2c Tax Incidence : General Equilibrium

Partial equilibrium tax incidence misses out on a lot of important aspects of economic activity. Among those aspects : markets are interrelated, so that prices of all goods are determined **simultaneously**, not one market at a time ; behind the suppliers of goods and services are the owners of the factors used in producing the goods and services, who may be affected by a tax ; people are simultaneously buying goods and services, and also selling factors they own, such as the labour they supply.

General equilibrium models can help deal with all these aspects. A general equilibrium model is a comprehensive model of the whole economy. But there is a price to pay for making models so general : in order to analyze these models theoretically, they have to be made very simple. We can, and do, construct more realistic models. But these realistic models are complicated. It is very difficult, in particular, often, to understand how the models work, or what particular assumption may be responsible for the results.

So most of the theoretical results in general equilibrium tax incidence are obtained using some fairly simple general equilibrium models. The most commonly used model of general equilibrium tax incidence is called the **Harberger** model. But Harberger was really only responsible for introducing this model into the analysis of tax incidence. It's a model which has been used in the theory of international trade for many years : that's where Harberger got it. So this model, and what can be done with it, should be familiar to those of you who have taken AP/ECON 3150.

It is assumed that there are two goods X and Y. These two goods are produced using two factors of production K and L. The goods in each industry are produced by many small firms, each of which is a price taker, in both output markets and input markets. So — if there are no taxes — the owner of a firm in the X industry, faces a given price p_x for the output that she sells. She can hire workers at a market wage of w per person–hour, and can rent machinery at a market rental rate of r per machine–hour. So she chooses how much labour and capital to employ (at given rates w and r per worker–hour and machine–hour respectively), and then sells the output they produce at a price of p_x per unit. She chooses the quantities of labour and machinery to hire so as to maximize her profits, $p_x x - wq_L - rq_K$ if x is the quantity of output produced, q_L is the quantity of labour she hires, and q_K the quantity of machinery ; here output x depends (and increases with) the quantities used of the inputs, q_L and q_K.

So firms are buying inputs, and selling output. Individuals are selling the inputs (labour and/or capital) that they own, and using the income to buy goods and services from the firms.

We can add different kinds of taxes into this model. That's the whole point of the exercise. So firms may have to pay a tax on the labour, or on the capital, that they hire. Buyers may have to pay a tax on the labour income, or the capital income they earn, and they may have to pay taxes on some of the goods that they purchase.

With two goods, and two factors of production, there are actually ten possible taxes that can be analyzed. These will all be treated as *ad valorem* taxes ; it is much more simple that way. There are 2 ad valorem **excise** taxes possible, τ_X and τ_Y on consumption of good X or good Y. There are 2 **general factor income taxes** possible, τ_L and τ_K on all income earned from supplying labour and from all income earned from supplying capital. There are then 4 **specific factor taxes**, on the use of a particular factor in a specific sector : for example, a tax τ_{KX} on the use of capital, but only levied on firms in the X industry (along with τ_{KY}, τ_{LX} and τ_{LY}). That makes 8 taxes so far. Then there is a **general income tax** τ on all income, from whatever source. Unlike the Canadian income tax, this tax is a constant proportion of income, to make the analysis easier. The tenth tax is a tax τ_c on all consumption by individuals, levied at the same percentage on purchases of X and of Y, in other words, a **general sales tax**.

Now it turns out that there are some relations among the 10 taxes. First of all, if the government were to levy a partial factor tax τ_{LX} on the use of labour in the X industry, and it also were to levy a partial factor tax τ_{LY} on the use of labour in the Y industry, then these two taxes together are exactly equivalent to a general tax on labour income, wherever it is earned.

So

1 Specific factor taxes, levied at the same rate τ in all industries which use the factor are exactly equivalent to a general factor income tax on all income earned by suppliers of the factor.

Less obviously, perhaps, suppose that the government levies specific factor taxes on the use of each factor in the X industry, and at the same rate. That is, it sets $\tau_{LX} = \tau_{KX}$. Then these two specific factor taxes in the X industry are exactly equivalent in their impact, to a tax τ_X on purchases of good X.

Why? The two taxes drive up all costs of firms in the X industry by some proportion $\tau_{LX} = \tau_{KX}$, and that's equivalent to a general tax on the cost of good X. And from the partial equilibrium section, we know that in a competitive industry a tax on costs of firms in an industry is exactly the same in its incidence as a tax on buyers' purchases of the output of that industry. So

2 Specific factor taxes, levied at the same rate τ on all the factors of production used in an industry, are exactly equivalent to an excise tax at the rate τ on the output of that industry.

More obviously perhaps, a tax on the consumption of good X, and a tax at the same rate on the consumption of good Y, are equivalent to a general tax on all consumption. That is

3 Excise taxes, levied at the same rate τ on all goods and services consumed by people, are equivalent to a general sales tax at the rate τ.

A tax on labour income, and a tax on capital income at the same rate, together are exactly equivalent, more or less by definition, to a general income tax at that rate.

4 Factor income taxes, levied at the same rate τ on all possible sources of income, are equivalent to a general income tax at the rate τ.

Finally, there is a relationship between a general sales tax and a general income tax. A general sales tax raises the prices of all goods and services by some proportion. A general income tax lowers income by some proportion. From the perspective of any individual, both taxes just shift in her budget line parallel. They do not change **relative** prices of different consumption goods : a general sales tax raises them all by the same proportion, and an income tax does not change them. Algebraically, the budget constraint of the consumer is

$$p_x x + p_y y = M$$

if M is her income, and if there are no taxes. A general sales tax at the rate τ_c changes that budget constraint to

$$(1+\tau_c)p_x x + (1+\tau_c)p_y y = M \tag{GST}$$

Now divide both sides by $1+\tau_c$ to get

$$p_x x + p_y y = \frac{M}{1+\tau_c} \tag{PIT}$$

In other words, the general sales tax just shifts in the budget set ; it is exactly the same as lowering the person's income.

By how much is the person's income effectively lowered? Suppose that the consumption tax rate τ_c is 25 percent. That is the same as dividing income by 1.25. In other words, in this case, the budget line equation becomes

$$p_x x + p_y y = \frac{M}{1.25} = (0.8)M$$

That's exactly what an income tax at the rate of 20 percent does : reduces the person's income to 80 percent of what it was. The general sales tax does the same thing by raising all prices by 25 percent. And both taxes, the general sales tax at a rate of 25 percent, and the 20 percent income tax, raise the same amount of revenue. If a person's income is \$50,000 a year, then a 20 percent income tax collects \$10,000 from her. A general sales tax? Well, suppose this person buys nothing but cases of premium beer, at a (net of tax) price of \$40 a case. The 25 percent tax raises that beer price to \$50. The person buys 1000 cases with her income of \$50,000, and the government is collecting 10 bucks a case : same revenue.

More generally, since

$$\frac{1}{1+\tau_c} = \frac{1+\tau_c}{1+\tau_c} - \frac{\tau_c}{1+\tau_c} = 1 - \frac{\tau_c}{1+\tau_c}$$

then equations "GST" and "PIT" imply a general sales tax at the rate τ_c is exactly equivalent to a general income tax at the rate $\tau_c/(1+\tau_c)$.

5 A general sales tax at the rate τ_c on all consumption is the same as a general income tax at the rate $\tau_c/(1+\tau_c)$ on all sources of income.

Result 5 actually is a pretty important and general one. Raising the prices of everything you spend your income on is exactly the same as lowering your income. The one complication worth noting probably is the fact that the world does go on for more than one period. When people can transfer income between years, earning it in one year and spending it another, then we have to be a little more careful in looking at the consumer's budget problem. But, done right, the equivalence still holds. In particular, if the return to savings is not taxed (and the cost of borrowing not subsidized), then

$5'$ A tax on lifetime expenditure at the rate τ_c is equivalent to a tax on lifetime earnings at the rate $\tau/(1+\tau_c)$.

The basic model used by Harberger needs several assumptions. Many can be altered or relaxed without changing the basic results, but it is best to start with the basic model.

First, it is assumed that there are **constant returns to scale** in technology in each industry. That means that doubling the quantity of all inputs will exactly double the quantity produced of the output of the industry. You may recall that constant returns to scale means that there are **zero economic profits** in perfect competition : if each factor its paid the value of its marginal product, then the cost of labour plus the cost of machinery exactly equals the value of production. Constant returns to scale also mean that the isoquants for an industry all look similar ; in particular, the cost–minimizing capital–labour ratio in an industry depends only on the prices of labour and of capital, not on the level of output of the industry.

Of course, the technology of food production will in general differ from the technology of clothing production. Although production in each industry exhibits constant returns to scale, the isoquants for food production may look very different from those in clothing production. Of particular importance for the incidence of sector–specific taxes is the capital–labour ratio in each industry, and how that ratio differs between the industry. We say that the food industry is more **labour intensive** — which is equivalent to less capital intensive — than the clothing industry if this capital–labour ratio is lower in the food industry. What is this ratio? In each industry, it is simply the ratio of the number of hours of machinery used per unit of output, to the number of person–hours of labour used. That is, if we draw the isoquant for an industry, say the food industry, and then find the cost–minimizing input combination, then the capital–labour ratio is the slope of a line through this cost–minimizing combination, if we graph capital usage on the vertical and labour usage on the horizontal. If you draw a few diagrams, you can see that food production will be more labour intensive (that is, less capital intensive) if the isoquants for food production are more steep than those for clothing production (when capital is graphed on the vertical axis, labour on the horizontal).

So an industry being more labour intensive means that, relatively speaking, labour is more important, and capital less important in the production technology of that industry. An alternative definition is that labour costs account for a higher fraction of the labour intensive industry's overall costs than they do for the less labour intensive industry.

Harberger's second set of assumptions concern the behaviour of suppliers of the inputs to production, labour and capital. There are two assumptions here, really. One is that the total quantity of labour available in the economy is fixed, as is the total quantity of capital. This fixity means that this is a "short run" model, in that people are not accumulating or decumulating capital. It also implies a closed economy, in that the quantities available of labour and capital are not being varied by migration, or trade. The other assumption, which is crucial, is that each input is **perfectly mobile** between industries. In this sense, then, the Harberger model is a long–run model. It is being assumed that workers can move between industries, if the wage they can earn in one industry were to diverge from the other.

In fact, this perfect mobility among industries implies that the net return to capital must be the same in each industry, and the net wage earned by workers must be the same in each industry. If capital earned a higher net return in food production than in clothing production, some owners of capital would move some capital from the clothing industry to the food industry. The assumption that capital is "perfectly" mobile between industries means that capital owners will reallocate their investments until the net return is the same in each sector. Similarly, perfect mobility of labour means that workers will move from one industry to the other, as soon as there is any difference in the net wage they can earn. This mobility implies that the wage, net of all taxes, must be the same in both industries in equilibrium.

Virtually by assumption, then, there is no such thing in the Harberger model as "capital in the food industry", for the purpose of tax incidence analysis. A tax may be born in some part by owners of capital, but it is owners of capital **everywhere**. Since capital earns the same net after–tax return in both sectors, there is no way that owners of capital in one sector may do better than owners of capital in the other sector in the long–run.

So this free mobility among industries is an essential component of the Harberger model, and it means that there really are only two groups to consider : owners of capital and owners of labour.

Harberger assumes that all firms are perfectly competitive, in input and output markets. That means, for example, that food firms hire labour up until the level at which the wage (including any taxes) they must pay per hour equals the value of labour's marginal product : the marginal product of labour times the price (net of an output taxes) of food.

The main goal of Harberger's work was to analyze the incidence of taxes on the **source** side : the relative tax burden on owners of labour and of capital. So he wanted to keep the other side, the "use" side as simple as possible. To do so, he assumed that all owners of labour, and all owners of capital, have the same pattern of consumption. That is, what some individual consumes is assumed to depend only on the prices of commodities. The fraction of their income that different people spend on food, for instance, is assumed to be the same for all people. That fraction will vary with the (tax inclusive) price of food, but will not vary across individuals. So there is really no issue here of a tax being born by "heavy" consumers of food, since everyone is assumed to spend the same fraction of her income on food. The only issue left is how the relative returns to labour and capital are affected by some tax.

Under all these basic assumptions, the incidence of many of the taxes is very straightforward. The only complicated ones are partial factor taxes, which were Harberger's particular concern.

Since factor supplies are fixed, a tax on capital earnings in all sectors will be born entirely by capital owners. Likewise, a general tax on labour income will be born by workers, in proportion to their earnings. A general income tax is a combination of a tax on labour earnings, with a tax on capital earnings at the same rate. So, in the Harberger model, a general income tax will be born by labour and by capital, in proportion to their share of total income : a 15% income tax will reduce the real income of workers and capital owners both by 15%.

A general sales tax is equivalent to a general income tax. So in the Harberger model, a general sales tax too will be born by labour and capital owners, in proportion to their share of national income.

Due to Harberger's assumption about consumption patterns, an excise tax on good X cannot be shifted forward to "consumers of good X". That is because his assumption about consumption patterns implies that each person in the economy spends the same proportion of her income on good X. By assumption there is no particular group who are disproportionately "consumers of good X".

So an excise tax must be shifted backwards on to factors used in the production of good X. The factor which will bear the cost of an excise tax will be the factor which is used most intensively in the production of good X. That is, if (and only if) $L_X/K_X > L_Y/K_Y$, then an excise tax on good X will lower the real return to labour, relative to the real return to capital. More generally, an excise tax on the output of industry X will be born disproportionately by the factor which is relatively most important in the production of good X.

How would an excise tax lower the return of the factor used most intensively in the production of the taxed good? The details are exactly the same as he **output effect** of a specific factor tax, which is described immediately below.

But with a partial factor tax, matters are more complicated than with an excise tax. Consider a tax on the use of labour in the food industry. The impact of this tax can be divided into two effects, an **output effect** and a **substitution effect**. An excise tax on food would have only the output effect, not the substitution effect. But before going through these effects, remember the assumption of perfect intersectoral mobility of factors. That means that we cannot conclude that some portion of this partial factor tax is born by "labour in the food industry". Labour is assumed mobile, so the portion of the tax which is born by labour is born equally by all labour owners, in either sector.

Since this is a partial factor tax, it really has two aspects. On the one hand, it is levied only on labour used in the food industry, not on capital. To that extent it is a labour tax. The fact that, in the food industry, labour use is now taxed, and capital use is not, means that firms will want to substitute capital for labour. This leads to the substitution effect : the substitution of capital for labour in the food industry leads to a reduction in the demand for labour by firms and an increase in the demand for capital. As a result, the net return to labour falls and the net return to capital tends to rise. This is just a tendency, not a sure thing, since there is this other effect, the output effect. The substitution just described was a movement along an isoquant. Moving along an isoquant for food production means output is being held constant, which is why this is just the factor substitution effect.

But this is also a tax which is levied only on the food industry, not on the clothing industry. Taxing factor usage by food producing firms will raise their costs. Perfect competition implies that these higher production costs lead to higher output prices. (Recall that Harberger assumed constant returns to scale in food production, so that firms' supply curves are horizontal.) So because the partial factor tax is directed only at food production, the price of food goes up, because of this added cost. Buyers will react to this price change. They will now substitute in their consumption, buying more clothing, and less food, because of the increase in the relative cost of food. So output of food should shrink, and output of clothing should increase.

How does the economy adjust production of food and clothing due to the increased clothing demand, and the reduced food demand? Inputs are perfectly mobile between industries. So some suppliers of labour and capital will move from the food industry to the clothing industry. This shift in factor allocation lies behind the output effect. Suppose, for example, that the food industry is more labour intensive than the clothing industry. Food production is shrinking : that means a big decrease in food firms' demand for labour, and a not–so–big decrease in food firms' capital demand. Why? Because each unit produced of food uses lots of labour and not so much capital — because the food industry was assumed to be **labour intensive**. What's happening in the clothing industry? The expanded production means lots more demand for capital, and not so much more demand for labour, since clothing production has just been assumed more capital intensive. Overall, then, the demand for labour in the economy seems to have shrunk : a big reduction in demand for labour in the food industry and a not–so–big increase in the clothing industry. Just the reverse happened to the demand for capital : a big increase in demand in the clothing industry and a not–so–big reduction in demand in the food industry. These changes in factor demands are reflected in the prices of the factors : an increase in the return to capital and a fall in the return to labour.

This whole story depended on food being more labour intensive in production. If it had been more capital intensive, then the output effect would have led to a fall in the return to capital and an increase in the return to labour. The output effect of a partial factor tax, then, is a fall in the relative price of the factor used **more intensively** in the taxed sector.

The overall effect of the partial factor tax is the sum of the factor substitution effect and the output effect. The example was a tax on labour in the food industry. If the food industry were more labour intensive, then the two effects reinforce each other. Each effect implied a reduction in labour's relative price. But if food production were more capital intensive than clothing production, then the two effects work in opposite directions : the factor substitution effect works to lower the wage, and the output effect would work to raise it. The overall outcome is ambiguous.

In other words, it might be capital, not labour, which is bearing most of the cost of a tax on labour use only, if the tax were levied only on the output of the most capital intensive industry in the economy.

We can actually say a bit more about the magnitudes of these effects. The more substitutable labour is for capital, the bigger is the factor substitution effect. The following tendencies will make the output effect bigger in absolute value : bigger differences in factor intensities between industries ; more elastic substitution by consumers between food and clothing ; less substitutability between labour and capital in either the food or the clothing industry.

Looking in more detail at these two effects, consider first

The Output Effect of a Partial Factor Tax

Let a_{XL} denote the number of person–hours of labour (input L) which are needed to produce one kilogramme of food (good X). Similarly, let a_{XK} denote the number of machine–hours of K needed to produce one kilogramme of X, a_{YL} denote the quantity of L needed to produce one unit of good Y, and a_{YK} denote the quantity of K needed to produce 1 unit of Y.

Good X is "relatively more labour–intensive" compared to good Y if (and only if)

$$\frac{a_{XL}}{a_{XK}} > \frac{a_{YL}}{a_{YK}}$$

For example, if producing one unit of food needs 10 hours of labour, and 1 hour of machinery, while producing one unit of clothing needs 12 hours of labour and 30 hours of machinery, it seems natural to think of labour as being relatively more important as an input to food than to clothing. In this case

$$\frac{a_{XL}}{a_{XK}} = 10 > 0.4 = \frac{a_{YL}}{a_{YK}}$$

[Good X being relatively more labour intensive than good Y is the same thing as labour accounting for a larger share of the costs of firms in industry X than labour's share in the costs in industry Y.[5]]

What is the cost of producing one kilogramme of food? Let w be the wage per hour of input L, and let r be the cost per hour of using machinery. Since each kilogramme of food requires a_{XL} hours of labour, and a_{XK} hours of machinery services, then the cost of producing the kilogramme of food is

$$wa_{XL} + ra_{XK}$$

In perfect competition, firms make zero economic profits. So — if there are no taxes to complicate matters — this cost of producing a unit of good X must equal the price the firm gets from selling a unit of good X. If p_X is the price per kilogramme of food, then this zero–profit condition implies that

$$wa_{XL} + ra_{XK} = p_x \tag{O1}$$

[5] at least when there are no industry–specific factor taxes

Similarly, the unit cost of good Y must equal its price

$$wa_{YL} + ra_{YK} = p_y \tag{$O2$}$$

Now we can use equation ($O2$) to solve for r in terms of the other variables. Re–arranging equation ($O2$) yields

$$r = \frac{p_y - wa_{YL}}{a_{YK}} \tag{$O2'$}$$

If we now substitute for r in equation (O1), we get

$$wa_{XL} + a_{XK}\left[\frac{p_y - wa_{YL}}{a_{YK}}\right] = p_x \tag{$O3$}$$

Equation (O3) can be re–arranged as

$$w\left[a_{XL} - \frac{a_{XK}a_{YL}}{a_{YK}}\right] = p_x - \frac{a_{XK}}{a_{YK}}p_y \tag{$O3'$}$$

Dividing both sides of equation ($O3'$) by a_{XK} yields

$$w\left[\frac{a_{XL}}{a_{XK}} - \frac{a_{YL}}{a_{YK}}\right] = \frac{1}{a_{XK}}p_x - \frac{1}{a_{YK}}p_y \tag{$O3''$}$$

The coefficient on w is exactly the measure of relative labour intensity ; this coefficient $(a_{XL}/a_{XK}) - (a_{YL}/a_{YK})$ is positive if and only if production in the X industry uses a higher labour–to–machinery ratio than production in the Y industry.

Suppose now that there were **no substitutability** possible in production. That is, suppose that the coefficients a_{XL}, a_{XK}, a_{YL} and a_{YK} were constant.

(In general that's not the case ; as the prices of labour and machinery change, firms will change their techniques of production. That is, if w were to increase, we'd expect a_{XL} and a_{YL} to fall, and a_{XK} and a_{YK} to rise, as firms substituted machines for the now–more–expensive labour. But this variability of these coefficients would complicate my analysis a little.)

Then equation ($O3''$) says that the wage rate will go up whenever p_x goes up or p_y goes down — provided that the X industry is more labour intensive.

Figure 3 shows the effect of an increase in the price of good X on wages, when it is more labour intensive. The figure depicts the the (w, r) pairs which yield zero profits in each industry, when $a_{XL} = 3$, $a_{XK} = 1$, $a_{YL} = 1$ and $a_{YK} = 2$, when p_Y is 1. The zero profit curve for the X industry has slope -3, and the zero profit curve for the Y industry has slope $-1/2$. The intersection of the zero profit curve for the X industry, and the zero profit curve for the Y industry, is the equilibrium (w, r) pair. The figure shows that an increase in p_X from 1 to 2 will increase w, and decrease r, because the X industry is more labour intensive.

If the two industries did not have fixed coefficients in production, then the zero profit curves for each industry would not be straight lines. But it still would be true that the more labour intensive industry would have a steeper zero profit curve, so that an increase in the price of the more labour intensive good must increase w and decrease r.

In other words :

The return to labour tends to move in the same direction as the relative price of the commodity which uses labour most intensively in production.

This statement in italics is really the basis of Harberger's "output effect". For example, suppose that there is a tax on the use of some factor in the X industry, but not in the Y industry. (The tax could be on the use of labour in the X industry, or on the use of machinery in the X industry, or both. What matters for the output effect is that it is only levied in one industry.) From the basic partial equilibrium analysis, a tax which drives up costs in one industry will result in a decline in demand for the output of that industry. This decline in demand will also drive down the (net of tax) price of the output in that industry. In other words, any tax which is directed only at the X industry will result in a lower relative price for X. Whether this change in output prices will decrease or increase the return to labour will depend — entirely — on which industry is the more labour intensive in production.

Formally, suppose that the coefficients a_{LX}, a_{KX}, a_{LY} and a_{KY} were fixed, that is that production in each industry took place using **fixed proportions**. That would mean that there is no substitution effect, since fixed proportions means that substitution between labour and capital would be impossible. So all we would have, if a specific factor tax were levied, would be the **output effect**. Now there are fixed total supplies L and K of labour and capital. How much labour will be used by the X industry? It's $a_{LX}X$, since each unit of output in the X industry uses a_{LX} hours of labour, and the total output produced is X. That means that the total demand for labour, by both industries together, is $a_{LX}X + a_{LY}Y$. This demand for labour must equal the total quantity of labour which is available, L, which is fixed. A similar condition must hold in the capital market. Since the amount used of any factor must equal the amount supplied, then the following two equations must hold :

$$a_{LX}X + a_{LY}Y = L \qquad (eqL)$$

$$a_{KX}X + a_{KY}Y = K \qquad (eqK)$$

When these coefficients a_{LX}, a_{KX}, a_{LY} and a_{KY} are fixed, then equations (eqL) and (eqK) can be solved to determine X and Y as functions of K and L. In other words, with constant a_{LX}, a_{KX}, a_{LY} and a_{KY} , equations (eqL) and (eqK) are two equations in 2 unknowns X and Y, and can be solved as

$$X = \frac{a_{KY}L - a_{LY}K}{A} \qquad (X1)$$

$$Y = \frac{a_{LX}L - a_{KX}K}{A} \qquad (Y1)$$

where

$$A = a_{LX}a_{KY} - a_{LY}a_{KX}$$

Note that the determinant A is positive if and only if $a_{LX}/a_{KX} > a_{LY}/a_{KY}$, that is if and only if the X industry is more labour intensive. [6]

Next, note that only relative prices matter. And quantities demanded by consumers depend on relative prices of the commodities. So

$$\frac{X}{Y} = \phi(\frac{P_Y}{P_X}) \qquad (demand))$$

describes completely consumer's demands for final goods, where P_X and P_Y are the prices actually paid by consumers (including any taxes), and $\phi(P_Y/P_X)$ is some increasing function : the higher the relative price of good Y, the more that consumers will shift purchases to good X.

Given that X and Y are determined by equations $(X1)$ and $(Y1)$, then the relative price ratio P_Y/P_X is determined completely from equation (*demand*).

So now suppose that there is a specific factor tax on the use of labour in the X industry. It is more convenient to represent the tax here as a **unit tax** at the rate t. So the cost of labour, per hour, by firms in the X industry, is $w + t$, while the cost of labour to firms in the Y industry (which do not have to pay this tax) is w. If this unit tax on labour in the X industry is the only tax in the economy, then the following two equations must hold.

$$a_{LX}(w + t) + a_{KX}r = P_X \qquad (X2)$$

$$a_{LY}w + a_{KY}r = P_Y \qquad (Y2)$$

Why? Perfect competition and constant returns to scale imply that each firm makes zero profits. Equations $(X2)$ and $(Y2)$ say that the cost per unit produced must equal the price received per unit. Now equations $(X2)$ and $(Y2)$ can be written in matrix form as

$$\begin{pmatrix} a_{LX} & a_{KX} \\ a_{LY} & a_{KY} \end{pmatrix} \begin{pmatrix} w \\ r \end{pmatrix} = \begin{pmatrix} P_X \\ P_Y \end{pmatrix} - \begin{pmatrix} a_{LX} \\ 0 \end{pmatrix} t \qquad (matrix)$$

The determinant of the matrix in the equation above is just A, defined earlier

$$A = a_{LX}a_{KY} - a_{LY}a_{KX}$$

The output prices P_X and P_Y are determined completely by the equilibrium conditions $(X1)$, $(Y1)$ and (*demand*). [Actually, since only relative prices matter, we can **fix** one price, say the price of good Y, and the **numéraire**, and then P_X will be determined from equation (*demand*).]

[6] This won't work if both industries have the same labour intensity ; then $A = 0$ and equations $(X1)$ and $(Y1)$ don't make sense.

So — if the input demand coefficients a_{LX}, a_{KX}, a_{LY} and a_{KY} are constant — equation ($matrix$) says that we can write the prices of the inputs as

$$\begin{pmatrix} w \\ r \end{pmatrix} = \begin{pmatrix} a_{LX} & a_{KX} \\ a_{LY} & a_{KY} \end{pmatrix}^{-1} [\begin{pmatrix} P_X \\ P_Y \end{pmatrix} - \begin{pmatrix} a_{LX}t \\ 0 \end{pmatrix}] \qquad (matrix2)$$

which says, when the above equation is solved, that

$$w = \frac{a_{KY}P_X - a_{KX}P_Y}{A} - \frac{a_{LX}a_{KY}}{A}t \qquad (w)$$

The first term on the right side of equation (w) does not change with taxes (if a_{LX}, a_{KX}, a_{LY} and a_{KY} are constant). So w will **decrease** with the tax exactly when $A > 0$. Similarly, r will **increase** with the tax precisely when $A > 0$. Recall, however, that $A > 0$ exactly when the X industry is more labour intensive. Thus

If no input substitution were possible in either industry, then any specific factor tax will lower the relative return on the factor used most intensively in the taxed industry and raise the relative return on the factor used less intensively in the taxed industry.

So, the output effect of a tax levied in one industry depends entirely on the relative factor intensities of production in the two industries — not on which factor it is levied on.

In other words, if $(a_{XL}/a_{XK}) > (a_{YL}/a_{YK})$, then the output effect of a tax on capital use in the X industry will be bad for labour, and good for capital, even though it is levied on capital use. What matters for the output effect is in which industry the tax is levied, and which industry is the more labour intensive. [7]

Output Effect : *Any tax directed at a particular industry tends to drive down the net return to the factor used most intensively in that industry.*

The Substitution Effect of a Partial Factor Tax

In general, the coefficients a_{XL} and a_{XK} are not constant. The firm chooses the quantities of labour and machinery it hires so as to minimize the cost of production. Minimizing the cost of a producing a given quantity of food involves moving along an *isoquant* in L–K space. If the costs of labour and machinery are w and r per hour respectively, then the firm will want to choose an input combination (L, K) where the slope of its isoquant equals w/r (if we graph L on the horizontal axis, and K on the vertical).

So what would happen if, for example, a tax were levied on the use of labour in the food industry (which did not apply to the use of capital)? If the returns to labour and capital

[7] The statement above in italics will actually be true even when the coefficients a_{XL}, a_{XK}, a_{YL} and a_{YK} are not constant ; the mathematics just gets more complicated.

remained constant, the cost of labour to the firm would increase from w to $w(1 + \tau_{XL})$, if τ_{XL} were the rate of the *ad valorem* tax on the use of labour in the food industry. This would raise the ratio of the input prices, from w/r to $(w/r)(1 + \tau_{XL})$. The firm would move along the isoquant (as in figure 4), choosing a production plan which used relatively more machinery and relatively less labour.

This move along an isoquant is the Harberger's substitution effect. Remember that the isoquant depicts all the (L, K) combinations which can produce a given level of output. So here we are holding constant the output of food, and looking at the substitution of one input for another. (In doing the output effect, we ignored substitution of one input for another, and looked at the effects of the change in the demand for food.)

What we get, in the move up and to the left along the isoquant, was a decline in the demand for labour by firms in the industry,and an increase in the demand for machinery. This decline in labour demand in the X industry must drive down the net wage w — as a partial equilibrium diagram of the demand and supply of labour will show. Therefore

Substitution Effect : *A tax on the use of some input in an industry tends to drive down the wage of that input.*

The Harberger Model and the Corporate Income Tax

Now Harberger's particular interest was the incidence of the corporate income tax. Why did he think this model was appropriate to the analysis of this tax? Well, the corporate income tax is a tax on earnings of corporations, which might be thought of as the return to capital. So, it's somewhat plausible that the corporate income tax is a tax on the return to capital. But why a *partial* factor tax on capital, rather than a general factor tax? The corporate income tax is levied only on incorporated businesses, and many businesses are not incorporated. So Harberger's X sector is those industries in which most of the businesses are incorporated : industries such as manufacturing and petroleum refining. His Y sector is those industries in which few of the firms are incorporated : industries such as agriculture and fishing. Following his logic, then, the corporate income tax is a partial factor tax on the use of capital in the X sector. Theoretically, the incidence of such a tax might fall largely on labour, if the X sector happened to be very labour intensive relative to the predominantly unincorporated sectors. This seems not to be the case for the United States when Harberger did his analysis : his conclusion was that the corporate income tax is born largely by owners of capital. Of course, his assumption of perfect factor mobility means that the tax is born by owners of of capital in general, not just owners of incorporated firms.

The practical applicability of Harberger's model to the corporate income tax in Canada is probably quite limited. For one thing, it is not that clear that it is a tax on the return to capital, either in Canada or anywhere else. Much later in the term we will look briefly at some of the particular features of the corporation tax. But secondly, the assumption that the overall supply of capital is fixed seems quite unrealistic to a small open economy such as Canada, in which investors invest abroad, foreigners invest here, and both groups are likely to change their investment decisions in response to tax changes.