2b Tax Incidence : Partial Equilibrium

The most simple way of analyzing tax incidence is to use demand curves and supply curves. This analysis is an example of a **partial equilibrium** model. Partial equilibrium models look at **individual markets** in isolation. They are appropriate when the effects of the tax change are — for the most part — confined to one market. They are inappropriate when there are changes in other markets.

Why might partial equilibrium analysis be misleading in many cases? Recall that a demand curve for some commodity expresses the quantity consumers wish to buy of a good as a function of the price they pay for that good — **holding constant** a bunch of stuff. In particular, when drawing an ordinary [as opposed to a compensated] demand curve for socks, we are holding constant the prices buyers pay for all other goods (other than socks), and the incomes of all the buyers. Very often, a tax will change not only the price that consumers pay for socks, but the prices they pay for other goods. For example, an 8 percent sales tax should have some effects on the market for socks. But since a sales tax also applies to many more goods than socks, the sales tax will also probably affect the equilibrium price of shoes, pants, and many other commodities. All these changes will **shift** the demand curve for socks, if there are any important substitutabilities or complementarities in the demand for socks with the prices of other goods.

So partial equilibrium analysis of the market for some commodity, or group of commodities, will be useful, and reasonably accurate, only when looking at at tax which applies **only** to that group of commodities. It will not be useful to use partial equilibrium analysis to examine the incidence of a general sales tax.

Even when a tax is levied on just one commodity, partial equilibrium analysis is accurate only when the market for the commodity is relatively small compared to the economy as a whole. There are several reasons for that restriction. First of all, look at the demand side. Suppose we were examining the impact of a tax on food purchases, in a fairly poor country. Two facts about food consumption are relevant here. First, it is a very major part of the consumption expenditure of low–income people. Second, demand for food is quite inelastic with respect to price. So a tax on food will probably raise the after–tax price people pay for food. Given the low price elasticity, that means that people's overall expenditure on food will increase : inelastic means exactly that expenditure goes up when the price goes up. Well, the increase in food expenditure, when food is a significant part of consumers' costs, means a significant reduction in the income available for buying other goods and services. Hence, the tax is bound to shift demand curves for other goods and services significantly : it will have "general equilibrium" effects not captured in the partial equilibrium analysis.

A second reason why partial equilibrium analysis is only accurate when the market for the taxed commodity is "small" is the importance of the **sources** of income in tax incidence. Imagine a Brazilian trying to analyze the effects of a national production tax on coffee. Now Brazilians consume a lot of coffee, so we certainly expect to see effects from this tax on the retail market for coffee in Brazil. But coffee production is also a major industry in Brazil. This tax will also have a significant effect on people's incomes, in their roles as suppliers of labour in the coffee industry, or owners of coffee plantations. These effects will shift demand curves for many goods and services which Brazilians consume : down, if their real incomes fall. Again, there are general equilibrium effects not depicted in the partial equilibrium analysis of the retail market for coffee. Probably a tax on coffee production would lower significantly the value of agricultural land in many parts of Brazil, an effect not captured by a supply–demand diagram of the retail market for coffee.

The supply curve – demand curve diagram is an example of partial equilibrium analysis, the most important example. But there are other models which are also partial equilibrium in nature. Another, different, partial equilibrium model is that of a market with a monopoly supplier. This is still partial equilibrium, since we are looking at the one market in isolation. But since this analysis applies to a market which is not perfectly competitive, it certainly cannot be done with a supply curve in the picture.

So suppose that an **excise** tax is imposed on a perfectly competitive market. Remember : an excise tax is a tax on a single commodity, or a group of commodities. Taxes on gasoline, tobacco, or air travel are some Canadian excise taxes. These are the sort of taxes which can be analyzed using partial equilibrium analysis. There are really two ways to analyze the effects of this tax using a supply curve – demand curve diagram. But they will both give the same answer – if you do the analysis right.

The main distinction between the two approaches is which price is put on the vertical axis. In the absence of taxation, there is **one** price for the good, the (same) price paid by buyers and received by sellers. When there is an excise tax on the good, these two prices are different. It is crucial to keep in mind which price you mean when you do the analysis.

The first approach considered here is to look at a tax imposed on the buyers, so that the analysis will be done in term of the price which the buyers pay. Consider a **unit tax**, of t per unit sold, levied on the sellers. A unit tax means a tax which is assessed per unit sold : 20 cents per litre of gasoline, or 2 dollars per box of 20 cigarettes. This way of defining the tax can be contrasted with an **ad valorem** tax, which is assessed as a **percentage** of the sales price : 10 percent of your cable bill, or 7 percent of the cost of your hotel room. The difference is, the amount you pay per hotel room will vary with the price of the hotel room under ad valorem taxation, but not under a unit tax. That makes the analysis somewhat different for the two cases, although the results are not that different. (Most Canadian excise taxes are defined as ad valorem taxes — but the algebra may be slightly neater with unit taxes.)

With a unit tax of t per unit sold, the relationship between the **demand price** P^D paid by buyers and the **supply price** p_s received by sellers is

$$P^D = p_s + t$$

This is just a definition : the (tax–included) price paid by buyers is the (net of tax) price received by sellers, plus the tax due. The demand function $D(P^D)$ defines the quantity of the good buyers are willing to buy as a function of the price they have to pay P^D (and of other stuff which we are holding constant in partial equilibrium analysis). The supply function $S(p_s)$ defines the quantity of the good that sellers are willing to sell as a function of the price p_s that they receive (again, and also on other stuff being held constant). The presence or absence of the tax, or the level of the tax, does **not** alter either of those functions. As long as consumers' preferences and incomes, and the prices of other goods and services, do not change, then the quantity they are will to buy, as a function of the price they have to pay, remains $D(P^D)$. Similarly, the aggregate quantity that sellers are willing to sell remains $S(p_s)$. What changes with the tax is the relation between the two prices P^D and p_s.

So what is critical here is that buyers' and sellers' decisions depend on different prices, when there is a tax. Buyers care about the price that they have to pay, when all taxes are included. Sellers care about the price which they receive for the item, after any taxes have been remitted. If a shirt is marked with a price of \$20, and is subject to 13% HST, then the actual price a person pays, if she buys the shirt, is \$22.60 (\$20 plus the 13% in taxes) ; the price received by the vendor is the marked price of \$20, since the vendor does not get to keep the HST that she's collected. In equilibrium, the quantity supplied must equal the quantity demanded, so that the equation

$$D(P^D) = S(p_s)$$

must (always) hold. Those two equations

$$P^D = p_s + t \qquad (taxdef)$$

$$D(P^D) = S(p_s) \qquad (eq)$$

form two equations in two unknowns, and determine P^D and p_s as functions of the unit tax rate t.

How to solve them depends on how we substitute. One way is to treat everything in terms of the demand price P^D. If we substitute from the definition "taxdef" for p_s into the equilibrium condition "eq", then we get the single equilibrium condition

$$D(P^D) = S(P^D - t)$$

which tells us how the demand price P^D depends on the unit tax rate t in a perfectly competitive market. In fact, we can simply differentiate this equation, getting

$$[S'(P^D - t) - D'(P^D)]dP^D = S'(P^D - t)dt \qquad (D)$$

or

$$\frac{\partial P^D}{\partial t} = \frac{S'(P^D - t)}{S'(P^D - t) - D'(P^D)} \qquad (algebra)$$

Graphically, if we have the demand price P^D on the vertical, then the tax does not move the demand curve : that relation remains as it was. The supply curve shifts, not because anything about the behaviour of sellers has changed, but because the relation between P^D and p_s has changed. The supply curve shifts up by t. Suppose $(200, 10)$ were on the old supply curve, meaning sellers were willing to provide 200 units, in aggregate, if the price they received were 10. They still are willing to provide 200 units at a net price of 10. But now, for them to get a net price of 10, buyers must be paying a price of $10 + t$. So the point $(200, 10 + t)$ is on the new supply curve.

If the supply curve moves up (and left), then the new equilibrium point must move up and left, up the demand curve. The price P^D paid by buyers must go up, and the quantity sold in the market must fall, as a result of the tax. The price p_s received by sellers must fall. Why? The quantity sold has fallen, since the equilibrium moves left. Since the quantity sold equals $S(p_s)$, and the supply curve $S(p_s)$ slopes up, then p_s must have fallen. How much the demand price P^D rises, and the supply price p_s falls, depends on the slopes of the supply and demand curves, as the algebraic solution (equation "algebra") indicates.

In figure 1, the original equilibrium price, when there is no tax present, is 10, and the equilibrium quantity sold is 200. In this figure, the demand curve has the equation

$$Q^D = 300 - 10P^D$$

and the supply curve has the equation

$$Q_s = 2(p_s)^2$$

[You can check that $P^D = p_s = 10$, and $Q^D = Q_s = 200$ solves both of these equations.]

A unit tax of $t = 10$ shifts the supply curve up. The new equilibrium, determined by the intersection of the "new" (shifted up) supply curve with the demand curve, has $P^D = 17.8077$ and $Q_s = Q^D = 121.922$. [You can check that this price and quantity satisfy the equations $Q^D = 300 - P^D$ and $Q_s = 2(P^D - 10)^2$.] That means that the price paid by buyers has increased, because of the $10 unit tax, from 10 to 17.8077 ; the price received by sellers has decreased, from 10 to 7.8077. In this example, the buyers are bearing about 78 percent of the tax.

On the other hand, we can do the whole analysis in terms of the supply price, if the buyers are legally responsible for the tax. Substituting or P^D in the equilibrium condition gives the equation

$$D(p_s + t) = S(p_s)$$

so that we can again differentiate to get an algebraic solution, here

$$[S'(p_s) - D'(p_s + t)]dp_s = D'(p_s + t)dt$$

or

$$\frac{\partial p_s}{\partial t} = \frac{D'(p_s + t)}{S'(p_s) - D'(p_s + t)} \tag{S}$$

Graphically, since we are now doing things in terms of the supply price, the supply curve remains where it was, and the demand curve shifts down by t. If buyers were previous willing to buy, in aggregate, 200 units at a demand price of 10, then they still are willing to buy 200 units at an (everything included) price of 10, but now a demand price of 10 means that suppliers only receive $10 - t$. So $(200, 10)$ on the old demand curve means $(200, 10 - t)$ on the shifted demand curve. Since the demand curve shifts down, the new equilibrium is below and to the left of the previous equilibrium : the price received by sellers falls as a result of the introduction of the tax, and the quantity sold in the market falls. Since the quantity sold in the market falls, and the quantity sold equals the quantity demanded, then the price paid by buyers must have fallen.

Qualitatively, then, both ways of doing the analysis give us similar answers. But they actually give the **exact** same answer. That is what I meant when I said earlier that statutory incidence does not matter at all. Whether the suppliers are legally responsible (so that the method of figure 1 is most natural), or the demanders are legally responsible (so that figure 2 makes the most sense) has absolutely no effect on the impact of a given unit tax on the prices actually paid by buyers and the prices received by sellers.

In figure 2, the supply curve and the original demand curve are drawn exactly the same as those in figure 1. The tax is also the same. The demand curve has been shifted down by 10. In figure 2, the equilibrium (p_s, Q_s) combination is $(10, 200)$ when there is no tax (just as in figure 1), and $(7.8077, 121.922)$ when there is a tax. So, if we measure things in terms of the supply price (for instance, if demanders are legally responsible for the tax), the changes are **exactly** what they were in figure 1 : the sellers' price falls to 7.8077, the buyers' price rises to 17.8077, and the quantity sold falls to 121.922.

The two pictures, figures 1 and 2, really constitute a proof of a comment I made early in this section : when markets are perfectly competitive, then the statutory incidence of a unit excise tax does not matter for the economic incidence.

What does matter? The shapes of the supply and demand curves. In fact, equation "algebra" above can be used to get an exact formula for the incidence of a unit excise tax, in terms of the elasticities of supply and demand. The punch line is going to be : the less elastic side of the market bears more of the tax.

Repeating two of the equations derived above,

$$\frac{\partial P^D}{\partial t} = \frac{S'(P^D - t)}{S'(P^D - t) - D'(P^D)} \qquad (algebra)$$

$$\frac{\partial p_s}{\partial t} = \frac{D'(p_s + t)}{S'(p_s) - D'(p_s + t)} \qquad (S)$$

These equations are consistent : it does not matter which one you use.

Recall that, by definition,

$$P^D = p_s + t$$

If we differentiate this expression with respect to t, we get

$$\frac{\partial P^D}{\partial t} = \frac{\partial p_s}{\partial t} + 1$$

Now take equation "S" above, and add 1, to get

$$\frac{\partial p_s}{\partial t} + 1 = \frac{D'(p_s + t)}{S'(p_s) - D'(p_s + t)} \tag{$S+1$}$$

Since

$$1 = \frac{S'(p_s) - D'(p_s + t)}{S'(p_s) - D'(P_s + t)}$$

Therefore

$$\frac{\partial P^D}{\partial t} = \frac{\partial p_s}{\partial t} + 1 = \frac{D'(p_s + t) + S'(p_s) - D'(p_s + t)}{S'(p_s) - D'(P_s + t)}$$

which [I hope you can see] equals

$$\frac{S'(P^D - t)}{S'(P^D - t) - D'(P^D)}$$

which was equation "algebra", obtained from shifting the supply curve.

And that's the consistency check. Both formulas give the same answer.

Note that expression "algebra" says that, relatively speaking, the higher is S', and the lower in absolute value is D', the more that P^D goes up — that is, the more of the tax is born by buyers.

This formula can also be converted to elasticities.

Recall that the elasticity of supply is defined by

$$\epsilon_s \equiv S'(p_s)\frac{p_s}{Q}$$

and the elasticity of demand by

$$\epsilon_D \equiv -D'(P^D)\frac{P^D}{Q}$$

where Q is the quantity sold. Here I have defined both elasticities so that they are both **positive** in sign (by multiplying my demand derivative by -1). If we multiply every term in the expression for $\frac{\partial P^D}{\partial t}$ by the same thing, namely p_s/Q, then equation "algebra" becomes

$$\frac{\partial P^D}{\partial t} = \frac{\epsilon_s}{\epsilon_s + (p_s/P^D)\epsilon_D} \tag{\textit{elastic}}$$

(when we also multiply and divide the last term in the denominator by P^D). Except for that little p_s/P^D, this expression says that who bears the tax depends on the relative elasticities. Now, if the tax is pretty small, as a fraction of the price, then p_s/P^D is almost equal to 1, so that the expression on the right side of equation "elastic" is quite close to

$$\frac{\epsilon_s}{\epsilon_s + \epsilon_D}$$

which would say, for example, that each side of the market would bear half the tax if the supply and demand elasticities were equal in absolute value. Regardless of the value of p_s/P^D, this expression gives the limiting cases for horizontal or vertical demand or supply curves. If $\epsilon_s = 0$, or if $\epsilon_D = \infty$ then P^D is unaffected by the tax change ; sellers bear the entirety of the tax burden, and buyers none of it. If $\epsilon_s = \infty$, or if $\epsilon_D = 0$, then

$$\frac{\partial P^D}{\partial t} = 1$$

so that the price paid by buyers goes up by the full amount of the tax ; buyers bear 100% of the tax burden, and (since p_s does not change in this case) sellers bear none of it.

In figures 1 and 2, the equations of the demand and supply curves were

$$Q^D = 300 - 10P^D$$

$$Q_s = 2(p_s)^2$$

The original equilibrium, in which there was no tax, had a quantity of 200, and a price of 10. Now we can calculate the elasticities of demand and supply, using the usual formulas, at $(200, 10)$:

$$\epsilon_D = -\frac{\partial Q^D}{\partial P^D}\frac{P^D}{Q^D} = -(-10)\frac{10}{200} = 0.5$$

$$\epsilon_s = \frac{\partial Q_s}{\partial p_s}\frac{p_s}{Q_s} = 4(p_s)\frac{p_s}{Q_s} = 4(10)\frac{10}{200} = 2$$

So the elasticity formula says that the buyers should bear a proportion $2/(2+0.5) = 0.8$ of the tax.

Above, the actual calculation of the new equilibrium prices and quantity, after the imposition of a tax, was that buyers bore about 78 percent of the tax. So the elasticity formula comes very close to the actual answer, but is not exact. That is because elasticities in general, and in this case in particular, are not constant. As we move along the demand curve, or along the supply curve, the elasticities will change in value. As well the adjustment factor p_s/P^D in the elasticity formula will change as the tax changes. So, unless the change in the tax is infinitessimal, the elasticity formula provides an approximation to the effect of a tax change. Usually, the bigger the tax change, the worse the approximation. The case in figure 1 and 2 shows that it is usually a pretty good approximation : even though the tax imposed was a pretty big one, the answer using the elasticity formula (80 percent) was very close to the actual share (78 percent) born by buyers. The big advantage of using the elasticity formula (or the derivative formula "algebra") is that you do not have to calculate explicitly what the new equilibrium will be. This formula uses only information about the shapes of demand and supply curve at the current equilibrium, and enables one to predict fairly accurately the effect of a tax change.

As another example, suppose that the equations for the supply and demand curves were

$$D(P^D) = 24 - 2P^D$$

$$S(p_s) = (p_s)^2$$

Then the equilibrium if there were no tax is $p_s = P^D = 4$, and $Q = 16$. At this initial equilibrium

$$\epsilon_D = (2)\frac{4}{16} = 0.5$$

and

$$\epsilon_s = (2)(4)\frac{4}{16} = 2$$

so the elasticity formula says again (by coincidence) that the buyers should bear 80 percent of the tax, and sellers 20 percent.

Now suppose a unit tax of 50 cents is imposed. Solving for the new equilibrium on a spreadsheet gives the answer that now (to two decimal places),

$$p_s = 3.90$$

$$P_D = 4.40$$

$$Q = (3.90)^2 = 15.20 = (24 - 2 * (4.40))$$

In other words, the 50 cent tax has increased the buyers' price by 40 cents, and decreased the price received by sellers by 10 cents. If the tax were \$1, then the new equilibrium would be $p_s = 3.80$, $P^D = 4.80$, and $Q = 14.40$, again just what the elasticity approximation predicted — to two decimal places. If the tax were \$2, then the formula is not perfect, not even to two decimal places. Then we get $p_s = 3.58$, $P^D = 5.58$, and $Q = 12.83$, so that buyers now bear $1.58/2 = 79$ percent of the tax, not the 80 percent predicted by the elasticity formula.

There is not much difference, in a competitive market, between the analysis of a unit excise tax, and an ad valorem tax. Recall that a unit tax is collected as a fixed amount per unit sold, so that the tax does not vary with the price of the commodity. An ad valorem tax is collected as a percentage of the price of the commodity, so that the tax collected per unit will fall if the price of the unit falls. There's not really much of a difference. Algebraically, let τ be the rate of the ad valorem tax, expressed as a fraction of the supply price. So a ten percent ad valorem tax on restaurant meals would mean that $\tau = 0.1$. Then the equilibrium condition becomes

$$S(p_s) = D(p_s[1+\tau])$$

since now

$$P^D = p_s(1+\tau)$$

Differentiating that equilibrium condition, and using the definitions of elasticity, yields, for example, that

$$\frac{\partial p_s}{\partial \tau} = \frac{\epsilon_D}{\epsilon_s(1+\tau)+\epsilon_D} p_s$$

which again shows that the two sides of the market bear the tax in inverse relation to their elasticities.

In addition, if the demand curve, and the supply curve, were both straight lines, then the tax incidence can also be calculated in terms of the **slopes** of the lines. That is, suppose that

$$Q^d = A - BP^D$$

was the equation of the demand curve, and

$$q_s = -c + dp_s$$

was the equation of the supply curve, where A, B, c and d were positive constants.

The equilibrium condition here, that quantity supplied equal quantity demanded, can be written

$$A - BP^D = -c + d(P^D - t)$$

which can be re–arranged as

$$(B + d)P^D = A + c + dt$$

or

$$P^D = \frac{A + c}{B + d} + \frac{d}{B + d}t \qquad (linear)$$

Equation "linear" specifies exactly the incidence of any unit tax in this market. Whenever t goes up by \$1, equation "linear" says that the price P^D paid by buyers goes up by $d/(B + d)$ dollars. Since $p_s = P^D - t$, equation "linear" also says that

$$p_s = \frac{A + c}{B + d} - \frac{B}{B + d}t$$

These equations then say that the bigger is d relative to B, the more of the tax is born by buyers. It is perfectly consistent with the earlier elasticity formula : in this case

$$\epsilon_D = B\frac{P^D}{Q}$$

$$\epsilon_s = d\frac{p_s}{Q}$$

so that

$$\frac{\epsilon_D}{\epsilon_s} = (1 + t)\frac{B}{d}$$

B being large relative to d is the same thing as ϵ_D being large relative to ϵ_s.

Since $-(1/B)$ is the slope of the demand curve (when price is on the vertical axis, quantity on the horizontal), and $1/d$ is the slope of the supply curve, then equation "linear" can be used as an approximation of the tax incidence story, even when demand and supply curves are not exactly equal : take the slope of the demand curve, and let B be minus 1 over that slope, let d be 1 over the slope of the supply curve, and the share of the tax born by buyers is $d/(B + d)$.

For example, in the example used in figure 1, in which

$$Q^D = 300 - 10P^D$$

$$Q_s = 2(p_s)^2$$

then

$$B = -\frac{\partial Q^D}{\partial P^D} = 10$$

and

$$d = \frac{\partial Q_s}{\partial p_s} = 4p_s = 40$$

(at the original no–tax equilibrium in which $p_s = P^D = 10$), so that $d/(B + d) = 0.8$, and the linear approximation suggests that the buyers will bear 80 percent of a unit tax. Not a bad approximation, even though the supply curve is not a straight line.

Now a problem with this partial equilibrium analysis is that there are not very many important excise taxes out there in Canada. But the model can be used to analyze some other taxes, and get some insight. Partial equilibrium analysis need not be restricted to the markets for **final goods**. If the markets are competitive, then we can also use supply and demand curves for the market for **factors of production**. The most obvious candidate is the labour market. That is, the same supply and demand curves, and elasticity formulae, can be used to answer a question raised earlier : who really bears a payroll tax? As emphasized earlier, if labour markets are competitive, then that the statutory incidence of the payroll tax does not determine who bears the tax. However the payroll tax is assessed between workers and employers, what determines how the costs of the tax are born by these two groups is the relation between the overall elasticity of labour supply in Canada, and the overall elasticity of demand. The elasticity of labour demand might well be reasonably high : to the extent that Canadian employers compete on global markets, increases in wages may lead to significant reductions in their demands for workers. The empirical evidence suggests that the overall elasticity of labour supply in Canada is not very high. Those properties of the demand for labour in Canada, and the supply of labour in Canada, mean that workers probably do bear most of the cost of payroll taxes — even if employers are legally responsible for 50 percent.

Another application of simple supply–demand analysis is to a tax on capital. Capital can be regarded as another factor of production. Again, it may not be too unreasonable to view the Canadian capital market as quite competitive. There are many competing firms, each using capital as an input to production. But it is also pretty reasonable to assume that Canada is a small open economy. That means that the supply curve of capital to Canada is horizontal. That is, people and firms will invest in Canada only if they earn at least as large a return — net of all taxes — as they do anywhere else in the world. If Canada is so small that it has no influence on the world return to capital, then it must offer investors this world return to capital — or get no investment. An immediate implication of this model is that any tax on capital income in Canada must be born by demanders of capital, not suppliers. The horizontal supply curve means that any increase in the tax we levy on the return to capital can have no effect on the return earned by investors. If this net return fell, investors wouldn't invest. So the gross return must increase by exactly the amount of the tax increase, meaning that demanders of capital, who have to pay this gross return, will bear the entirety of the tax.

Partial equilibrium is not restricted to perfectly competitive markets. Tax incidence can be analyzed in less–than–perfectly competitive markets. Why is this still partial equilibrium analysis? Because the analysis still deals with one market in isolation. The extreme form of imperfect competition is a monopoly, with only one seller. If this monopoly must charge the same price to all buyers, then we know that it will choose an output level such that its marginal revenue equals its marginal cost.

So suppose that we put a unit tax on the monopoly. One way of analyzing the incidence of this tax is to treat the unit tax as an added cost of the monopoly. This way assumes, in essence, that the monopoly seller bears the statutory incidence of the tax. Then the monopoly's optimal choice of quantity level Q becomes

$$MR(Q) = MC(Q) + t$$

since a unit tax of t raises the cost to the monopoly of each unit by t. Another way would be to treat the tax as shifting the marginal revenue curve. If buyers had to pay the tax, then the monopoly's revenue becomes

$$(P(Q) - t)Q$$

where $P(Q)$ is buyers' inverse demand curve. Marginal revenue, you will recall (from AP/ECON 2300), is the derivative of revenue with respect to quantity sold, or

$$P'(Q)Q + P(Q) - t$$

So the first version has the marginal cost curve shifting up by $t$and the second has the marginal revenue curve shifting down by t. As in the case of perfect competition, either version will give the exact same solution. For a single price monopoly, the statutory incidence of the tax is irrelevant for the economic incidence.

Equally important, in general the monopoly will bear part of the tax. Even though the firm has market power, and even though it sets price, it will bear part of the cost of the tax. Suppose, for example, that the monopoly bore the statutory responsibility for the tax. If it wanted to, it could actually increase its price by the full amount of the tax, forcing buyers to bear the entirety of the tax. But it will not want to do so. Raising the price to buyers must lower its sales, and that hurts the monopoly. Remember that a single–price monopoly must choose an optimal output level on the elastic part of the demand curve. That means that any price increase will result in a major loss in sales.

Formally, we can simply differentiate the monopoly's optimality condition

$$MC(Q) + t = MR(Q)$$

to get

$$\frac{\partial Q}{\partial t} = -\frac{1}{MC'(Q) - MR'(Q)}$$

This equation tells us that the monopoly must lower its quantity sold in response to a tax increase, since $MC' - MR' > 0$ at the monopoly's optimum. (That's the second order condition for the monopoly's optimization problem, that the marginal revenue curve cut the marginal cost curve from above, coming from the left.) So the price paid by the consumers must increase. How much does the price paid by buyers increase? The price they pay is $P(Q)$, so that

$$\frac{\partial P^D}{\partial t} = -\frac{P'(Q)}{MC'(Q) - MR'(Q)}$$

Now we can get a pretty quick answer as to the incidence of this tax in at least one special case. Suppose that the demand curve is a **straight line**, and suppose as well that marginal cost did not vary with the quantity produced. Constant marginal cost means that $MC' = 0$ in the expression above. And when the demand curve is a straight line, then the marginal revenue curve is also a straight line — with a slope exactly **twice** the slope of the demand curve. (You can check that both Varian's and Nicholson's intermediate micro textbooks show this fact.) So if $MC' = 0$, and if $MR'(Q) = 2P'(Q)$, then

$$\frac{\partial P^D}{\partial t} = \frac{P'}{2P'} = 0.5$$

In other words, in this example, with a straight–line demand curve (and constant marginal costs), the buyers bear exactly half of the unit tax, regardless of what the slope of the demand curve actually is.

What would happen if this good — with this cost function and this demand curve — were supplied by a competitive industry, rather than by a monopolist? Recall that it was assumed that the marginal cost curve was horizontal in this example. If this good were supplied competitively, then the industry's supply curve would be the horizontal sum of firms' marginal costs curves — in other words, horizontal. With a horizontal supply curve, we know (I hope!) that buyers bear 100 percent of the tax. So this is an example in which, not only does the single–price monopoly bear some of the tax, but the monopoly bears more of the tax than sellers would bear if the industry were competitive. This example is special : it only works if the demand curve is a straight line and if marginal costs are constant. In fact, if the **elasticity** of demand were constant, instead of the slope (remember, slope and elasticity are not the same thing!), then it can show that now the buyers would bear **more** of the tax under monopoly than they would under perfect competition.

To see this, note that

$$MR = P'(Q)Q + P(Q)$$

Since

$$\epsilon_D = -\frac{\partial Q}{\partial P}\frac{P}{Q}$$

then

$$\frac{1}{\epsilon_D} = -P'(Q)\frac{Q}{P}$$

which means that

$$MR = P(Q)[1 - \frac{1}{\epsilon_D}] \qquad (mono1)$$

so that the monopoly's optimal solution can be written

$$P(Q)[1 - \frac{1}{\epsilon_D}] = MC + t$$

(That only makes sense if $\epsilon_D > 1$; otherwise the left hand side of the equation would be negative. That's why we know that single–price monopoly always chooses a point on the demand curve at which demand is elastic.) Re–writing this equation,

$$P(Q) = (MC + t)\frac{\epsilon_D}{\epsilon_D - 1} \qquad (mono2)$$

Now suppose that the demand curve exhibited a constant elasticity. This would be the case if the demand curve could be written

$$Q(P) = AP^{-b}$$

for positive constants A and b[2].

Then equation "mono2" says that if t goes up by \$1, then the price $P(Q)$ goes up by $\epsilon_D/(\epsilon_D - 1)$ dollars. That amount,

$$\frac{\epsilon_D}{\epsilon_D - 1}$$

must be greater than 1, if the elasticity ϵ_D must be greater than 1 (which it must be, for the monopoly to have a solvable optimization problem). For instance, if $\epsilon_D = 3$, then $\epsilon_D/(\epsilon_D - 1) = 1.5$.

So, if the marginal cost is constant[3], and if the demand curve has a constant elasticity — which is definitely something different than having a constant slope — then the monopoly will shift more than 100 percent of a unit tax on to buyers.

So the two main points of the section on monopoly : again, the statutory incidence does not matter for a unit tax, and buyers could bear more, or less, of the tax under monopoly than they would under perfect competition.

In between monopoly and perfect competition is the case of oligopoly, in which there are a few sellers. There are many different models of how oligoplists behave, and each gives a different implication for partial equilibrium tax incidence. For example, if the oligopoly is a successful cartel, able to coordinate actions so that firms all charge the monopoly price, then the tax incidence here

[2] with $b > 1$

[3] otherwise, we would have had to worry about MC varying in equation "mono2"

will be the same as in a monopoly. If they are extremely unsuccessful in coordination, and each set prices independently, then the oligopoly behaviour is described by Bertrand's model. If they behave this way, each setting prices independently, then the oligopoly price in equilibrium will be the competitive price. So here, the incidence of the tax would be the same as it would be if there were many competitive firms in the industry.

Or perhaps the firms are unsuccessful in coordination, and each sets its quantity independently. This form of oligopoly behaviour is called Cournot behaviour. In the Cournot model, the outcome of a tax change is different than under monopoly or under perfect competition. For example, suppose that marginal costs are constant, that the demand curve is linear, and that all firms are identical. With the linear demand curve, and the constant marginal costs, we already know what happens in perfect competition and in monopoly. Buyers bear 100 percent and 50 percent of the tax, respectively. What happens in an oligopoly with 4 firms, for example, under these assumptions, if firms set quantities non–cooperatively — before and after the tax increase? In this case buyers bear 80 percent of the cost, somewhere in between monopoly and perfect competition. In this model, the buyers can be shown to bear a fraction $n/(n+1)$ of the tax increase, where n is the total number of firms. Note that this general rule includes perfect competition and monopoly as special cases, with $n = \infty$ and $n = 1$ respectively.

One more topic in the section on partial equilibrium tax incidence is slightly different. The topic is the **capitalization** of taxes. The point is perhaps a simple one, but it may be an important one for such taxes as the local property tax. Suppose that in October 2015, the government of the region of York announces a tax of \$100 per hectare per year, on all land in the region, however the land is being used. This hypothetical tax will start on July 1 2016, and will be collected each July 1 thereafter, from whomever is the registered owner of the land on July 1.

Notice that the statutory incidence of this tax is on the owner of the land on July 1 of the tax year. Notice also that the tax is being levied on something that is perfectly inelastic supply. So competitive supply–demand analysis would then suggest that, for example, the incidence of the 2017 installment of this tax would be born completely by whomever owns the land on July 1 2017.

That's not correct. If the tax is announced in October 2015, then people who own land (in October 2015) cannot avoid paying the tax simply by selling the land to someone else. Potential buyers will pay attention to the taxes for which they will be liable. This means that the future tax liabilities are will be **capitalized** into the price of the land at the time of the announcement of the tax. That is, the market price of some piece of land, should always equal the **present value** of the future returns from that land, at least in a competitive land market. The concept of present value should be a familiar one (from AP/ECON 2300). If x_t is the annual cash flow from owning this piece of land in the year t, and r is the annual interest rate, then the price of the land at the beginning of 2012 is

$$P = x_{2012} + x_{2013}/(1+r) + x_{2014}/(1+r)^2 + \cdots$$

The idea that the price of land — or any other asset — must equal the present value of the net cash flow from that asset, is a central and important one in microeconomics. An immediate implication of this notion is that an anticipated reduction in all future annual cash flow of $100 per hectare per year, will lower the current price of the land by the present value of that reduction : $\frac{100}{r}$ dollars in this case, if the interest rate is expected to be constant for all future periods.[4]

So who bears a tax on the return to some asset? The tax liability is capitalized into the selling price of the asset. It is the owner of the asset **at the time the tax is announced** who bears the cost of the tax. The principle is really a simple one. If buyers of assets have their eyes open, they cannot be forced to bear the consequences of any anticipated event. The price at which they are willing to buy must decrease, after a tax increase, in order to make them willing to buy.

The reality can be more complicated than the theory here. The result so far is that the owner of the tax at the time the tax is announced will bear the tax. But taxes, and other economic policies, are often not complete surprises. What is meant by "announced"? When the law actually is ratified? When it was passed? When it was introduced in the legislature? When it was first suggested?

The fact that tax changes (or other policy changes) are not introduced suddenly complicates this notion of capitalization, but does not affect its validity. In essence, each step of the way, as a tax increase goes from rumour to law, increases the probability that the tax increase will happen. (Or there may be decreases, if the tax increase gets defeated eventually.) And at each of the steps, the cost of the increased probability of the tax is born by the owners of the asset at the time of the step.

[4] This is actually a slight approximation. The formula would be exact if the tax were due exactly a year from now, exactly two years from now, and so on. Since the tax is due July 1 of each year, then the present value of the costs is actually somewhat greater than $100/r$, since the first payment is less than a year from now.