good.

Is that assumption valid? **No**. Raising the tax on coffee will shift out the compensated demand curve for tea. That will affect the magnitude of the deadweight loss in the market for tea, if there already was a tax on tea. In particular, if tea and coffee are substitutes, then increasing the tax on coffee actually *reduces* the excess burden in the market for tea.

This is again an important point, one worth repeating, and emphasizing : if we already have a tax on some good, then increasing the tax on a substitute for the that good will actually reduce the excess burden in the market for the first good.

To see this, suppose that we tax both good X and good Y. Then the definition of the excess burden becomes

$$EB = [E(p_x+t_x, p_y+t_y, u) - E(p_x, p_y, u)] - [t_x X^H(p_x+t_x, p_y+t_y, u) + t_y Y^H(p_x+t_x, p_y+t_y, u)] \qquad (46)$$

The first term in squared brackets in expression (46) is the cost to the consumer of **both taxes put together** : how much more it would cost to get her to the given level of utility when both goods are taxed. The second term in square brackets is the total tax revenue from both taxes combined.

If we change the tax rate on good Y — and recognize that quantity demanded of one good may depend on the prices of other goods — then we get (from (46)),

$$\frac{\partial EB}{\partial t_y} = \frac{\partial E(P_X, P_Y, u)}{\partial P_Y} - t_x \frac{\partial X^H(P_X, P_Y, u)}{\partial P_Y} - t_y \frac{\partial Y^H(P_X, P_Y, u)}{\partial P_Y} - Y^H(P_X, P_Y, u) \qquad (47)$$

where $P_Y = p_y + t_y$ is the tax–included price of good Y. As before, Shepard's Lemma implies that the first and last terms on the right side of equation (47) cancel out, so that

$$\frac{\partial EB}{\partial t_y} = -t_x \frac{\partial X^H(P_X, P_Y, u)}{\partial P_Y} - t_y \frac{\partial Y^H(P_X, P_Y, u)}{\partial P_Y} \qquad (48)$$

Now suppose we start from a situation in which good X has a high tax — $t_X > 0$ — but in which we are not taxing good Y —- $t_y = 0$. Then equation (48) becomes

$$\frac{\partial EB}{\partial t_y} = -t_x \frac{\partial X^H(P_X, P_Y, u)}{\partial P_Y} \qquad (49)$$

If goods X and Y are **net substitutes**, then expression (49) must be **negative**. By definition, goods X and Y are net substitutes if (and only if)

$$\frac{\partial X^H(P_X, P_Y, u)}{\partial P_Y} > 0$$

So what does equation (49) imply?

RESULT If we were taxing only tea (and not coffee), then the overall excess burden will **decrease** if we introduce a tax on coffee (and keep the tax on tea where it was), provided that tea and coffee are net substitutes for each other.

Conversely, if we had a tax on tea in place, we would also reduce the overall excess burden of the tax system if we introduced a small **subsidy** on a **net complement** to tea (such as, perhaps, lemons), holding the existing tax on tea where it was.

That is, although we have zero excess burden if we have no taxes, and we must have a positive excess burden if we start to tax at least one good,

(i) it is **not** true that increasing tax rates on some goods must increase the overall excess burden of the tax system, if we already have taxes in place which we can't get rid of

or

(ii) while efficiency requires that the MRS between any 2 goods must equal the MRT between those goods, if we have a tax in place on good X, which we can't remove, it is **not** true that we would then be best off ensuring that $MRS_{YZ} = MRT_{YZ}$ for any other pair of goods Y and Z.